14.581 International Trade
— Lecture 3: Ricardian Theory (II)—

Putting Ricardo to Work

- Ricardian model has long been perceived has useful pedagogic tool, with little empirical content:
  - Great to explain undergrads why there are gains from trade
  - But grad students should study richer models (Feenstra's first textbook had a total of 3 pages on the Ricardian model!)
- Eaton and Kortum (2002) have lead to "Ricardian revival"
  - Same basic idea as in Wilson (1980): Who cares about the pattern of trade for counterfactual analysis?
  - But more structure: Small number of parameters, so well-suited for quantitative work
- **Goals of this lecture:**
  1. Present EK model
  2. Introduce tools for welfare and counterfactual analysis

Basic Assumptions

- N countries, $i = 1, ..., N$
- Continuum of goods $u \in [0, 1]$
- Preferences are CES with elasticity of substitution σ:

$$U_i = \left(\int_0^1 q_i(u)^{(\sigma-1)/\sigma} du \right)^{\sigma/(\sigma-1)},$$

- One factor of production (labor)
- There may also be intermediate goods (more on that later)
- $c_i \equiv$ unit cost of the "common input" used in production of all goods
  - Without intermediate goods, c_i is equal to wage w_i in country i

Basic Assumptions (Cont.)

- Constant returns to scale:
  - $Z_i(u)$ denotes productivity of (any) firm producing u in country i
  - $Z_i(u)$ is drawn independently (across goods and countries) from a **Fréchet distribution**:

$$\Pr(Z_i \leq z) = F_i(z) = e^{-T_i z^{-\theta}},$$

    with $\theta > \sigma - 1$ (important restriction, see below)
  - Since goods are symmetric except for productivity, we can forget about index u and keep track of goods through $\boldsymbol{Z} \equiv (Z_1, ..., Z_N)$.
- Trade is subject to iceberg costs $d_{ni} \geq 1$
  - d_{ni} units need to be shipped from i so that 1 unit makes it to n
- All markets are perfectly competitive

- Let $P_{ni}(\boldsymbol{Z}) \equiv c_i d_{ni}/Z_i$ be the unit cost at which country i can serve a good $\boldsymbol{Z}$ to country n and let $G_{ni}(p) \equiv \Pr(P_{ni}(\boldsymbol{Z}) \leq p)$. Then:

$$G_{ni}(p) = \Pr\left(Z_i \geq c_i d_{ni}/p\right) = 1 - F_i(c_i d_{ni}/p)$$

- Let $P_n(\boldsymbol{Z}) \equiv \min\{P_{n1}(\boldsymbol{Z}), ..., P_{nN}(\boldsymbol{Z})\}$ and let $G_n(p) \equiv \Pr(P_n(\boldsymbol{Z}) \leq p)$ be the price distribution in country n. Then:

$$G_n(p) = 1 - \exp[-\Phi_n p^{\theta}]$$

where

$$\Phi_n \equiv \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta}$$

Four Key Results

A - The Price Distribution (Cont.)

- To show this, note that (suppressing notation $\boldsymbol{Z}$ from here onwards)

$$\begin{aligned}\Pr(P_n &\leq p) = 1 - \Pi_i \Pr(P_{ni} \geq p) \\ &= 1 - \Pi_i \left[1 - G_{ni}(p)\right]\end{aligned}$$

- Using

$$G_{ni}(p) = 1 - F_i(c_i d_{ni}/p)$$

then

$$\begin{aligned}1 - \Pi_i \left[1 - G_{ni}(p)\right] &= 1 - \Pi_i F_i(c_i d_{ni}/p) \\ &= 1 - \Pi_i e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}} \\ &= 1 - e^{-\Phi_n p^{\theta}}\end{aligned}$$

Four Key Results

B - The Allocation of Purchases

- Consider a particular good. Country n buys the good from country i if $i = \arg\min\{p_{n1}, ..., p_{nN}\}$. The probability of this event is simply country i's contribution to country n's price parameter Φ_n,

$$\pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}$$

- To show this, note that

$$\pi_{ni} = \Pr\left(P_{ni} \leq \min_{s \neq i} P_{ns}\right)$$

- If $P_{ni} = p$, then the probability that country i is the least cost supplier to country n is equal to the probability that $P_{ns} \geq p$ for all $s \neq i$

Four Key Results

B - The Allocation of Purchases (Cont.)

- The previous probability is equal to

$$\Pi_{s \neq i} \Pr(P_{ns} \geq p) = \Pi_{s \neq i} \left[1 - G_{ns}(p)\right] = e^{-\Phi_n^{-i} p^\theta}$$

where

$$\Phi_n^{-i} = \sum_{s \neq i} T_i \left(c_i d_{ni}\right)^{-\theta}$$

- Now we integrate over this for all possible p's times the density $dG_{ni}(p)$ to obtain

$$\int_0^\infty e^{-\Phi_n^{-i} p^\theta} T_i \left(c_i d_{ni}\right)^{-\theta} \theta p^{\theta-1} e^{-T_i (c_i d_{ni})^{-\theta} p^\theta} dp$$

$$= \left(\frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\Phi_n}\right) \int_0^\infty \theta \Phi_n e^{-\Phi_n p^\theta} p^{\theta-1} dp$$

$$= \pi_{ni} \int_0^\infty dG_n(p) dp = \pi_{ni}$$

Four Key Results

B - The Allocation of Purchases (Cont.)

- Close connection between EK and McFadden's logit model
- Take heteorogeneous consumers, indexed by u, with utility $U_n(u)$ from consuming good i:

$$U_i(u) = U_i - p_i + \varepsilon_i(u)$$

with $\varepsilon_i(u)$ i.i.d from **Gumbel distribution:**

$$\Pr(\varepsilon_i(u) \leq \varepsilon) = \exp(-\exp(-\theta\varepsilon))$$

- **Logit:** for each *consumer* u, choose *good* i that maximizes $U_i(u) \Rightarrow$

$$\pi_i = \frac{\exp[\theta(U_i - p_i)]}{\sum_j \exp[\theta(U_j - p_j)]}$$

- **EK:** for each *good* u, choose *source country* i that minimizes $\ln p_i(u) = \ln c_i - \ln Z_i(u)$. Then ln(**Fréchet**) =**Gumbel** $\Rightarrow$

$$\pi_i = \frac{\exp[\theta(-\ln c_i)]}{\sum_j \exp[\theta(-\ln c_j)]} = \frac{c_i^{-\theta}}{\sum_j c_j^{-\theta}}$$

- The price of a good that country n actually buys from any country i also has the distribution $G_n(p)$.
- To show this, note that if country n buys a good from country i it means that i is the least cost supplier. If the price at which country i sells this good in country n is q, then the probability that i is the least cost supplier is

$$\Pi_{s\neq i}\Pr(P_{ni}\geq q)=\Pi_{s\neq i}\left[1-G_{ns}(q)\right]=e^{-\Phi_n^{-i}q^{\theta}}$$

- The joint probability that country i has a unit cost q of delivering the good to country n **and** is the the least cost supplier of that good in country n is then

$$e^{-\Phi_n^{-i}q^{\theta}}dG_{ni}(q)$$

C - The Conditional Price Distribution (Cont.)

- Integrating this probability $e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q)$ over all prices $q \leq p$ and using $G_{ni}(q) = 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^\theta}$ then

$$\int_0^p e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q)$$
$$= \int_0^p e^{-\Phi_n^{-i} q^\theta} \theta T_i (c_i d_{ni})^{-\theta} q^{\theta - 1} e^{-T_i (c_i d_{ni})^{-\theta} p^\theta} dq$$
$$= \left(\frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \right) \int_0^p e^{-\Phi_n q^\theta} \theta \Phi_n q^{\theta - 1} dq$$
$$= \pi_{ni} G_n(p)$$

- Given that $\pi_{ni} \equiv$ probability that for any particular good country i is the least cost supplier in n, then conditional distribution of the price charged by i in n for the goods that i actually sells in n is

$$\frac{1}{\pi_{ni}} \int_0^p e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q) = G_n(p)$$

Four Key Results

C - The Conditional Price Distribution (Cont.)

- In Eaton and Kortum (2002):
  1. All the adjustment is at the extensive margin: countries that are more distant, have higher costs, or lower $T's$, simply sell a smaller range of goods, but the average price charged is the same.
  2. The share of spending by country n on goods from country i is the same as the probability π_{ni} calculated above.
- We will establish a similar property in models of monopolistic competition with Pareto distributions of firm-level productivity

- The exact price index for a CES utility with elasticity of substitution $\sigma < 1 + \theta$, defined as

$$p_n \equiv \left(\int_0^1 p_n(u)^{1-\sigma} du \right)^{1/(1-\sigma)},$$

is given by

$$p_n = \gamma \Phi_n^{-1/\theta}$$

where

$$\gamma = \left[\Gamma \left(\frac{1-\sigma}{\theta} + 1 \right) \right]^{1/(1-\sigma)},$$

where Γ is the Gamma function, *i.e.* $\Gamma(a) \equiv \int_0^\infty x^{a-1} e^{-x} dx$.

Four Key Results

D - The Price Index (Cont.)

- To show this, note that

$$\begin{aligned} p_n^{1-\sigma} &= \int_0^1 p_n(u)^{1-\sigma} du = \\ \int_0^\infty p^{1-\sigma} dG_n(p) &= \int_0^\infty p^{1-\sigma} \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta} dp. \end{aligned}$$

- Defining $x = \Phi_n p^\theta$, then $dx = \Phi_n \theta p^{\theta-1}$, $p^{1-\sigma} = (x/\Phi_n)^{(1-\sigma)/\theta}$, and

$$\begin{aligned} p_n^{1-\sigma} &= \int_0^\infty (x/\Phi_n)^{(1-\sigma)/\theta} e^{-x} dx \\ &= \Phi_n^{-(1-\sigma)/\theta} \int_0^\infty x^{(1-\sigma)/\theta} e^{-x} dx \\ &= \Phi_n^{-(1-\sigma)/\theta} \Gamma\left(\frac{1-\sigma}{\theta} + 1\right) \end{aligned}$$

- This implies $p_n = \gamma \Phi_n^{-1/\theta}$ with $\frac{1-\sigma}{\theta} + 1 > 0$ or $\sigma - 1 < \theta$ for gamma function to be well defined

Equilibrium

- Let X_{ni} be total spending in country n on goods from country i
- Let $X_n \equiv \sum_i X_{ni}$ be country n's total spending
- We know that $X_{ni}/X_n = \pi_{ni}$, so

$$X_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_j T_j(w_j d_{nj})^{-\theta}} X_n \qquad (*)$$

- Suppose that there are no intermediate goods so that $c_i = w_i$.
- In equilibrium, total income in country i must be equal to total spending on goods from country i so

$$w_i L_i = \sum_n X_{ni}$$

- Trade balance further requires $X_n = w_n L_n$ so that

$$w_i L_i = \sum_n \frac{T_i(w_i d_{ni})^{-\theta}}{\sum_j T_j(w_j d_{nj})^{-\theta}} w_n L_n$$

Equilibrium (Cont.)

- This provides system of $N-1$ independent equations (Walras' Law) that can be solved for wages $(w_1, ..., w_N)$ up to a choice of numeraire
- Everything is as if countries were exchanging labor
  - Fréchet distributions imply that labor demands are iso-elastic
  - Armington model leads to similar eq. conditions under assumption that each country is exogenously specialized in a differentiated good
  - In the Armington model, the labor demand elasticity simply coincides with elasticity of substitution σ.
    - See Anderson and van Wincoop (2003)
- Iso-elastic case is what trade economists refer to as a ''gravity model'' with (*)=''gravity equation''
  - We'll come back to gravity models many times in this class

Gains from Trade

- Consider again the case where $c_i = w_i$
- From (*), we know that

$$\pi_{nn} = \frac{X_{nn}}{X_n} = \frac{T_n w_n^{-\theta}}{\Phi_n}$$

- We also know that $p_n = \gamma \Phi_n^{-1/\theta}$, so **real wages** are equal to

$$\omega_n \equiv w_n / p_n = \gamma^{-1} T_n^{1/\theta} \pi_{nn}^{-1/\theta}.$$

- Under autarky we have $\omega_n^A = \gamma^{-1} T_n^{1/\theta}$, hence the **gains from trade** are given by

$$GT_n \equiv \omega_n / \omega_n^A = \pi_{nn}^{-1/\theta}$$

- Trade elasticity θ and share of expenditure on domestic goods π_{nn} are sufficient statistics to compute GT

Gains from Trade (Cont.)

- A typical value for π_{nn} (manufacturing) is 0.7. With $\theta = 5$ this implies $GT_n = 0.7^{-1/5} = 1.074$ or 7.4% gains. Belgium has $\pi_{nn} = 0.2$, so its gains are $GT_n = 0.2^{-1/5} = 1.38$ or 38%.
- One can also use the previous approach to measure the welfare gains associated with any foreign shock, not just moving to autarky:

$$\omega_n' / \omega_n = \left(\pi_{nn}' / \pi_{nn} \right)^{-1/\theta}$$

- For more general counterfactual scenarios, however, one needs to know both π_{nn}' and π_{nn}.

Adding an Input-Output Loop

- Imagine that intermediate goods are used to produce a composite good with a CES production function with elasticity $\sigma > 1$. This composite good can be either consumed or used to produce intermediate goods (input-output loop).
- Each intermediate good is produced from labor and the composite good with a Cobb-Douglas technology with labor share β. We can then write $c_i = w_i^{\beta} p_i^{1-\beta}$.

Adding an Input-Output Loop (Cont.)

- The analysis above implies

$$\pi_{nn} = \gamma^{-\theta} T_n \left(\frac{c_n}{p_n}\right)^{-\theta}$$

and hence

$$c_n = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n$$

- Using $c_n = w_n^{\beta} p_n^{1-\beta}$ this implies

$$w_n^{\beta} p_n^{1-\beta} = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n$$

so

$$w_n / p_n = \gamma^{-1/\beta} T_n^{-1/\theta\beta} \pi_{nn}^{-1/\theta\beta}$$

- The gains from trade are now

$$\omega_n / \omega_n^A = \pi_{nn}^{-1/\theta\beta}$$

- Standard value for β is $1/2$ (Alvarez and Lucas, 2007). For $\pi_{nn} = 0.7$ and $\theta = 5$ this implies $GT_n = 0.7^{-2/5} = 1.15$ or 15% gains.

Adding Non-Tradables

- Assume now that the composite good cannot be consumed directly.
- Instead, it can either be used to produce intermediates (as above) or to produce a consumption good (together with labor).
- The production function for the consumption good is Cobb-Douglas with labor share α.
- This consumption good is assumed to be non-tradable.

Adding Non-Tradables (Cont.)

- The price index computed above is now p_{gn}, but we care about $\omega_n \equiv w_n / p_{fn}$, where

$$p_{fn} = w_n^{\alpha} p_{gn}^{1-\alpha}$$

- This implies that

$$\omega_n = \frac{w_n}{w_n^{\alpha} p_{gn}^{1-\alpha}} = \left(w_n / p_{gn}\right)^{1-\alpha}$$

- Thus, the gains from trade are now

$$\omega_n / \omega_n^A = \pi_{nn}^{-\eta/\theta}$$

  where

$$\eta \equiv \frac{1-\alpha}{\beta}$$

- Alvarez and Lucas argue that $\alpha = 0.75$ (share of labor in services). Thus, for $\pi_{nn} = 0.7$, $\theta = 5$ and $\beta = 0.5$, this implies $GT_n = 0.7^{-1/10} = 1.036$ or 3.6% gains

Comparative statics (Dekle, Eaton and Kortum, 2008)

- Go back to the simple EK model above ($\alpha = 0$, $\beta = 1$). We have

$$X_{ni} = \frac{T_i (w_i d_{ni})^{-\theta} X_n}{\sum_{i=1}^{N} T_i (w_i d_{ni})^{-\theta}}$$

$$\sum_n X_{ni} = w_i L_i$$

- As we have already established, this leads to a system of non-linear equations to solve for wages,

$$w_i L_i = \sum_n \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_k T_k (w_k d_{nk})^{-\theta}} w_n L_n .$$

Comparative statics (Dekle, Eaton and Kortum, 2008)

- Consider a shock to labor endowments, trade costs, or productivity. One could compute the original equilibrium, the new equilibrium and compute the changes in endogenous variables.
- But there is a simpler way that uses only information for observables in the initial equilibrium, trade shares and GDP; the trade elasticity, θ; and the exogenous shocks. First solve for changes in wages by solving

$$\hat{w}_i \hat{L}_i Y_i = \sum_n \frac{\pi_{ni} \hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta}} \hat{w}_n \hat{L}_n Y_n$$

and then get changes in trade shares from

$$\hat{\pi}_{ni} = \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta}}.$$

- From here, one can compute welfare changes by using the formula above, namely $\hat{\omega}_n = \left(\hat{\pi}_{nn}\right)^{-1/\theta}$.

Comparative statics (Dekle, Eaton and Kortum, 2008)

- To show this, note that trade shares are

$$\pi_{ni} = \frac{T_i \left(w_i d_{ni}\right)^{-\theta}}{\sum_k T_k \left(w_k d_{nk}\right)^{-\theta}} \text{ and } \pi'_{ni} = \frac{T'_i \left(w'_i d'_{ni}\right)^{-\theta}}{\sum_k T'_k \left(w'_k d'_{nk}\right)^{-\theta}}.$$

- Letting $\hat{x} \equiv x'/x$, then we have

$$\begin{aligned}\hat{\pi}_{ni} &= \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k T'_k \left(w'_k d'_{nk}\right)^{-\theta} / \sum_j T_j \left(w_j d_{nj}\right)^{-\theta}} \\ &= \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta} T_k \left(w_k d_{nk}\right)^{-\theta} / \sum_j T_j \left(w_j d_{nj}\right)^{-\theta}} \\ &= \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta}}.\end{aligned}$$

Comparative statics (Dekle, Eaton and Kortum, 2008)

- On the other hand, for equilibrium we have

$$w_i' L_i' = \sum_n \pi_{ni}' w_n' L_n' = \sum_n \hat{\pi}_{ni} \pi_{ni} w_n' L_n'$$

- Letting $Y_n \equiv w_n L_n$ and using the result above for $\hat{\pi}_{ni}$ we get

$$\hat{w}_i \hat{L}_i Y_i = \sum_n \frac{\pi_{ni} \hat{T}_i \left(\hat{w}_i \hat{d}_{ni} \right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk} \right)^{-\theta}} \hat{w}_n \hat{L}_n Y_n$$

- This forms a system of N equations in N unknowns, $\hat{w}_i$, from which we can get $\hat{w}_i$ as a function of shocks and initial observables (establishing some numeraire). Here π_{ni} and Y_i are data and we know $\hat{d}_{ni}$, $\hat{T}_i$, $\hat{L}_i$, as well as θ.

Comparative statics (Dekle, Eaton and Kortum, 2008)

- To compute the implications for welfare of a foreign shock, simply impose that $\hat{L}_n = \hat{T}_n = 1$, solve the system above to get $\hat{w}_i$ and get the implied $\hat{\pi}_{nn}$ through

$$\hat{\pi}_{ni} = \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni} \right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk} \right)^{-\theta}}.$$

  and use the formula to get

$$\hat{\omega}_n = \hat{\pi}_{nn}^{-1/\theta}$$

- Of course, if it is not the case that $\hat{L}_n = \hat{T}_n = 1$, then one can still use this approach, since it is easy to show that in autarky one has $w_n / p_n = \gamma^{-1} T_n^{1/\theta}$, hence in general

$$\hat{\omega}_n = \left(\hat{T}_n \right)^{1/\theta} \hat{\pi}_{nn}^{-1/\theta}$$

Extensions of EK

- **Bertrand Competition**: Bernard, Eaton, Jensen, and Kortum (2003)
  - Bertrand competition $\Rightarrow$ variable markups at the firm-level
  - Measured productivity varies across firms $\Rightarrow$ one can use firm-level data to calibrate model
- **Multiple Sectors**: Costinot, Donaldson, and Komunjer (2012)
  - $T_i^k \equiv$ fundamental productivity in country i and sector k
  - One can use EK's machinery to study pattern of trade, not just volumes
- **Non-homothetic preferences:** Fieler (2011)
  - Rich and poor countries have different expenditure shares
  - Combined with differences in θ^k across sectors k, one can explain pattern of North-North, North-South, and South-South trade