14.581 International Trade
— Lecture 4: Ricardian Theory (II)—
Ricardian model has long been perceived has useful pedagogic tool, with little empirical content:

- Great to explain undergrads why there are gains from trade
- But grad students should study richer models (Feenstra’s graduate textbook has a total of 3 pages on the Ricardian model!)

Eaton and Kortum (2002) have lead to “Ricardian revival”

- Same basic idea as in Wilson (1980): Who cares about the pattern of trade for counterfactual analysis?
- But more structure: Small number of parameters, so well-suited for quantitative work

**Goals of this lecture:**

1. Present EK model
2. Discuss estimation of its key parameter
3. Introduce tools for welfare and counterfactual analysis
Basic Assumptions

- $N$ countries, $i = 1, \ldots, N$
- Continuum of goods $u \in [0, 1]$
- Preferences are CES with elasticity of substitution $\sigma$:
  \[
  U_i = \left( \int_0^1 q_i(u)^{(\sigma-1)/\sigma} \, du \right)^{\sigma/(\sigma-1)},
  \]
- One factor of production (labor)
- There may also be intermediate goods (more on that later)
- $c_i \equiv$ unit cost of the “common input” used in production of all goods
  - Without intermediate goods, $c_i$ is equal to wage $w_i$ in country $i$
Basic Assumptions (Cont.)

- **Constant returns to scale:**
  - $Z_i(u)$ denotes productivity of (any) firm producing $u$ in country $i$
  - $Z_i(u)$ is drawn independently (across goods and countries) from a **Fréchet distribution**:
    \[
    \Pr(Z_i \leq z) = F_i(z) = e^{-T_i z^{-\theta}},
    \]
    with $\theta > \sigma - 1$ (important restriction, see below)
  - Since goods are symmetric except for productivity, we can forget about index $u$ and keep track of goods through $Z \equiv (Z_1, ..., Z_N)$.

- Trade is subject to iceberg costs $d_{ni} \geq 1$
  - $d_{ni}$ units need to be shipped from $i$ so that 1 unit makes it to $n$

- All markets are perfectly competitive
Four Key Results
A - The Price Distribution

- Let $P_{ni}(Z) \equiv c_i d_{ni} / Z_i$ be the unit cost at which country $i$ can serve a good $Z$ to country $n$ and let $G_{ni}(p) \equiv \Pr(P_{ni}(Z) \leq p)$. Then:

$$G_{ni}(p) = \Pr(Z_i \geq c_i d_{ni} / p) = 1 - F_i(c_i d_{ni} / p)$$

- Let $P_n(Z) \equiv \min\{P_{n1}(Z), \ldots, P_{nN}(Z)\}$ and let $G_n(p) \equiv \Pr(P_n(Z) \leq p)$ be the price distribution in country $n$. Then:

$$G_n(p) = 1 - \exp[-\Phi_n p^\theta]$$

where

$$\Phi_n \equiv \sum_{i=1}^N T_i(c_i d_{ni})^{-\theta}$$
To show this, note that (suppressing notation $Z$ from here onwards)

$$
\Pr(P_n \leq p) = 1 - \Pi_i \Pr(P_{ni} \geq p) = 1 - \Pi_i [1 - G_{ni}(p)]
$$

Using

$$
G_{ni}(p) = 1 - F_i(c_id_{ni}/p)
$$

then

$$
1 - \Pi_i [1 - G_{ni}(p)] = 1 - \Pi_i F_i(c_id_{ni}/p) = 1 - \Pi_i e^{-T_i(c_id_{ni})^{-\theta}p^\theta} = 1 - e^{-\Phi_n p^\theta}
$$
Consider a particular good. Country $n$ buys the good from country $i$ if $i = \arg\min\{p_{n1}, ..., p_{nN}\}$. The probability of this event is simply country $i$’s contribution to country $n$’s price parameter $\Phi_n$,

$$\pi_{ni} = \frac{T_i(c_id_{ni})^{-\theta}}{\Phi_n}$$

To show this, note that

$$\pi_{ni} = \Pr\left(P_{ni} \leq \min_{s \neq i} P_{ns}\right)$$

If $P_{ni} = p$, then the probability that country $i$ is the least cost supplier to country $n$ is equal to the probability that $P_{ns} \geq p$ for all $s \neq i$. 
The previous probability is equal to

$$\Pi_{s \neq i} \Pr(P_{ns} \geq p) = \Pi_{s \neq i} [1 - G_{ns}(p)] = e^{-\Phi_{n}^{-i}p^\theta}$$

where

$$\Phi_{n}^{-i} = \sum_{s \neq i} T_i (c_i d_{ni})^{-\theta}$$

Now we integrate over this for all possible $p$'s times the density $dG_{ni}(p)$ to obtain

$$\int_{0}^{\infty} e^{-\Phi_{n}^{-i}p^\theta} T_i (c_i d_{ni})^{-\theta} \theta p^{\theta-1} e^{-T_i(c_i d_{ni})^{-\theta}p^\theta} dp$$

$$= \left( \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_{n}} \right) \int_{0}^{\infty} \theta \Phi_{n} e^{-\Phi_{n}p^\theta} p^{\theta-1} dp$$

$$= \pi_{ni} \int_{0}^{\infty} dG_{n}(p) dp = \pi_{ni}$$
Close connection between EK and McFadden’s logit model

Take heterogeneous consumers, indexed by $u$, with utility $U_n(u)$ from consuming good $i$:

$$U_i(u) = U_i - p_i + \varepsilon_i(u)$$

with $\varepsilon_i(u)$ i.i.d from **Gumbel distribution**:

$$\Pr(\varepsilon_i(u) \leq \varepsilon) = \exp(-\exp(-\theta \varepsilon))$$

**Logit**: for each consumer $u$, choose good $i$ that maximizes $U_i(u) \Rightarrow$

$$\pi_i = \frac{\exp(\theta (U_i - p_i))}{\sum_j \exp(\theta (U_j - p_j))}$$

**EK**: for each good $u$, choose source country $i$ that minimizes $\ln p_i(u) = \ln c_i - \ln Z_i(u)$. Then $\ln(\text{Fréchet}) = \text{Gumbel} \Rightarrow$

$$\pi_i = \frac{\exp(\theta (-\ln c_i))}{\sum_j \exp(\theta (-\ln c_j))} = \frac{c_i^{-\theta}}{\sum_j c_j^{-\theta}}$$
The price of a good that country $n$ actually buys from any country $i$ also has the distribution $G_n(p)$.

To show this, note that if country $n$ buys a good from country $i$ it means that $i$ is the least cost supplier. If the price at which country $i$ sells this good in country $n$ is $q$, then the probability that $i$ is the least cost supplier is

$$\Pi_{s \neq i} \Pr(P_{ni} \geq q) = \Pi_{s \neq i} [1 - G_{ns}(q)] = e^{-\Phi_n^{-i} q^\theta}$$

The joint probability that country $i$ has a unit cost $q$ of delivering the good to country $n$ and is the the least cost supplier of that good in country $n$ is then

$$e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q)$$
Integrating this probability $e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q)$ over all prices $q \leq p$ and using $G_{ni}(q) = 1 - e^{-T_i(c_id_{ni})^{-\theta} p^\theta}$ then

$$
\int_0^p e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q) = \int_0^p e^{-\Phi_n^{-i} q^\theta} \theta T_i(c_id_{ni})^{-\theta} q^{\theta-1} e^{-T_i(c_id_{ni})^{-\theta} p^\theta} dq
$$

$$= \left( \frac{T_i(c_id_{ni})^{-\theta}}{\Phi_n} \right) \int_0^p e^{-\Phi_n q^\theta} \theta \Phi_n q^{\theta-1} dq = \pi_{ni} G_n(p)
$$

Given that $\pi_{ni} \equiv$ probability that for any particular good country $i$ is the least cost supplier in $n$, then conditional distribution of the price charged by $i$ in $n$ for the goods that $i$ actually sells in $n$ is

$$
\frac{1}{\pi_{ni}} \int_0^p e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q) = G_n(p)
$$
In Eaton and Kortum (2002):

1. All the adjustment is at the extensive margin: countries that are more distant, have higher costs, or lower $T'$s, simply sell a smaller range of goods, but the average price charged is the same.

2. The share of spending by country $n$ on goods from country $i$ is the same as the probability $\pi_{ni}$ calculated above.

We will establish a similar property in models of monopolistic competition with Pareto distributions of firm-level productivity.
The exact price index for a CES utility with elasticity of substitution \( \sigma < 1 + \theta \), defined as

\[
p_n \equiv \left( \int_0^1 p_n(u)^{1-\sigma} \, du \right)^{1/(1-\sigma)},
\]

is given by

\[
p_n = \gamma \Phi_{\eta}^{-1/\theta}
\]

where

\[
\gamma = \left[ \Gamma\left( \frac{1-\sigma}{\theta} + 1 \right) \right]^{1/(1-\sigma)}
\]

where \( \Gamma \) is the Gamma function, \textit{i.e.} \( \Gamma(a) \equiv \int_0^\infty x^{a-1} e^{-x} \, dx \).
To show this, note that
\[ p_n^{1-\sigma} = \frac{1}{\theta} \int_0^1 p_n(u)^{1-\sigma} du = \]
\[ \int_0^\infty p^{1-\sigma} dG_n(p) = \int_0^\infty p^{1-\sigma} \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta} dp. \]

Defining \( x = \Phi_n p^\theta \), then \( dx = \Phi_n \theta p^{\theta-1}, p^{1-\sigma} = \left( x / \Phi_n \right)^{(1-\sigma)/\theta} \), and
\[ p_n^{1-\sigma} = \int_0^\infty \left( x / \Phi_n \right)^{(1-\sigma)/\theta} e^{-x} dx \]
\[ = \Phi_n^{-(1-\sigma)/\theta} \int_0^\infty x^{(1-\sigma)/\theta} e^{-x} dx \]
\[ = \Phi_n^{-(1-\sigma)/\theta} \Gamma \left( \frac{1-\sigma}{\theta} + 1 \right) \]

This implies \( p_n = \gamma \Phi_n^{-1/\theta} \) with \( \frac{1-\sigma}{\theta} + 1 > 0 \) or \( \sigma - 1 < \theta \) for gamma function to be well defined.
Let $X_{ni}$ be total spending in country $n$ on goods from country $i$.

Let $X_n \equiv \sum_i X_{ni}$ be country $n$’s total spending.

We know that $X_{ni}/X_n = \pi_{ni}$, so

$$X_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_j T_j(w_j d_{nj})^{-\theta}} X_n \tag{*}$$

Suppose that there are no intermediate goods so that $c_i = w_i$.

In equilibrium, total income in country $i$ must be equal to total spending on goods from country $i$ so

$$w_i L_i = \sum_n X_{ni}$$

Trade balance further requires $X_n = w_n L_n$ so that

$$w_i L_i = \sum_n \frac{T_i(w_i d_{ni})^{-\theta}}{\sum_j T_j(w_j d_{nj})^{-\theta}} w_n L_n$$
This provides system of \( N - 1 \) independent equations (Walras’ Law) that can be solved for wages \((w_1, \ldots, w_N)\) up to a choice of numeraire.

Everything is as if countries were exchanging labor:
- Fréchet distributions imply that labor demands are iso-elastic.
- Armington model leads to similar eq. conditions under assumption that each country is exogenously specialized in a differentiated good.
- In the Armington model, the labor demand elasticity simply coincides with elasticity of substitution \( \sigma \).
  - See Anderson and van Wincoop (2003)
- Iso-elastic case is what trade economists refer to as a “‘gravity model’” with \((*)=‘‘gravity equation’’\)
  - We’ll come back to gravity models many times in this class.
How to Estimate the Trade Elasticity?

- As we will see, trade elasticity $\theta =$ key structural parameter for welfare and counterfactual analysis in EK model (and other gravity models)
- From (*) we also get that country $i$’s share in country $n$’s expenditures normalized by its own share is
  \[ S_{ni} \equiv \frac{X_{ni}}{X_{ni}/X_i} = \frac{\Phi_i}{\Phi_n} d_{ni}^{-\theta} = \left( \frac{p_i d_{ni}}{p_n} \right)^{-\theta} \]
- This shows the importance of trade costs in determining trade volumes. Note that if there are no trade barriers (i.e, frictionless trade), then $S_{ni} = 1$.
- If we had data on $d_{ni}$, we could run a regression of $\ln S_{ni}$ on $\ln d_{ni}$ with importer and exporter dummies to recover $\theta$
  - But how do we get $d_{ni}$?
EK use price data to measure \( p_i d_{ni} / p_n \):

- They use retail prices in 19 OECD countries for 50 manufactured products from the UNICP 1990 benchmark study.
- They interpret these data as a sample of the prices \( p_i(j) \) of individual goods in the model.
- They note that for goods that \( n \) imports from \( i \) we should have \( p_n(j) / p_i(j) = d_{ni} \), whereas goods that \( n \) doesn’t import from \( i \) can have \( p_n(j) / p_i(j) \leq d_{ni} \).
- Since every country in the sample does import manufactured goods from every other, then \( \max_j \{ p_n(j) / p_i(j) \} \) should be equal to \( d_{ni} \).
- To deal with measurement error, they actually use the second highest \( p_n(j) / p_i(j) \) as a measure of \( d_{ni} \).
Let $r_{ni}(j) \equiv \ln p_n(j) - \ln p_i(j)$. They calculate $\ln(p_n / p_i)$ as the mean across $j$ of $r_{ni}(j)$. Then they measure $\ln(p_i d_{ni} / p_n)$ by

$$D_{ni} = \frac{\max 2_j \{r_{ni}(j)\}}{\sum_j r_{ni}(j) / 50}$$

Given $S_{ni} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$ they estimate $\theta$ from $\ln(S_{ni}) = -\theta D_{ni}$. Method of moments: $\theta = 8.28$. OLS with zero intercept: $\theta = 8.03$. 

Figure 2.—Trade and prices.
Simonovska and Waugh (2014, JIE) argue that EK’s procedure suffers from upward bias:

- Since EK are only considering 50 goods, maximum price gap may still be strictly lower than trade cost
- If we underestimate trade costs, we overestimate trade elasticity
- Simulation based method of moments leads to a $\theta$ closer to 4.

An alternative approach is to use tariffs (Caliendo and Parro, 2015, RES). If $d_{ni} = t_{ni}\tau_{ni}$ where $t_{ni}$ is one plus the ad-valorem tariff (they actually do this for each 2 digit industry) and $\tau_{ni}$ is assumed to be symmetric, then

$$\frac{X_{ni}X_{ij}X_{jn}}{X_{nj}X_{ji}X_{in}} = \left(\frac{d_{ni}d_{ij}d_{jn}}{d_{nj}d_{ji}d_{in}}\right)^{-\theta} = \left(\frac{t_{ni}t_{ij}t_{jn}}{t_{nj}t_{ji}t_{in}}\right)^{-\theta}$$

They can then run an OLS regression and recover $\theta$. Their preferred specification leads to an estimate of 8.22.
Shapiro (2014) uses time-variation in freight costs (again for each 2-digit industry):

$$\ln X_{ni}^t = \alpha_{ni} + \beta_{nt} + \gamma_{it} - \theta \ln(1 + s_{ni}^t) + \varepsilon_{ni}^t$$

- $s_{ni}^t \equiv$ total shipping costs between $i$ and $n$ in (Q1 and Q4 of) year $t$
- $\alpha_{ni} \equiv$ importer-exporter fixed effect; $\beta_{nt} \equiv$ importer-year fixed effect; $\gamma_{it} \equiv$ exporter-year fixed-effect
- To deal with measurement error in freight costs, he instruments shipping costs from Q1 and Q4 with shipping costs from Q2 and Q3
- IV estimate of trade elasticity $\equiv 7.91$.

Head and Mayer (2015) offer a review of trade elasticity estimates:
- Typical value is around 5
- BTW should we expect aggregate $= \text{sector-level elasticities}$?
Gains from Trade

- Consider again the case where $c_i = w_i$
- From (*), we know that
  \[ \pi_{nn} = \frac{X_{nn}}{X_n} = \frac{T_n w_n^{-\theta}}{\Phi_n} \]
- We also know that $p_n = \gamma \Phi_n^{-1/\theta}$, so
  \[ \omega_n \equiv w_n / p_n = \gamma^{-1} T_n^{1/\theta} \pi_{nn}^{-1/\theta} . \]
- Under autarky we have $\omega_n^A = \gamma^{-1} T_n^{1/\theta}$, hence the gains from trade are given by
  \[ GT_n \equiv \omega_n / \omega_n^A = \pi_{nn}^{-1/\theta} \]
- Trade elasticity $\theta$ and share of expenditure on domestic goods $\pi_{nn}$ are sufficient statistics to compute $GT$
A typical value for \( \pi_{nn} \) (manufacturing) is 0.7. With \( \theta = 5 \) this implies \( GT_n = 0.7^{-1/5} = 1.074 \) or 7.4\% gains. Belgium has \( \pi_{nn} = 0.2 \), so its gains are \( GT_n = 0.2^{-1/5} = 1.38 \) or 38\%.

One can also use the previous approach to measure the welfare gains associated with any foreign shock, not just moving to autarky:

\[
\frac{\omega'_n}{\omega_n} = \left( \frac{\pi'_n}{\pi_{nn}} \right)^{-1/\theta}
\]

For more general counterfactual scenarios, however, one needs to know both \( \pi'_{nn} \) and \( \pi_{nn} \).
Imagine that intermediate goods are used to produce a composite good with a CES production function with elasticity $\sigma > 1$. This composite good can be either consumed or used to produce intermediate goods (input-output loop).

Each intermediate good is produced from labor and the composite good with a Cobb-Douglas technology with labor share $\beta$. We can then write $c_i = w_i^\beta p_i^{1-\beta}$. 

$\text{14.581 (Week 3)}$

Ricardian Theory (II)
Adding an Input-Output Loop (Cont.)

The analysis above implies

$$\pi_{nn} = \gamma^{-\theta} T_n \left( \frac{c_n}{p_n} \right)^{-\theta}$$

and hence

$$c_n = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n$$

Using $c_n = w_n^\beta p_n^{1-\beta}$ this implies

$$w_n^\beta p_n^{1-\beta} = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n$$

so

$$w_n / p_n = \gamma^{-1/\beta} T_n^{-1/\theta \beta} \pi_{nn}^{-1/\theta \beta}$$

The gains from trade are now

$$\omega_n / \omega_n^A = \pi_{nn}^{-1/\theta \beta}$$

Standard value for $\beta$ is 1/2 (Alvarez and Lucas, 2007). For $\pi_{nn} = 0.7$ and $\theta = 5$ this implies $GT_n = 0.7^{-2/5} = 1.15$ or 15% gains.
Assume now that the composite good cannot be consumed directly.
Instead, it can either be used to produce intermediates (as above) or to produce a consumption good (together with labor).
The production function for the consumption good is Cobb-Douglas with labor share $\alpha$.
This consumption good is assumed to be non-tradable.
The price index computed above is now $p_{gn}$, but we care about
\[ \omega_n \equiv \frac{w_n}{p_{fn}}, \text{ where} \]
\[ p_{fn} = w_n^\alpha p_{gn}^{1-\alpha} \]

This implies that
\[ \omega_n = \frac{w_n}{w_n^\alpha p_{gn}^{1-\alpha}} = \left(\frac{w_n}{p_{gn}}\right)^{1-\alpha} \]

Thus, the gains from trade are now
\[ \frac{\omega_n}{\omega^A} = \pi_{nn}^{-\eta}/\theta \]

where
\[ \eta \equiv 1 - \frac{\alpha}{\beta} \]

Alvarez and Lucas argue that $\alpha = 0.75$ (share of labor in services).
Thus, for $\pi_{nn} = 0.7$, $\theta = 5$ and $\beta = 0.5$, this implies
\[ GT_n = 0.7^{-1/10} = 1.036 \text{ or } 3.6\% \text{ gains} \]
Go back to the simple EK model above ($\alpha = 0$, $\beta = 1$). We have

$$X_{ni} = \frac{T_i (w_i d_{ni})^{-\theta} X_n}{\sum_{i=1}^{N} T_i (w_i d_{ni})^{-\theta}}$$

$$\sum_n X_{ni} = w_i L_i$$

As we have already established, this leads to a system of non-linear equations to solve for wages,

$$w_i L_i = \sum_n \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_k T_k (w_k d_{nk})^{-\theta}} w_n L_n.$$
Comparative statics (Dekle, Eaton and Kortum, 2008)

- Consider a shock to labor endowments, trade costs, or productivity. One could compute the original equilibrium, the new equilibrium and compute the changes in endogenous variables.
- But there is a simpler way that uses only information for observables in the initial equilibrium, trade shares and GDP; the trade elasticity, $\theta$; and the exogenous shocks. First solve for changes in wages by solving

$$\hat{w}_i \hat{L}_i Y_i = \sum_n \frac{\pi_{ni} \hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}} \hat{w}_n \hat{L}_n Y_n$$

and then get changes in trade shares from

$$\hat{\pi}_{ni} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}}.$$

- From here, one can compute welfare changes by using the formula above, namely $\hat{\omega}_n = (\hat{\pi}_{nn})^{-1/\theta}$. 
Comparative statics (Dekle, Eaton and Kortum, 2008)

- To show this, note that trade shares are

\[ \pi_{ni} = \frac{T_i (w_id_{ni})^{-\theta}}{\sum_k T_k (w_k d_{nk})^{-\theta}} \quad \text{and} \quad \pi'_{ni} = \frac{T'_i (w'_id'_{ni})^{-\theta}}{\sum_k T'_k (w'_k d'_{nk})^{-\theta}}. \]

- Letting \( \hat{x} \equiv x'/x \), then we have

\[ \hat{\pi}_{ni} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k T'_k (w'_k d'_{nk})^{-\theta} / \sum_j T_j (w_j d_{nj})^{-\theta}} \]

\[ = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta} T_k (w_k d_{nk})^{-\theta} / \sum_j T_j (w_j d_{nj})^{-\theta}} \]

\[ = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}}. \]
Comparative statics (Dekle, Eaton and Kortum, 2008)

- On the other hand, for equilibrium we have
  \[ w'_i L'_i = \sum_n \pi'_ni w'_n L'_n = \sum_n \hat{\pi}_{ni} \pi_{ni} w'_n L'_n \]

- Letting \( Y_n \equiv w_n L_n \) and using the result above for \( \hat{\pi}_{ni} \) we get
  \[ \hat{w}_i \hat{L}_i Y_i = \sum_n \frac{\pi_{ni} \hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}} \hat{w}_n \hat{L}_n Y_n \]

- This forms a system of \( N \) equations in \( N \) unknowns, \( \hat{w}_i \), from which we can get \( \hat{w}_i \) as a function of shocks and initial observables (establishing some numeraire). Here \( \pi_{ni} \) and \( Y_i \) are data and we know \( \hat{d}_{ni}, \hat{T}_i, \hat{L}_i \), as well as \( \theta \).
Comparative statics (Dekle, Eaton and Kortum, 2008)

- To compute the implications for welfare of a foreign shock, simply impose that $\hat{L}_n = \hat{T}_n = 1$, solve the system above to get $\hat{w}_i$ and get the implied $\hat{\pi}_{nn}$ through

$$\hat{\pi}_{ni} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}}.$$

and use the formula to get

$$\hat{\omega}_n = \hat{\pi}_{nn}^{-1/\theta}.$$

- Of course, if it is not the case that $\hat{L}_n = \hat{T}_n = 1$, then one can still use this approach, since it is easy to show that in autarky one has $w_n/p_n = \gamma^{-1} T_n^{1/\theta}$, hence in general

$$\hat{\omega}_n = (\hat{T}_n)^{1/\theta} \hat{\pi}_{nn}^{-1/\theta}.$$
Extensions of EK

  - Bertrand competition $\Rightarrow$ variable markups at the firm-level
  - Measured productivity varies across firms $\Rightarrow$ one can use firm-level data to calibrate model

- **Multiple Sectors**: Costinot, Donaldson, and Komunjer (2012)
  - $T_{ik} \equiv$ fundamental productivity in country $i$ and sector $k$
  - One can use EK’s machinery to study pattern of trade, not just volumes

- **Non-homothetic preferences**: Fieler (2011)
  - Rich and poor countries have different expenditure shares
  - Combined with differences in $\theta^k$ across sectors $k$, one can explain pattern of North-North, North-South, and South-South trade