14.581 International Trade
— Lecture 8: Factor Proportion Theory (I) —

Today's Plan

1. Factor Proportion Theory
2. Ricardo-Viner model
    1. Basic environment
    2. Comparative statics
3. Two-by-Two Heckscher-Ohlin model
    1. Basic environment
    2. Classical results:
        1. Factor Price Equalization Theorem
        2. Stolper-Samuelson (1941) Theorem
        3. Rybczynski (1965) Theorem

Factor Proportion Theory

- The law of comparative advantage establishes the relationship between relative autarky prices and trade flows
  - But where do relative autarky prices come from?
- Factor proportion theory emphasizes **factor endowment differences**
- **Key elements:**
  1. Countries differ in terms of factor abundance [i.e *relative* factor supply]
  2. Goods differ in terms of factor intensity [i.e *relative* factor demand]
- Interaction between 1 and 2 will determine differences in relative autarky prices, and in turn, the pattern of trade

Factor Proportion Theory

- In order to shed light on factor endowments as a source of CA, we will assume that:
  1. Production functions are identical around the world
  2. Households have identical homothetic preferences around the world
- We will first focus on two special models:
  - **Ricardo-Viner** with 2 goods, 1 "mobile" factor (labor) and 2 "immobile" factors (sector-specific capital)
  - **Heckscher-Ohlin** with 2 goods and 2 "mobile" factors (labor and capital)
- The second model is often thought of as a long-run version of the first (Neary 1978)
  - In the case of Heckscher-Ohlin, what it is the time horizon such that one can think of total capital as fixed in each country, though freely mobile across sectors?

Ricardo-Viner Model

Basic environment

- Consider an economy with:
  - Two goods, $g = 1, 2$
  - Three factors with endowments l, k_1, and k_2
- Output of good g is given by

$$y_g = f^g(l_g, k_g),$$

where:

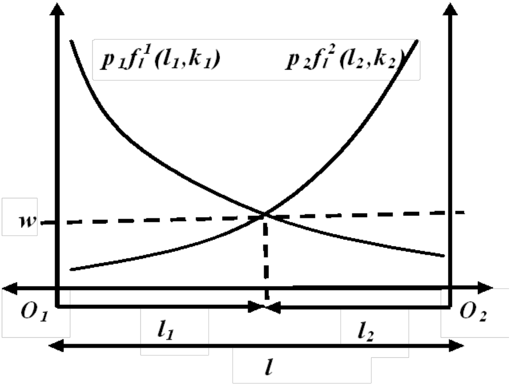
  - l_g is the (endogenous) amount of labor in sector g
  - f^g is homogeneous of degree 1 in (l_g, k_g)
- **Comments:**
  - l is a "mobile" factor in the sense that it can be employed in all sectors
  - k_1 and k_2 are "immobile" factors in the sense that they can only be employed in one of them
  - Model is isomorphic to DRS model: $y_g = f^g(l_g)$ with $f^g_{ll} < 0$
  - Payments to specific factors under CRS $\equiv$ profits under DRS

- We denote by:
  - p_1 and p_2 the prices of goods 1 and 2
  - w, r_1, and r_2 the prices of l, k_1, and k_2
- For now, (p_1, p_2) is exogenously given: **"small open economy"**
  - So no need to look at good market clearing
- **Profit maximization:**

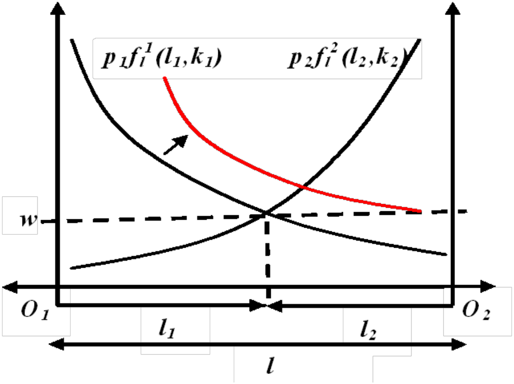
$$p_g f_l^g (l_g, k_g) = w \quad (1)$$
$$p_g f_k^g (l_g, k_g) = r_g \quad (2)$$

- **Labor market clearing:**

$$l = l_1 + l_2 \quad (3)$$



- Equations (1) and (3) jointly determine labor allocation and wage
- How do we recover payments to the specific factor from this graph?



- Consider a TOT shock such that p_1 increases:
  - $w \nearrow$, $l_1 \nearrow$, and $l_2 \searrow$
  - Condition (2) $\Rightarrow r_1 / p_1 \nearrow$ whereas r_2 (and a fortiori r_2 / p_1) $\searrow$

Ricardo-Viner Model

Comparative statics

- One can use the same type of arguments to analyze consequences of:
  - Productivity shocks
  - Changes in factor endowments
- In all cases, results are intuitive:
  - "Dutch disease" (Boom in export sectors, Bids up wages, which leads to a contraction in the other sectors)
  - Useful political-economy applications (Grossman and Helpman 1994)
- Easy to extend the analysis to more than 2 sectors:
  - Plot labor demand in one sector vs. rest of the economy

Ricardo-Viner Model

Equilibrium (II): two-country world

- Predictions on the pattern of trade in a two-country world depend on whether differences in factor endowments come from:
  - Differences in the relative supply of specific factors
  - Differences in the relative supply of mobile factors
- Accordingly, any change in factor prices is possible as we move from autarky to free trade (see Feenstra Problem 3.1 p. 98)

- Consider an economy with:
  - Two goods, $g = 1, 2$,
  - Two factors with endowments l and k
- Output of good g is given by

$$y_g = f^g\left(l_g, k_g\right),$$

  where:
  - l_g, k_g are the (endogenous) amounts of labor and capital in sector g
  - f^g is homogeneous of degree 1 in $\left(l_g, k_g\right)$

Two-by-Two Heckscher-Ohlin Model

Back to the dual approach

- $c_g(w,r) \equiv$ unit cost function in sector g

$$c_g(w,r) = \min_{l,k} \left\{ wl + rk \middle| f^g(l,k) \geq 1 \right\},$$

  where w and r the price of labor and capital

- $a_{fg}(w,r) \equiv$ unit demand for factor f in the production of good g
- Using the Envelope Theorem, it is easy to check that:

$$a_{lg}(w,r) = \frac{dc_g(w,r)}{dw} \text{ and } a_{kg}(w,r) = \frac{dc_g(w,r)}{dr}$$

- $A(w,r) \equiv [a_{fg}(w,r)]$ denotes the matrix of total factor requirements

- Like in RV model, we first look at the case of a **"small open economy"**
  - So no need to look at good market clearing
- **Profit-maximization:**

$$\begin{aligned} p_g &\leq wa_{lg}(w,r) + ra_{kg}(w,r) \text{ for all } g = 1,2 && (4) \\ p_g &= wa_{lg}(w,r) + ra_{kg}(w,r) \text{ if } g \text{ is produced in equilibrium} && (5) \end{aligned}$$

- **Factor market-clearing**:

$$\begin{aligned} l &= y_1 a_{l1}(w,r) + y_2 a_{l2}(w,r) && (6) \\ k &= y_1 a_{k1}(w,r) + y_2 a_{k2}(w,r) && (7) \end{aligned}$$

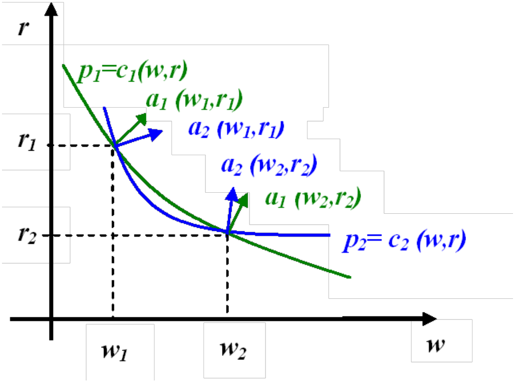
- **Question:**
  Can trade in goods be a (perfect) substitute for trade in factors?
- First classical result from the HO literature answers by the affirmative
- To establish this result formally, we'll need the following definition:
- **Definition**. *Factor Intensity Reversal (FIR) does not occur if:* (i) $a_{l1}(w,r)/a_{k1}(w,r) > a_{l2}(w,r)/a_{k2}(w,r)$ *for all* (w,r)*; or* (ii) $a_{l1}(w,r)/a_{k1}(w,r) < a_{l2}(w,r)/a_{k2}(w,r)$ *for all* (w,r).

Two-by-Two Heckscher-Ohlin Model

Factor Price Insensitivity (FPI)

- **Lemma** *If both goods are produced in equilibrium and FIR does not occur, then factor prices $\omega \equiv (w, r)$ are uniquely determined by good prices $p \equiv (p_1, p_2)$*
- **Proof:** If both goods are produced in equilibrium, then $p = A'(\omega)\omega$. By Gale and Nikaido (1965), this equation admits a unique solution if $a_{fg}(\omega) > 0$ for all f, g and $\det[A(\omega)] \neq 0$ for all ω, which is guaranteed by no FIR.
- **Comments:**
  - Good prices rather than factor endowments determine factor prices
  - In a closed economy, good prices and factor endowments are, of course, related, but not for a small open economy
  - All economic intuition can be gained by simply looking at Leontieff case
  - Proof already suggests that "dimensionality" will be an issue for FIR

- Link between no FIR and FPI can be seen graphically:



- If iso-cost curves cross more than once, then FIR must occur

- The previous lemma directly implies (Samuelson 1949) that:
- **FPE Theorem** *If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices*
- **Comments:**
  - Trade in goods can be a "perfect substitute" for trade in factors
  - Countries with different factor endowments can sustain same factor prices through different allocation of factors across sectors
  - Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries...
  - For next results, we'll maintain assumption that both goods are produced in equilibrium, but won't need free trade and same technology

- **Stolper-Samuelson Theorem** *An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduced the real return to the other factor*
- **Proof:** W.l.o.g. suppose that (i) $a_{l1}(\omega)/\,a_{k1}(\omega) > a_{l2}(\omega)/\,a_{k2}(\omega)$ and (ii) $\widehat{p}_2 > \widehat{p}_1$. Differentiating the zero-profit condition (5), we get

$$\widehat{p}_g = \theta_{lg}\widehat{w} + (1 - \theta_{lg})\,\widehat{r}, \tag{8}$$

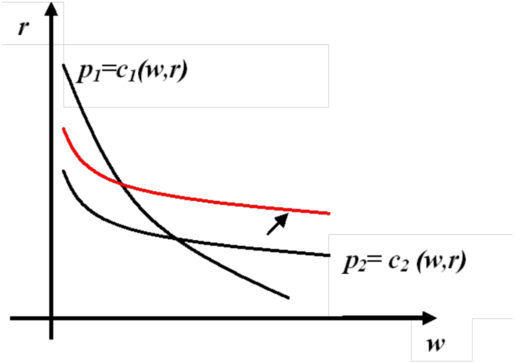
where $\widehat{x} = d\ln x$ and $\theta_{lg} \equiv wa_{lg}(\omega)\,/\,c_g(\omega)$. Equation $(8) + (ii)$ imply

$$\widehat{w} > \widehat{p}_2 > \widehat{p}_1 > \widehat{r} \text{ or } \widehat{r} > \widehat{p}_2 > \widehat{p}_1 > \widehat{w}$$

By (i), $\theta_{l2} < \theta_{l1}$. So (ii) further requires $\widehat{r} > \widehat{w}$. Combining the previous inequalities, we get

$$\widehat{r} > \widehat{p}_2 > \widehat{p}_1 > \widehat{w}$$

- **Comments:**
  - Previous "hat" algebra is often referred to "Jones' (1965) algebra"
  - The chain of inequalities $\widehat{r} > \widehat{p}_2 > \widehat{p}_1 > \widehat{w}$ is referred as a "magnification effect"
  - SS predict both winners and losers from change in relative prices
  - Like FPI and FPE, SS entirely comes from zero-profit condition (+ no joint production)
  - Like FPI and FPE, sharpness of the result hinges on "dimensionality"
  - In the empirical literature, people often talk about "Stolper-Samuelson effects" whenever looking at changes in relative factor prices (though changes in relative good prices are rarely observed)



- Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontieff case:
    - In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies

- Previous results have focused on the implication of *zero profit condition*, Equation (5), for *factor prices*
- We now turn our attention to the implication of *factor market clearing*, Equations (6) and (7), for *factor allocation*
- **Rybczynski Theorem** *An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry*

- **Proof:** W.l.o.g. suppose that (i) $a_{l1}(\omega)/a_{k1}(\omega) > a_{l2}(\omega)/a_{k2}(\omega)$ and (ii) $\widehat{k} > \widehat{l}$. Differentiating factor market clearing conditions (6) and (7), we get

$$\widehat{l} = \lambda_{l1}\widehat{y}_1 + (1-\lambda_{l1})\,\widehat{y}_2 \tag{9}$$

$$\widehat{k} = \lambda_{k1}\widehat{y}_1 + (1-\lambda_{k1})\,\widehat{y}_2 \tag{10}$$

where $\lambda_{l1} \equiv a_{l1}(\omega)\,y_1/l$ and $\lambda_{k1} \equiv a_{k1}(\omega)\,y_1/k$. Equation (8) + (ii) imply

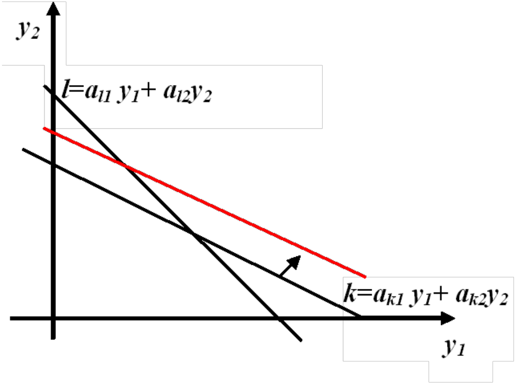
$$\widehat{y}_1 > \widehat{k} > \widehat{l} > \widehat{y}_2 \text{ or } \widehat{y}_2 > \widehat{k} > \widehat{l} > \widehat{y}_1$$

By (i), $\lambda_{k1} < \lambda_{l1}$. So (ii) further requires $\widehat{y}_2 > \widehat{y}_1$. Combining the previous inequalities, we get

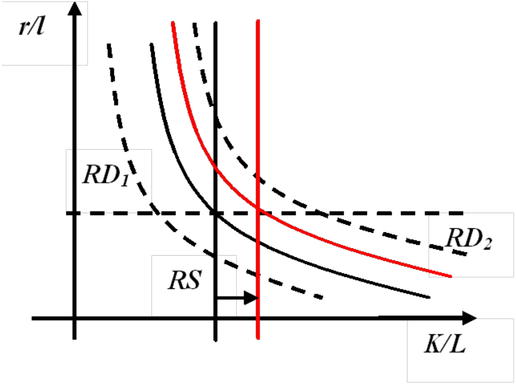
$$\widehat{y}_2 > \widehat{k} > \widehat{l} > \widehat{y}_1$$

- Like for FPI and FPE Theorems:
    - (p_1, p_2) is exogenously given $\Rightarrow$ factor prices and factor requirements are not affected by changes factor endowments
    - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy
- Like for SS Theorem, we have a "magnification effect"
- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on "dimensionality"

- Since good prices are fixed, it is as if we were in Leontieff case



- Rybczynski effect can also be illustrated using relative factor supply and relative factor demand:



- *Cross-sectoral reallocations* are at the core of HO predictions:
  - For relative factor prices to remain constant, *aggregate* relative demand must go up, which requires expansion capital intensive sector