

# 14.581 International Trade

## — Lecture 7: Factor Proportion Theory —

- ① Factor Proportion Theory
- ② Ricardo-Viner model
  - ① Basic environment
  - ② Comparative statics
- ③ Heckscher-Ohlin model
  - ① Basic environment
  - ② Classical results
  - ③ High-dimensional issues
  - ④ Quantitative issues

# Factor Proportion Theory

- The law of comparative advantage establishes the relationship between relative autarky prices and trade flows
  - But where do relative autarky prices come from?
- Factor proportion theory emphasizes **factor endowment differences**
- **Key elements:**
  - ① Countries differ in terms of factor abundance [i.e. *relative* factor supply]
  - ② Goods differ in terms of factor intensity [i.e. *relative* factor demand]
- Interaction between 1 and 2 will determine differences in relative autarky prices, and in turn, the pattern of trade

# Factor Proportion Theory

- In order to shed light on factor endowments as a source of CA, we will assume that:
  - 1 Production functions are identical around the world
  - 2 Households have identical homothetic preferences around the world
- We will first focus on two special models:
  - **Ricardo-Viner** with 2 goods, 1 “mobile” factor (labor) and 2 “immobile” factors (sector-specific capital)
  - **Heckscher-Ohlin** with 2 goods and 2 “mobile” factors (labor and capital)
- The second model is often thought of as a long-run version of the first (Neary 1978)
  - In the case of Heckscher-Ohlin, what is the time horizon such that one can think of total capital as fixed in each country, though freely mobile across sectors?

# Ricardo-Viner Model

## Basic environment

- Consider an economy with:
  - Two goods,  $g = 1, 2$
  - Three factors with endowments  $l$ ,  $k_1$ , and  $k_2$
- Output of good  $g$  is given by

$$y_g = f^g(l_g, k_g),$$

where:

- $l_g$  is the (endogenous) amount of labor in sector  $g$
- $f^g$  is homogeneous of degree 1 in  $(l_g, k_g)$
- **Comments:**
  - $l$  is a “mobile” factor in the sense that it can be employed in all sectors
  - $k_1$  and  $k_2$  are “immobile” factors in the sense that they can only be employed in one of them
  - Model is isomorphic to DRS model:  $y_g = f^g(l_g)$  with  $f_{ll}^g < 0$
  - Payments to specific factors under CRS  $\equiv$  profits under DRS

# Ricardo-Viner Model

## Equilibrium (I): small open economy

- We denote by:
  - $p_1$  and  $p_2$  the prices of goods 1 and 2
  - $w$ ,  $r_1$ , and  $r_2$  the prices of  $l$ ,  $k_1$ , and  $k_2$
- For now,  $(p_1, p_2)$  is exogenously given: **“small open economy”**
  - So no need to look at good market clearing
- **Profit maximization:**

$$p_g f_l^g(l_g, k_g) = w \quad (1)$$

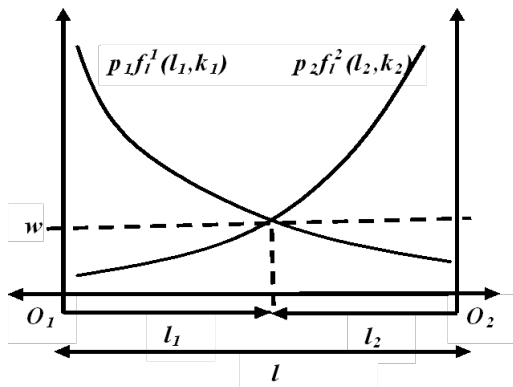
$$p_g f_k^g(l_g, k_g) = r_g \quad (2)$$

- **Labor market clearing:**

$$l = l_1 + l_2 \quad (3)$$

# Ricardo-Viner Model

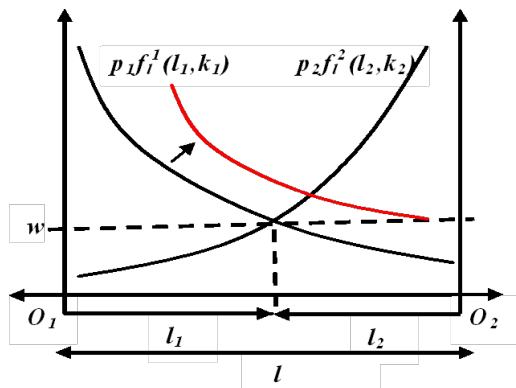
## Graphical analysis



- Equations (1) and (3) jointly determine labor allocation and wage
- How do we recover payments to the specific factor from this graph?

# Ricardo-Viner Model

## Comparative statics



- Consider a TOT shock such that  $p_1$  increases:
  - $w \nearrow$ ,  $l_1 \nearrow$ , and  $l_2 \searrow$
  - Condition (2)  $\Rightarrow r_1/p_1 \nearrow$  whereas  $r_2$  (and a fortiori  $r_2/p_1$ )  $\searrow$



# Ricardo-Viner Model

## Comparative statics

- One can use the same type of arguments to analyze consequences of:
  - Productivity shocks
  - Changes in factor endowments
- In all cases, results are intuitive:
  - “Dutch disease” (Boom in export sectors, Bids up wages, which leads to a contraction in the other sectors)
  - Useful political-economy applications (Grossman and Helpman 1994)
- Easy to extend the analysis to more than 2 sectors:
  - Plot labor demand in one sector vs. rest of the economy
  - Convenient for empirical work (Kovak 2013)

# Ricardo-Viner Model

## Equilibrium (II): two-country world

- Predictions on the pattern of trade in a two-country world depend on whether differences in factor endowments come from:
  - Differences in the relative supply of specific factors
  - Differences in the relative supply of mobile factors
- Accordingly, any change in factor prices is possible as we move from autarky to free trade (see Feenstra Problem 3.1 p. 98)

# Two-by-Two Heckscher-Ohlin Model

## Basic environment

- Consider an economy with:
  - Two goods,  $g = 1, 2$ ,
  - Two factors with endowments  $l$  and  $k$
- Output of good  $g$  is given by

$$y_g = f^g(l_g, k_g),$$

where:

- $l_g, k_g$  are the (endogenous) amounts of labor and capital in sector  $g$
- $f^g$  is homogeneous of degree 1 in  $(l_g, k_g)$

# Two-by-Two Heckscher-Ohlin Model

Back to the dual approach

- $c_g(w, r) \equiv$  unit cost function in sector  $g$

$$c_g(w, r) = \min_{l, k} \{wl + rk \mid f^g(l, k) \geq 1\},$$

where  $w$  and  $r$  the price of labor and capital

- $a_{fg}(w, r) \equiv$  unit demand for factor  $f$  in the production of good  $g$
- Using the Envelope Theorem, it is easy to check that:

$$a_{lg}(w, r) = \frac{dc_g(w, r)}{dw} \quad \text{and} \quad a_{kg}(w, r) = \frac{dc_g(w, r)}{dr}$$

- $A(w, r) \equiv [a_{fg}(w, r)]$  denotes the matrix of total factor requirements

# Two-by-Two Heckscher-Ohlin Model

Equilibrium conditions (I): small open economy

- Like in RV model, we first look at the case of a “**small open economy**”

- So no need to look at good market clearing

- **Profit-maximization:**

$$p_g \leq wa_{lg}(w, r) + ra_{kg}(w, r) \text{ for all } g = 1, 2 \quad (4)$$

$$p_g = wa_{lg}(w, r) + ra_{kg}(w, r) \text{ if } g \text{ is produced in equilibrium} \quad (5)$$

- **Factor market-clearing:**

$$l = y_1 a_{l1}(w, r) + y_2 a_{l2}(w, r) \quad (6)$$

$$k = y_1 a_{k1}(w, r) + y_2 a_{k2}(w, r) \quad (7)$$

# Two-by-Two Heckscher-Ohlin Model

## Factor Price Equalization

- **Question:**

*Can trade in goods be a (perfect) substitute for trade in factors?*

- First classical result from the HO literature answers by the affirmative
- To establish this result formally, we'll need the following definition:
- **Definition.** *Factor Intensity Reversal (FIR) does not occur if: (i)  $a_{l1}(w, r) / a_{k1}(w, r) > a_{l2}(w, r) / a_{k2}(w, r)$  for all  $(w, r)$ ; or (ii)  $a_{l1}(w, r) / a_{k1}(w, r) < a_{l2}(w, r) / a_{k2}(w, r)$  for all  $(w, r)$ .*

# Two-by-Two Heckscher-Ohlin Model

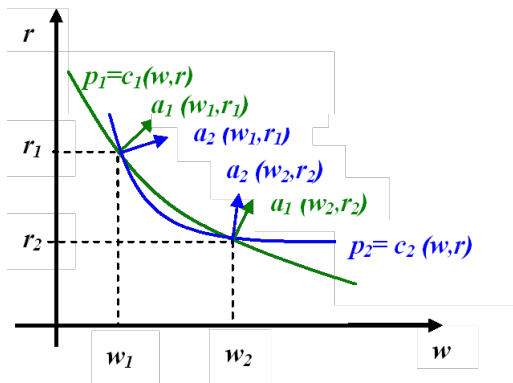
## Factor Price Insensitivity (FPI)

- **Lemma** *If both goods are produced in equilibrium and FIR does not occur, then factor prices  $\omega \equiv (w, r)$  are uniquely determined by good prices  $p \equiv (p_1, p_2)$*
- **Proof:** If both goods are produced in equilibrium, then  $p = A'(\omega)\omega$ . By Gale and Nikaido (1965), this equation admits a unique solution if  $a_{fg}(\omega) > 0$  for all  $f, g$  and  $\det [A(\omega)] \neq 0$  for all  $\omega$ , which is guaranteed by no FIR.
- **Comments:**
  - Good prices rather than factor endowments determine factor prices
  - In a closed economy, good prices and factor endowments are, of course, related, but not for a small open economy
  - All economic intuition can be gained by simply looking at Leontieff case
  - Proof already suggests that “dimensionality” will be an issue for FIR

# Two-by-Two Heckscher-Ohlin Model

## Factor Price Insensitivity (FPI): graphical analysis

- Link between no FIR and FPI can be seen graphically:



- If iso-cost curves cross more than once, then FIR must occur



# Heckscher-Ohlin Model

## Factor Price Equalization (FPE) Theorem

- The previous lemma directly implies (Samuelson 1949) that:
- **FPE Theorem** *If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices*
- **Comments:**
  - Trade in goods can be a “perfect substitute” for trade in factors
  - Countries with different factor endowments can sustain same factor prices through different allocation of factors across sectors
  - Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries...
  - For next results, we'll maintain assumption that both goods are produced in equilibrium, but won't need free trade and same technology

# Heckscher-Ohlin Model

## Stolper-Samuelson (1941) Theorem

- **Stolper-Samuelson Theorem** *An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduced the real return to the other factor*
- **Proof:** W.l.o.g. suppose that (i)  $a_{l1}(\omega)/a_{k1}(\omega) > a_{l2}(\omega)/a_{k2}(\omega)$  and (ii)  $\hat{p}_2 > \hat{p}_1$ . Differentiating the zero-profit condition (5), we get

$$\hat{p}_g = \theta_{lg} \hat{w} + (1 - \theta_{lg}) \hat{r}, \quad (8)$$

where  $\hat{x} = d \ln x$  and  $\theta_{lg} \equiv wa_{lg}(\omega) / c_g(\omega)$ . Equation (8) + (ii) imply

$$\hat{w} > \hat{p}_2 > \hat{p}_1 > \hat{r} \text{ or } \hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$$

By (i),  $\theta_{l2} < \theta_{l1}$ . So (ii) further requires  $\hat{r} > \hat{w}$ . Combining the previous inequalities, we get

$$\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$$

# Heckscher-Ohlin Model

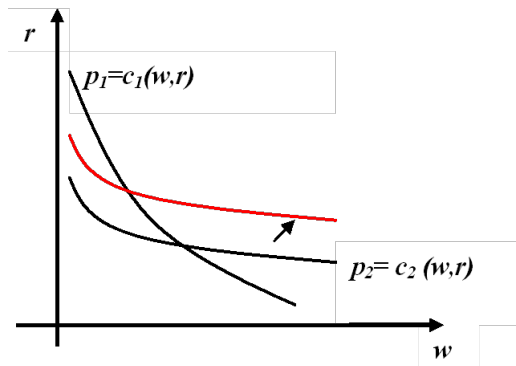
## Stolper-Samuelson (1941) Theorem

- **Comments:**

- Previous “hat” algebra is often referred to “Jones’ (1965) algebra”
- The chain of inequalities  $\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$  is referred as a “magnification effect”
- SS predict both winners and losers from change in relative prices
- Like FPI and FPE, SS entirely comes from zero-profit condition (+ no joint production)
- Like FPI and FPE, sharpness of the result hinges on “dimensionality”
- In the empirical literature, people often talk about “Stolper-Samuelson effects” whenever looking at changes in relative factor prices (though changes in relative good prices are rarely observed)

# Heckscher-Ohlin Model

Stolper-Samuelson (1941) Theorem: graphical analysis



- Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontieff case:
  - In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies

# Two-by-Two Heckscher-Ohlin Model

## Rybczynski (1965) Theorem

- Previous results have focused on the implication of *zero profit condition*, Equation (5), for *factor prices*
- We now turn our attention to the implication of *factor market clearing*, Equations (6) and (7), for *factor allocation*
- **Rybczynski Theorem** *An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry*

# Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem

- **Proof:** W.l.o.g. suppose that (i)  $a_{l1}(\omega)/a_{k1}(\omega) > a_{l2}(\omega)/a_{k2}(\omega)$  and (ii)  $\hat{k} > \hat{l}$ . Differentiating factor market clearing conditions (6) and (7), we get

$$\hat{l} = \lambda_{l1}\hat{y}_1 + (1 - \lambda_{l1})\hat{y}_2 \quad (9)$$

$$\hat{k} = \lambda_{k1}\hat{y}_1 + (1 - \lambda_{k1})\hat{y}_2 \quad (10)$$

where  $\lambda_{l1} \equiv a_{l1}(\omega)y_1/l$  and  $\lambda_{k1} \equiv a_{k1}(\omega)y_1/k$ . Equation (8) + (ii) imply

$$\hat{y}_1 > \hat{k} > \hat{l} > \hat{y}_2 \text{ or } \hat{y}_2 > \hat{k} > \hat{l} > \hat{y}_1$$

By (i),  $\lambda_{k1} < \lambda_{l1}$ . So (ii) further requires  $\hat{y}_2 > \hat{y}_1$ . Combining the previous inequalities, we get

$$\hat{y}_2 > \hat{k} > \hat{l} > \hat{y}_1$$

# Two-by-Two Heckscher-Ohlin Model

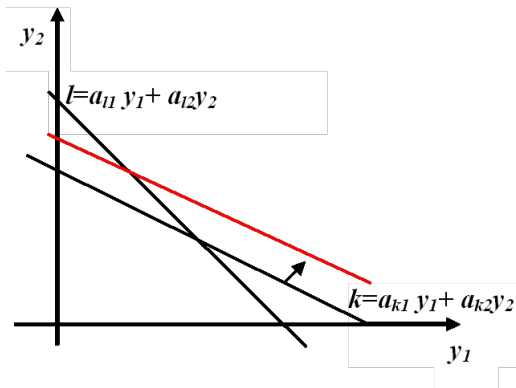
Rybczynski (1965) Theorem

- Like for FPI and FPE Theorems:
  - $(p_1, p_2)$  is exogenously given  $\Rightarrow$  factor prices and factor requirements are not affected by changes factor endowments
  - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy
- Like for SS Theorem, we have a “magnification effect”
- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on “dimensionality”

# Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem: graphical analysis (I)

- Since good prices are fixed, it is as if we were in Leontieff case

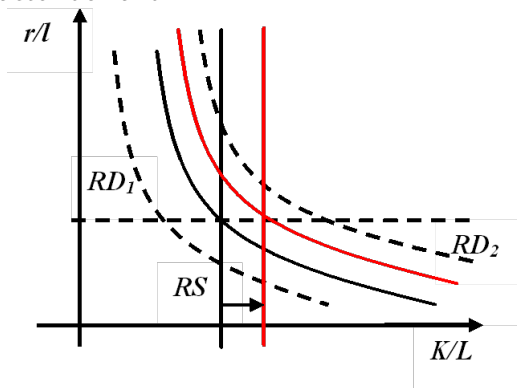




# Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem: graphical analysis (II)

- Rybczynski effect can also be illustrated using relative factor supply and relative factor demand:



- *Cross-sectoral reallocations* are at the core of HO predictions:
  - For relative factor prices to remain constant, *aggregate* relative demand must go up, which requires expansion capital intensive sector

# Two-by-Two-by-Two Heckscher-Ohlin Model

## Pattern of Trade

- Previous results hold for small open economies
  - relative good prices were taken as exogenously given
- We now turn world economy with two countries, North and South
- We maintain the two-by-two HO assumptions:
  - there are two goods,  $g = 1,2$ , and two factors,  $k$  and  $l$
  - identical technology around the world,  $y_g = f_g(k_g, l_g)$
  - identical homothetic preferences around the world,  $d_g^c = \alpha_g(p)I^c$
- **Question**  
What is the pattern of trade in this environment?

# Two-by-Two-by-Two Heckscher-Ohlin Model

## Strategy

- Start from **Integrated Equilibrium**  $\equiv$  competitive equilibrium that would prevail if *both* goods and factors were freely traded
- Consider **Free Trade Equilibrium**  $\equiv$  competitive equilibrium that prevails if goods are freely traded, but factors are not
- Ask: Can free trade equilibrium reproduce integrated equilibrium?
- *If factor prices are equalized through trade*, the answer is yes
- In this situation, one can then use homotheticity to go from differences in factor endowments to pattern of trade

# Two-by-Two-by-Two Heckscher-Ohlin Model

## Integrated equilibrium

- **Integrated equilibrium** corresponds to  $(p, \omega, y)$  such that:

$$(ZP) : p = A'(\omega) \omega \quad (11)$$

$$(GM) : y = \alpha(p) (\omega' v) \quad (12)$$

$$(FM) : v = A(\omega) y \quad (13)$$

where:

- $p \equiv (p_1, p_2)$ ,  $\omega \equiv (w, r)$ ,  $A(\omega) \equiv [a_{fg}(\omega)]$ ,  $y \equiv (y_1, y_2)$ ,  $v \equiv (l, k)$ ,  
 $\alpha(p) \equiv [\alpha_1(p), \alpha_2(p)]$
- $A(\omega)$  derives from cost-minimization
- $\alpha(p)$  derives from utility-maximization

# Two-by-Two-by-Two Heckscher-Ohlin Model

## Free trade equilibrium

- **Free trade equilibrium** corresponds to  $(p^t, \omega^n, \omega^s, y^n, y^s)$  such that:

$$(ZP) : \quad p^t \leq A'(\omega^c) \omega^c \text{ for } c = n, s \quad (14)$$

$$(GM) : \quad y^n + y^s = \alpha(p^t) (\omega^n v^n + \omega^s v^s) \quad (15)$$

$$(FM) : \quad v^c = A(\omega^c) y^c \text{ for } c = n, s \quad (16)$$

where (14) holds with equality if good is produced in country  $c$

- **Definition** *Free trade equilibrium replicates integrated equilibrium if  $\exists (y^n, y^s) \geq 0$  such that  $(p, \omega, \omega, y^n, y^s)$  satisfy conditions (14)-(16)*

# Two-by-Two-by-Two Heckscher-Ohlin Model

## Factor Price Equalization (FPE) Set

- **Definition**  $(v^n, v^s)$  are in the FPE set if  $\exists (y^n, y^s) \geq 0$  such that condition (16) holds for  $\omega^n = \omega^s = \omega$ .
- **Lemma** If  $(v^n, v^s)$  is in the FPE set, then free trade equilibrium replicates integrated equilibrium
- **Proof:** By definition of the FPE set,  $\exists (y^n, y^s) \geq 0$  such that

$$v^c = A(\omega) y^c$$

So Condition (16) holds. Since  $v = v^n + v^s$ , this implies

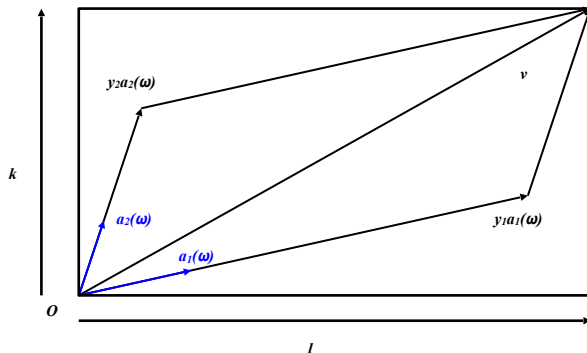
$$v = A(\omega) (y^n + y^s)$$

Combining this expression with condition (13), we obtain  $y^n + y^s = y$ . Since  $\omega^{n'} v^n + \omega^{s'} v^s = \omega' v$ , Condition (15) holds as well. Finally, Condition (11) directly implies (14) holds.

# Two-by-Two-by-Two Heckscher-Ohlin Model

Integrated equilibrium: graphical analysis

- Factor market clearing in the integrated equilibrium:



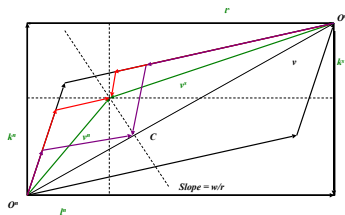




# Two-by-Two-by-Two Heckscher-Ohlin Model

## Heckscher-Ohlin Theorem: graphical analysis

- Suppose that  $(v^n, v^s)$  is in the FPE set
- **HO Theorem** *In the free trade equilibrium, each country will export the good that uses its abundant factor intensively*



- Outside the FPE set, additional technological and demand considerations matter (e.g. FIR or no FIR)

# Two-by-Two-by-Two Heckscher-Ohlin Model

## Heckscher-Ohlin Theorem: alternative proof

- HO Theorem can also be derived using Rybczynski effect:
  - ① Rybczynski theorem  $\Rightarrow y_2^n/y_1^n > y_2^s/y_1^s$  for any  $p$
  - ② Homotheticity  $\Rightarrow c_2^n/c_1^n = c_2^s/c_1^s$  for any  $p$
  - ③ This implies  $p_2^n/p_1^n < p_2^s/p_1^s$  under autarky
  - ④ Law of comparative advantage  $\Rightarrow$  HO Theorem

# Two-by-Two-by-Two Heckscher-Ohlin Model

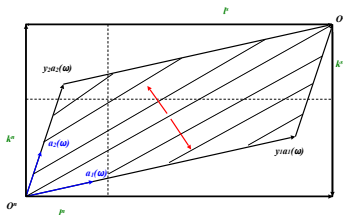
## Trade and inequality

- Predictions of HO and SS Theorems are often combined:
  - HO Theorem  $\Rightarrow p_2^n / p_1^n < p_2 / p_1 < p_2^s / p_1^s$
  - SS Theorem  $\Rightarrow$  *Moving from autarky to free trade, real return of abundant factor increases, whereas real return of scarce factor decreases*
  - If North is skill-abundant relative to South, inequality increases in the North and decreases in the South
- So why may we observe a rise in inequality in the South in practice?
  - Southern countries are not moving from autarky to free trade
  - Technology is not identical around the world
  - Preferences are not homothetic and identical around the world
  - There are more than two goods and two countries in the world

# Two-by-Two-by-Two Heckscher-Ohlin Model

## Trade volumes

- Let us define trade volumes as the sum of exports plus imports
- Inside FPE set, iso-volume lines are parallel to diagonal (HKa p.23)
  - the further away from the diagonal, the larger the trade volumes
  - factor abundance rather than country size determines trade volume

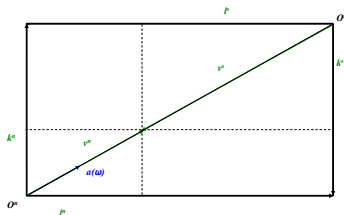


- If country size affects trade volumes in practice, what should we infer?

# High-Dimensional Predictions

FPE (I): More factors than goods

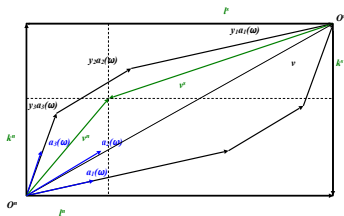
- Suppose now that there are  $F$  factors and  $G$  goods
- By definition,  $(v^n, v^s)$  is in the FPE set if  $\exists (y^n, y^s) \geq 0$  s.t.  
 $v^c = A(\omega) y^c$  for  $c = n, s$
- If  $F = G$  (“even case”), the situation is qualitatively similar
- If  $F > G$ , the FPE set will be “measure zero”:  
 $\{v \mid v = A(\omega) y^c \text{ for } y^c \geq 0\}$  is a  $G$ -dimensional cone in  $F$ -dimensional space
- **Example:** “Macro” model with 1 good and 2 factors



# High-Dimensional Predictions

FPE (II): More goods than factors

- If  $F < G$ , there will be indeterminacies in production,  $(y^n, y^s)$ , and so, trade patterns, but FPE set will still have positive measure
- **Example:** 3 goods and 2 factors



- By the way, are there more goods than factors in the world?

# High-Dimensional Predictions

Stolper-Samuelson-type results (I): “Friends and Enemies”

- SS Theorem was derived by differentiating zero-profit condition
- With an arbitrary number of goods and factors, we still have

$$\hat{p}_g = \sum_f \theta_{fg} \hat{w}_f \quad (17)$$

where  $w_f$  is the price of factor  $f$  and  $\theta_{fg} \equiv w_f a_{fg}(\omega) / c_g(\omega)$

- Now suppose that  $\hat{p}_{g_0} > 0$ , whereas  $\hat{p}_g = 0$  for all  $g \neq g_0$
- Equation (17) immediately implies the existence of  $f_1$  and  $f_2$  s.t.

$$\begin{aligned} \hat{w}_{f_1} &\geq \hat{p}_{g_0} > \hat{p}_g = 0 \text{ for all } g \neq g_0, \\ \hat{w}_{f_2} &< \hat{p}_g = 0 < \hat{p}_{g_0} \text{ for all } g \neq g_0. \end{aligned}$$

- So every good is “friend” to some factor and “enemy” to some other (Jones and Scheinkman 1977)

# High-Dimensional Predictions

## Stolper-Samuelson-type results (II): Correlations

- Ethier (1984) also provides the following variation of SS Theorem
- If good prices change from  $p^1$  to  $p^2$ , then the associated change in factor prices,  $\omega^2 - \omega^1$ , must satisfy

$$(p^2 - p^1)' A'(\omega^0) (\omega^2 - \omega^1) > 0, \text{ for some } \omega^0 \text{ between } \omega^1 \text{ and } \omega^2$$

- **Proof:**

- Define  $f(\omega) = (p^2 - p^1)' A'(\omega) \omega$ . Mean value theorem implies

$$f(\omega^2) = f(\omega^1) + (p^2 - p^1)' [A'(\omega^0) + dA'(\omega^0) \omega^0] (\omega^2 - \omega^1)$$

for some  $\omega^0$  between  $\omega^1$  and  $\omega^2$ . Cost-minimization at  $\omega^0$  requires

$$dA'(\omega^0) \omega^0 = 0$$



# High-Dimensional Predictions

## Stolper-Samuelson-type results (II): Correlations

- **Proof (Cont.):**

Combining the two previous expressions, we obtain

$$f(\omega^2) - f(\omega^1) = (p^2 - p^1)' A'(\omega^0) (\omega^2 - \omega^1)$$

From zero profit condition, we know that  $p^1 = A'(\omega^1) \omega^1$  and  $p^2 = A'(\omega^2) \omega^2$ . Thus

$$f(\omega^2) - f(\omega^1) = (p^2 - p^1)' (p^2 - p^1) > 0$$

The last two expressions imply

$$(p^2 - p^1)' A'(\omega^0) (\omega^2 - \omega^1) > 0$$

- **Interpretation:**

Tendency for changes in good prices to be accompanied by raises in prices of factors used intensively in goods whose prices have gone up

- What is  $\omega^0$ ?

# High-Dimensional Predictions

## Rybczynski-type results

- Rybczynski Theorem was derived by differentiating the factor market clearing condition
- If  $G = F > 2$ , same logic implies that increase in endowment of one factor decreases output of one good and increases output of another (Jones and Scheinkman 1977)
- If  $G < F$ , increase in endowment of one factor may increase output of all goods (Ricardo-Viner)
- In this case, we still have the following correlation (Ethier 1984)

$$(v^2 - v^1)' A(\omega^0) (y^2 - y^1) = (v^2 - v^1)' (v^2 - v^1) > 0$$

- If  $G > F$ , indeterminacies in production imply that we cannot predict changes in output vectors

# High-Dimensional Predictions

## Heckscher-Ohlin-type results

- Since HO Theorem derives from Rybczynski effect + homotheticity, problems of generalization in the case  $G < F$  and  $F > G$  carry over to the Heckscher-Ohlin Theorem
- If  $G = F > 2$ , we can invert the factor market clearing condition

$$y^c = A^{-1}(\omega) v^c$$

- By homotheticity, the vector of consumption in country  $c$  satisfies

$$d^c = s^c d$$

where  $s^c \equiv c$ 's share of world income, and  $d \equiv$  world consumption

- Good and factor market clearing requires

$$d = y = A^{-1}(\omega) v$$

- Combining the previous expressions, we get net exports

$$t^c \equiv y^c - d^c = A^{-1}(\omega) (v^c - s^c v)$$

# High-Dimensional Predictions

## Heckscher-Ohlin-Vanek Theorem

- Without assuming that  $G = F$ , we can still derive sharp predictions if we focus on the *factor content of trade* rather than *commodity trade*
- We define the *net exports of factor  $f$*  by country  $c$  as

$$\tau_f^c = \sum_g a_{fg}(\omega) t_g^c$$

- In matrix terms, this can be rearranged as

$$\tau^c = A(\omega) t^c$$

- **HOV Theorem** *In any country  $c$ , net exports of factors satisfy*

$$\tau^c = v^c - s^c v$$

- So countries should export the factors in which they are abundant compared to the world:  $v_f^c > s^c v_f$
- Assumptions of HOV Theorem are extremely strong: identical technology, FPE, homotheticity
  - One shouldn't be too surprised if it performs miserably in practice...

- Stolper-Samuelson offers sharp insights about distributional consequences of international trade, but...
  - Theoretical insights are only *qualitative*
  - Theoretical insights crucially rely on  $2 \times 2$  assumptions
- Alternatively one may want to know the *quantitative* importance of international trade:
  - Given the amount of trade that we actually observe in the data, how large are the effects of international trade on the skill premium?
  - In a country like the United States, how much higher or smaller would the skill premium be in the absence of trade?

# Quantitative Issues

## Eaton and Kortum (2002) Revisited

- Eaton and Kortum (2002)—as well as other gravity models—offer a simple starting point to think about these issues
- Consider multi-sector-multi-factor EK (e.g. Chor JIE 2010)
  - many varieties with different productivity levels  $z(\omega)$  in each sector  $s$
  - same factor intensity across varieties within sectors
  - different factor intensities across sectors
- Unit costs of production in country  $i$  and sector  $s$  are proportional to:

$$c_{i,s} = \left[ \left( \mu_s^H \right)^\rho \left( w_i^H \right)^{1-\rho} + \left( \mu_s^L \right)^\rho \left( w_i^L \right)^{1-\rho} \right]^{1/(1-\rho)} \quad (18)$$

where:

- $w_i^H, w_i^L \equiv$  wages of skilled and unskilled workers.
- $\rho \equiv$  elasticity of substitution between skilled and unskilled

# Quantitative Issues

Dekle, Eaton, and Kortum (2008) Revisited

- Suppose, like in EK, that productivity draws across varieties within sectors are independently drawn from a Fréchet
- Then one can show that the following gravity equation holds:

$$X_{ij,s} = \frac{T_i (\tau_{ij,s} c_{i,s})^{-\theta_s}}{\sum_{l=1}^n T_l (\tau_{lj,s} c_{l,s})^{-\theta_s}} E_{j,s}, \quad (19)$$

where  $E_{j,s} \equiv$  total expenditure on goods from sector  $s$  in country  $j$

- Two key equations, (18) and (19), are CES:
  - One can use DEK's strategy to do welfare and counterfactual analysis
  - But one can also discuss the consequences of changes in variable trade costs,  $\tau_{ij,s}$ , or technology,  $T_i$ , on skill premium
  - How large are GT compared to distributional consequences?
  - Some preliminary answers in Costinot and Rodriguez-Clare (2014); more in Burstein and Vogel (2016), Galle, Rodriguez-Clare and Yi (2017)
  - See also Burstein, Hanson, Tian, and Vogel (2018) for immigration