14.581 International Trade
— Lecture 9: Factor Proportion Theory (II) —

Today's Plan

1. Two-by-two-by-two Heckscher-Ohlin model
    1. Integrated equilibrium
    2. Heckscher-Ohlin Theorem
2. High-dimensional issues
    1. Classical theorems revisited
    2. Heckscher-Ohlin-Vanek Theorem
3. Quantitative Issues

Two-by-two-by-two Heckscher-Ohlin model

Basic environment

- Results derived in previous lecture hold for small open economies
  - relative good prices were taken as exogenously given
- We now turn world economy with two countries, North and South
- We maintain the two-by-two HO assumptions:
  - there are two goods, $g = 1{,}2$, and two factors, k and l
  - identical technology around the world, $y_g = f_g(k_g, l_g)$
  - identical homothetic preferences around the world, $d_g^c = \alpha_g(p) I^c$
- **Question**
  What is the pattern of trade in this environment?

Two-by-two-by-two Heckscher-Ohlin model

Strategy

- Start from **Integrated Equilibrium** $\equiv$ competitive equilibrium that would prevail if *both* goods and factors were freely traded
- Consider **Free Trade Equilibrium** $\equiv$ competitive equilibrium that prevails if goods are freely traded, but factors are not
- Ask: Can free trade equilibrium reproduce integrated equilibrium?
- *If factor prices are equalized through trade*, the answer is yes
- In this situation, one can then use homotheticity to go from differences in factor endowments to pattern of trade

Two-by-two-by-two Heckscher-Ohlin model

Integrated equilibrium

- **Integrated equilibrium** corresponds to (p, ω, y) such that:

$$
\begin{aligned}
(ZP) &\quad : \quad p = A'(\omega)\,\omega && (1)\\
(GM) &\quad : \quad y = \alpha(p)\left(\omega' v\right) && (2)\\
(FM) &\quad : \quad v = A(\omega)\,y && (3)
\end{aligned}
$$

  where:

  - $p \equiv (p_1, p_2)$, $\omega \equiv (w, r)$, $A(\omega) \equiv \left[a_{fg}(\omega)\right]$, $y \equiv (y_1, y_2)$, $v \equiv (l, k)$, $\alpha(p) \equiv [\alpha_1(p), \alpha_2(p)]$
  - $A(\omega)$ derives from cost-minimization
  - $\alpha(p)$ derives from utility-maximization

- **Free trade equilibrium** corresponds to $\left(p^t, \omega^n, \omega^s, y^n, y^s\right)$ such that:

$$\begin{aligned}
(ZP) &: \quad p^t \leq A'\left(\omega^c\right)\omega^c \text{ for } c = n, s && (4) \\
(GM) &: \quad y^n + y^s = \alpha\left(p^t\right)\left(\omega^{n\prime} v^n + \omega^{s\prime} v^s\right) && (5) \\
(FM) &: \quad v^c = A\left(\omega^c\right) y^c \text{ for } c = n, s && (6)
\end{aligned}$$

  where (4) holds with equality if good is produced in country c

- **Definition** *Free trade equilibrium replicates integrated equilibrium if* $\exists\ \left(y^n, y^s\right) \geq 0$ *such that* $\left(p, \omega, \omega, y^n, y^s\right)$ *satisfy conditions* (4)-(6)

- **Definition** (v^n, v^s) *are in the FPE set if* $\exists$ $(y^n, y^s) \geq 0$ *such that condition* (6) *holds for* $\omega^n = \omega^s = \omega$.
- **Lemma** *If* (v^n, v^s) *is in the FPE set, then free trade equilibrium replicates integrated equilibrium*
- **Proof:** By definition of the FPE set, $\exists$ $(y^n, y^s) \geq 0$ such that

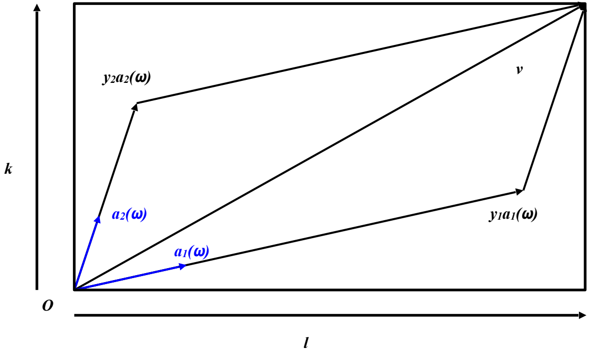
$$v^c = A(\omega) y^c$$

So Condition (6) holds. Since $v = v^n + v^s$, this implies

$$v = A(\omega)(y^n + y^s)$$

Combining this expression with condition (3), we obtain $y^n + y^s = y$. Since $\omega^{n\prime} v^n + \omega^{s\prime} v^s = \omega' v$, Condition (5) holds as well. Finally, Condition (1) directly implies (4) holds.

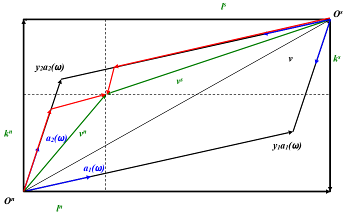
- Factor market clearing in the integrated equilibrium:



Two-by-two-by-two Heckscher-Ohlin model

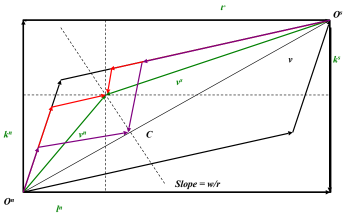
The "Parallelogram"

- **FPE set** $\equiv (v^n, v^s)$ inside the parallelogram



- When v^n and v^s are inside the parallelogram, we say that they belong to the same **diversification cone**
- This is a very different way of approaching FPE than FPE Theorem
  - Here, we have shown that there can be FPE iff factor endowments are not too dissimilar, whether or not there are no FIR
  - Instead of taking prices as given—whether or not they are consistent with integrated equilibrium—we take factor endowments as primitives

- Suppose that (v^n, v^s) is in the FPE set
- **HO Theorem** *In the free trade equilibrium, each country will export the good that uses its abundant factor intensively*



- Outside the FPE set, additional technological and demand considerations matter (e.g. FIR or no FIR)

- HO Theorem can also be derived using Rybczynski effect:
  1. Rybczynski theorem $\Rightarrow y_2^n / y_1^n > y_2^s / y_1^s$ for any p
  2. Homotheticity $\Rightarrow c_2^n / c_1^n = c_2^s / c_1^s$ for any p
  3. This implies $p_2^n / p_1^n < p_2^s / p_1^s$ under autarky
  4. Law of comparative advantage $\Rightarrow$ HO Theorem

Two-by-two-by-two Heckscher-Ohlin model

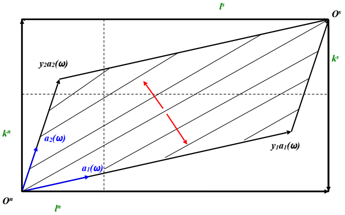
Trade and inequality

- Predictions of HO and SS Theorems are often combined:
  - HO Theorem $\Rightarrow p_2^n / p_1^n < p_2 / p_1 < p_2^s / p_1^s$
  - SS Theorem $\Rightarrow$ *Moving from autarky to free trade, real return of abundant factor increases, whereas real return of scarce factor decreases*
  - If North is skill-abundant relative to South, inequality increases in the North and decreases in the South
- So why may we observe a rise in inequality in the South in practice?
  - Southern countries are not moving from autarky to free trade
  - Technology is not identical around the world
  - Preferences are not homothetic and identical around the world
  - There are more than two goods and two countries in the world

Two-by-two-by-two Heckscher-Ohlin model

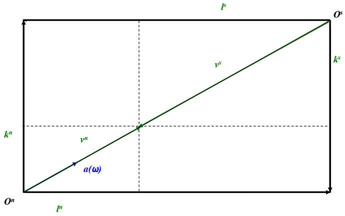
Trade volumes

- Let us define trade volumes as the sum of exports plus imports
- Inside FPE set, iso-volume lines are parallel to diagonal (HKa p.23)
  - the further away from the diagonal, the larger the trade volumes
  - factor abundance rather than country size determines trade volume



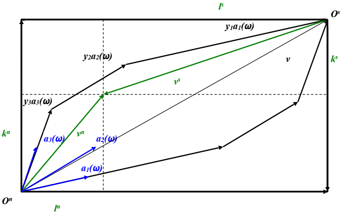
- If country size affects trade volumes in practice, what should we infer?

- Suppose now that there are F factors and G goods
- By definition, (v^n, v^s) is in the FPE set if $\exists\,(y^n, y^s) \geq 0$ s.t. $v^c = A(\omega)\, y^c$ for $c = n, s$
- If $F = G$ ("even case"), the situation is qualitatively similar
- If $F > G$, the FPE set will be "measure zero": $\{v | v = A(\omega)\, y^c \text{ for } y^c \geq 0\}$ is a G-dimensional cone in F-dimensional space
- **Example:** "Macro" model with 1 good and 2 factors



- So why does FPE always hold in a R-R model, even if $F > G$?

- If $F < G$, there will be indeterminacies in production, (y^n, y^s), and so, trade patterns, but FPE set will still have positive measure
- **Example:** 3 goods and 2 factors



- By the way, are there more goods than factors in the world?

- SS Theorem was derived by differentiating zero-profit condition
- With an arbitrary number of goods and factors, we still have

$$\widehat{p}_g = \sum_f \theta_{fg} \widehat{w}_f \tag{7}$$

where w_f is the price of factor f and $\theta_{fg} \equiv w_f a_{fg}(\omega) / c_g(\omega)$

- Now suppose that $\widehat{p}_{g_0} > 0$, whereas $\widehat{p}_g = 0$ for all $g \neq g_0$
- Equation (7) immediately implies the existence of f_1 and f_2 s.t.

$$\begin{aligned} \widehat{w}_{f_1} &\geq \widehat{p}_{g_0} > \widehat{p}_g = 0 \text{ for all } g \neq g_0, \\ \widehat{w}_{f_2} &< \widehat{p}_g = 0 < \widehat{p}_{g_0} \text{ for all } g \neq g_0. \end{aligned}$$

- So every good is "friend" to some factor and "enemy" to some other (Jones and Scheinkman 1977)

- Ethier (1984) also provides the following variation of SS Theorem
- If good prices change from p^1 to p^2, then the associated change in factor prices, $\omega^2 - \omega^1$, must satisfy

$$\left(p^2 - p^1\right)' A'\left(\omega^0\right)\left(\omega^2 - \omega^1\right) > 0, \text{ for some } \omega^0 \text{ between } \omega^1 \text{ and } \omega^2$$

- **Proof:**
- Define $f(\omega) = \left(p^2 - p^1\right)' A'(\omega)\,\omega$. Mean value theorem implies

$$f\left(\omega^2\right) = f\left(\omega^1\right) + \left(p^2 - p^1\right)'\left[A'\left(\omega^0\right) + dA'\left(\omega^0\right)\omega^0\right]\left(\omega^2 - \omega^1\right)$$

for some ω^0 between ω^1 and ω^2. Cost-minimization at ω^0 requires

$$dA'\left(\omega^0\right)\omega^0 = 0$$

- **Proof (Cont.):**
  Combining the two previous expressions, we obtain

$$f\left(\omega^2\right) - f\left(\omega^1\right) = \left(p^2 - p^1\right)' A'\left(\omega^0\right)\left(\omega^2 - \omega^1\right)$$

  From zero profit condition, we know that $p^1 = A'\left(\omega^1\right)\omega^1$ and $p^2 = A'\left(\omega^2\right)\omega^2$. Thus

$$f\left(\omega^2\right) - f\left(\omega^1\right) = \left(p^2 - p^1\right)'\left(p^2 - p^1\right) > 0$$

  The last two expressions imply

$$\left(p^2 - p^1\right)' A'\left(\omega^0\right)\left(\omega^2 - \omega^1\right) > 0$$

- **Interpretation:**
  Tendency for changes in good prices to be accompanied by raises in prices of factors used intensively in goods whose prices have gone up
- What is ω^0?

High-Dimensional Predictions

Rybczynski-type results

- Rybczynski Theorem was derived by differentiating the factor market clearing condition
- If $G = F > 2$, same logic implies that increase in endowment of one factor decreases output of one good and increases output of another (Jones and Scheinkman 1977)
- If $G < F$, increase in endowment of one factor may increase output of all goods (Ricardo-Viner)
- In this case, we still have the following correlation (Ethier 1984)

$$\left(v^2 - v^1\right)' A\left(\omega^0\right)\left(y^2 - y^1\right) = \left(v^2 - v^1\right)'\left(v^2 - v^1\right) > 0$$

- If $G > F$, indeterminacies in production imply that we cannot predict changes in output vectors

- Since HO Theorem derives from Rybczynski effect + homotheticity, problems of generalization in the case $G < F$ and $F > G$ carry over to the Heckscher-Ohlin Theorem
- If $G = F > 2$, we can invert the factor market clearing condition

$$y^c = A^{-1}(\omega) v^c$$

- By homotheticity, the vector of consumption in country c satisfies

$$d^c = s^c d$$

where $s^c \equiv c$'s share of world income, and $d \equiv$ world consumption
- Good and factor market clearing requires

$$d = y = A^{-1}(\omega) v$$

- Combining the previous expressions, we get net exports

$$t^c \equiv y^c - d^c = A^{-1}(\omega)(v^c - s^c v)$$

- Without assuming that $G = F$, we can still derive sharp predictions if we focus on the *factor content of trade* rather than *commodity trade*
- We define the *net exports of factor* f by country c as

$$\tau_f^c = \sum_g a_{fg}(\omega)\, t_g^c$$

- In matrix terms, this can be rearranged as

$$\tau^c = A(\omega)\, t^c$$

- **HOV Theorem** *In any country* c*, net exports of factors satisfy*

$$\tau^c = v^c - s^c v$$

- So countries should export the factors in which they are abundant compared to the world: $v_f^c > s^c v_f$
- Assumptions of HOV Theorem are extremely strong: identical technology, FPE, homotheticity
  - One shouldn't be too surprised if it performs miserably in practice...

Quantitative Issues

Basic Idea

- Stolper-Samuelson offers sharp insights about distributional consequences of international trade, but...
  - Theoretical insights are only *qualitative*
  - Theoretical insights crucially rely on 2×2 assumptions
- Alternatively one may want to know the *quantitative* importance of international trade:
  - Given the amount of trade that we actually observe in the data, how large are the effects of international trade on the skill premium?
  - In a country like the United States, how much higher or smaller would the skill premium be in the absence of trade?

- Eaton and Kortum (2002)—as well as other gravity models—offer a simple starting point to think about these issues
- Consider multi-sector-multi-factor EK (e.g. Chor JIE 2010)
  - many varieties with different productivity levels $z(\omega)$ in each sector s
  - same factor intensity across varieties within sectors
  - different factor intensities across sectors
- Unit costs of production in country i and sector s are proportional to:

$$c_{i,s} = \left[\left(\mu_s^H \right)^{\rho} \left(w_i^H \right)^{1-\rho} + \left(\mu_s^L \right)^{\rho} \left(w_i^L \right)^{1-\rho} \right]^{1/(1-\rho)} \quad (8)$$

where:

- w_i^H, $w_i^L \equiv$ wages of skilled and unskilled workers.
- $\rho \equiv$ elasticity of substitution between skilled and unskilled

- Suppose, like in EK, that productivity draws across varieties within sectors are independently drawn from a Fréchet
- Then one can show that the following gravity equation holds:

$$X_{ij,s} = \frac{T_i \left(\tau_{ij,s} c_{i,s}\right)^{-\theta_s}}{\sum_{l=1}^{n} T_l \left(\tau_{lj,s} c_{l,s}\right)^{-\theta_s}} E_{j,s}, \tag{9}$$

  where $E_{j,s} \equiv$ total expenditure on goods from sector s in country j
- Two key equations, (8) and (9), are CES:
  - One can use DEK's strategy to do welfare and counterfactual analysis
  - But one can also discuss the consequences of changes in variable trade costs, $\tau_{lj,s}$, or technology, T_i, on skill premium
  - How large are GT compared to distributional consequences?
  - Some preliminary answers in Costinot and Rodriguez Clare (2013)
  - Much more in Burstein and Vogel (JPE, 2016)