Today’s Plan

1. Taxonomy of neoclassical trade models
2. Standard Ricardian model: DFS 1977
   1. Free trade equilibrium
   2. Comparative statics
3. Multi-country extensions
4. The origins of cross-country technological differences
In a neoclassical trade model, comparative advantage, i.e. differences in relative autarky prices, is the rationale for trade.

Differences in autarky prices may have two origins:

1. Demand (periphery of the field)
2. Supply (core of the field)

- **Ricardian theory**: Technological differences
- **Factor proportion theory**: Factor endowment differences
In order to shed light on the role of technological and factor endowment differences:

- Ricardian theory assumes only one aggregate factor of production
- Factor proportion theory rules out technological differences across countries

Neither set of assumptions is realistic, but both may be useful depending on the question one tries to answer:

- If you want to understand the impact of the rise of China on real incomes in the US, Ricardian theory is the natural place to start
- If you want to study its effects on the skill premium, more factors will be needed

Note that:

- Technological and factor endowment differences are exogenously given
- No relationship between technology and factor endowments (Skill-biased technological change?)
Consider a world economy with two countries: Home and Foreign. Asterisks denote variables related to the Foreign country. Ricardian models differ from other neoclassical trade models in that there only is one aggregate factor of production.

There can be many (nontradable) factors, but they can all be aggregated into a single composite input.

We denote by:

- \( L \) and \( L^* \) the endowments of labor (in efficiency units) in the two countries
- \( w \) and \( w^* \) the wages (in efficiency units) in the two countries
There is a continuum of goods indexed by $z \in [0, 1]$

Since there are CRS, we can define the (constant) unit labor requirements in both countries: $a(z)$ and $a^*(z)$

$a(z)$ and $a^*(z)$ capture all we need to know about technology in the two countries

W.l.o.g, we order goods such that $A(z) \equiv \frac{a^*(z)}{a(z)}$ is decreasing

- Hence Home has a comparative advantage in the low-$z$ goods
- For simplicity, we'll assume strict monotonicity
Previous supply-side assumptions are all we need to make qualitative predictions about pattern of trade.

Let $p(z)$ denote the price of good $z$ under free trade.

Profit-maximization requires

$$p(z) - wa(z) \leq 0, \text{ with equality if } z \text{ produced at home (1)}$$

$$p(z) - w^*a^*(z) \leq 0, \text{ with equality if } z \text{ produced abroad (2)}$$

**Proposition** There exists $\tilde{z} \in [0, 1]$ such that Home produces all goods $z < \tilde{z}$ and Foreign produces all goods $z > \tilde{z}$.
Standard Ricardian Model

Free trade equilibrium (I): Efficient international specialization

- **Proof:** By contradiction. Suppose that there exists $z' < z$ such that $z$ produced at Home and $z'$ is produced abroad. (1) and (2) imply

\[
\begin{align*}
  p(z) - wa(z) &= 0 \\
  p(z') - wa(z') &\leq 0 \\
  p(z') - w^* a^*(z') &= 0 \\
  p(z) - w^* a^*(z) &\leq 0
\end{align*}
\]

This implies

\[
w a(z) w^* a^*(z') = p(z) p(z') \leq w a(z') w^* a^*(z),
\]

which can be rearranged as

\[
a^*(z') / a(z') \leq a^*(z) / a(z)
\]

This contradicts $A$ strictly decreasing.
Proposition simply states that Home should produce and specialize in the goods in which it has a CA.

Note that:
- Proposition does not rely on continuum of goods
- Continuum of goods + continuity of $A$ is important to derive

$$A(\tilde{z}) = \frac{w}{w^*} \equiv \omega$$

Equation (3) is the first of DFS’s two equilibrium conditions:
- Conditional on wages, goods should be produced in the country where it is cheaper to do so.

To complete characterization of free trade equilibrium, we need look at the demand side to pin down the relative wage $\omega$. 
Consumers have \textbf{identical Cobb-Douglas} pref around the world.

We denote by $b(z) \in (0, 1)$ the share of expenditure on good $z$:

$$b(z) = \frac{p(z)c(z)}{wL} = \frac{p(z)c^*(z)}{w^*L^*}$$

where $c(z)$ and $c^*(z)$ are consumptions at Home and Abroad.

By definition, share of expenditure satisfy: $\int_0^1 b(z) \, dz = 1$.
Let us denote by \( \theta (\tilde{z}) \equiv \int_0^{\tilde{z}} b(z) \, dz \) the fraction of income spent \textit{(in both countries)} on goods produced at Home.

Trade balance requires

\[
\theta (\tilde{z}) \, w^* L^* = [1 - \theta (\tilde{z})] \, wL
\]

LHS \equiv\text{ Home exports; RHS} \equiv\text{ Home imports}

Previous equation can be rearranged as

\[
\omega = \frac{\theta (\tilde{z})}{1 - \theta (\tilde{z})} \left( \frac{L^*}{L} \right) \equiv B(\tilde{z}) \quad (4)
\]

Note that \( B' > 0 \): an increase in \( \tilde{z} \) leads to a trade surplus at Home, which must be compensated by an increase in Home’s relative wage \( \omega \).
Efficient international specialization, Equation (3), and trade balance, (4), jointly determine ($\tilde{z}, \omega$)
Since Ricardian model is a neoclassical model, general results derived in previous lecture hold.

However, one can directly show the existence of gains from trade in this environment.

**Argument:**

- Set $w = 1$ under autarky and free trade.
- Indirect utility of Home representative household only depends on $p(\cdot)$.
- For goods $z$ produced at Home under free trade: no change compared to autarky.
- For goods $z$ produced Abroad under free trade:
  \[ p(z) = w^*a^*(z) < a(z) \]
- Since all prices go down, indirect utility must go up.
What Are the Consequences of (Relative) Country Growth?

Suppose that $L^* / L$ goes up (rise of China):

- $\omega$ goes up and $\tilde{z}$ goes down
- At initial wages, an increase in $L^* / L$ creates a trade deficit Abroad, which must be compensated by an increase in $\omega$
What are the Consequences of (Relative) Country Growth?

- Increase in $L^*/L$ raises indirect utility, i.e. real wage, of representative household at Home and lowers it Abroad:
  - Set $w = 1$ before and after the change in $L^*/L$
  - For goods $z$ whose production remains at Home: no change in $p(z)$
  - For goods $z$ whose production remains Abroad:
    - $\omega \uparrow \Rightarrow w^* \downarrow \Rightarrow p(z) = w^*a^*(z) \downarrow$
  - For goods $z$ whose production moves Abroad:
    - $w^*a^*(z) \leq a(z) \Rightarrow p(z) \downarrow$
  - So Home gains. Similar logic implies welfare loss Abroad

**Comments:**

- In spite of CRS at the industry-level, everything is as if we had DRS at the country-level
- As Foreign’s size increases, it specializes in sectors in which it is relatively less productive (compared to Home), which worsens its terms-of-trade, and so, lowers real GDP per capita
- The flatter the $A$ schedule, the smaller this effect
What are the Consequences of Technological Change?

There are many ways to model technological change:

1. Global uniform technological change: for all \( z \), \( \hat{a}(z) = \hat{a}^*(z) = x > 0 \)
2. Foreign uniform technological change: for all \( z \), \( \hat{a}(z) = 0 \), but \( \hat{a}^*(z) = x > 0 \)
3. International transfer of the most efficient technology: for all \( z \), \( a(z) = a^*(z) \) (Offshoring?)

Using the same logic as in the previous comparative static exercise, one can easily check that:

1. Global uniform technological change increases welfare everywhere
2. Foreign uniform technological change increases welfare everywhere (For Foreign, this depends on Cobb-Douglas assumption)
3. If Home has the most efficient technology, \( a(z) < a^*(z) \) initially, then it will lose from international transfer (no gains from trade)
Suppose that there is $T > 0$ such that:
- Home’s income is equal to $wL + T$,
- Foreign’s income is equal to $w^*L^* - T$

If preferences are identical in both countries, transfers do not affect the trade balance condition:

$$\left[1 - \theta (\tilde{z})\right] (wL + T) - \theta (\tilde{z}) (w^*L^* - T) = T$$

$$\iff \theta (\tilde{z}) w^*L^* = \left[1 - \theta (\tilde{z})\right] wL$$

So there are no terms-of-trade effect

If Home consumption is biased towards Home goods, $\theta (z) > \theta^* (z)$ for all $z$, then transfer further improves Home’s terms-of-trade.

See Dekle, Eaton, and Kortum (2007) for a recent application.
Multi-country extensions

- DFS 1977 provides extremely elegant version of the Ricardian model:
  - Characterization of free trade equilibrium boils down to finding \((\bar{z}, \omega)\) using efficient international specialization and trade balance

- Problem is that this approach does not easily extend to economies with more than two countries
  - In the two-country case, each country specializes in the goods in which it has a CA compared to the other country
  - Who is the other country if there are more than 2?

- Multi-country extensions of the Ricardian model:
  1. Jones (1961)
  2. Costinot (2009)
Multi-country extensions
Jones (1961)

- Assume $N$ countries, $G$ goods
- **Trick:** restrict attention to “Class of Assignments” where
  - each country only produces one good
  - each good is produced by the same number of countries
- Characterize the properties of optimal assignment within a class
- **Main result:**
  *Optimal assignment of countries to goods within a class will minimize the product of their unit labor requirements*
Multi-country extensions

Costinot (2009)

- Assume \( N \) countries, \( G \) goods
- **Trick:** put enough structure on the variation of unit-labor requirements across countries and industries to bring back two-country intuition
- Suppose that:
  - countries \( i = 1, \ldots, N \) countries have characteristics \( \gamma^i \in \Gamma \)
  - goods \( g = 1, \ldots, G \) countries have characteristics \( \sigma^g \in \Gamma \)
- \( a(\sigma, \gamma) \equiv \) unit labor requirement in \( \sigma \)-sector and \( \gamma \)-country
Definition: \( a(\sigma, \gamma) \) is strictly log-submodular if for any \( \sigma > \sigma' \) and \( \gamma > \gamma' \), \( a(\sigma, \gamma) a(\sigma', \gamma') < a(\sigma, \gamma') a(\sigma', \gamma) \).

If \( a \) is strictly positive, this can be rearranged as

\[
a(\sigma, \gamma) / a(\sigma', \gamma) < a(\sigma, \gamma') / a(\sigma', \gamma')
\]

In other words, high-\( \gamma \) countries have a comparative advantage in high-\( \sigma \) sectors.

Example:

In Krugman (1986), \( a(\sigma^s, \gamma^c) \equiv \exp(-\sigma^s \gamma^c) \), where \( \sigma^s \) is an index of good s’s “technological intensity” and \( \gamma^c \) is a measure of country c’s closeness to the world “technological frontier”.
Proposition If \( a(\sigma, \gamma) \) is log-submodular, then high-\( \gamma \) countries specialize in high-\( \sigma \) sectors

Proof: By contradiction. Suppose that there exists \( \gamma > \gamma' \) and \( \sigma > \sigma' \) such that country \( \gamma \) produces good \( \sigma' \) and country \( \gamma' \) produces good \( \sigma \). Then profit maximization implies

\[
\begin{align*}
 p(\sigma') - w(\gamma) a(\sigma', \gamma) &= 0 \\
 p(\sigma) - w(\gamma) a(\sigma, \gamma) &\leq 0 \\
 p(\sigma) - w(\gamma') a(\sigma, \gamma') &= 0 \\
 p(\sigma') - w(\gamma') a(\sigma', \gamma') &\leq 0
\end{align*}
\]

This implies

\[
a(\sigma, \gamma') a(\sigma', \gamma) \leq a(\sigma, \gamma) a(\sigma', \gamma')
\]

which contradicts a log-submodular
Multi-country extensions
Wilson (1980)

- Same as in DFS 1977, but with multiple countries and more general preferences
- **Trick:** Although predicting the exact pattern of trade may be difficult, one does not need to know it to make comparative static predictions
- At the aggregate level, Ricardian model is similar to an exchange-economy in which countries trade their own labor for the labor of other countries
  - Since labor supply is fixed, changes in wages can be derived from changes in (aggregate) labor demand
  - Once changes in wages are known, changes in all prices, and hence, changes in welfare can be derived
Multi-country extension
Eaton and Kortum (2002)

- Same as Wilson (1980), but with functional form restrictions on $a(z)$
- **Trick:** For each country $i$ and each good $z$, they assume that productivity, $1/a(z)$, is drawn from a Fréchet distribution

$$F(1/a) = \exp \left( -T_i a^\theta \right)$$

- Like Wilson (and unlike Jones), no attempt at predicting which goods countries trade:
  - Instead focus on bilateral trade flows and their implications for wages
- Unlike Wilson, trade flows only depends on a few parameters $(T_i, \theta)$
  - Will allow for calibration and counterfactual analysis
- This paper has had a profound impact on the field:
  - We’ll study it in detail in the next lecture
One obvious limitation of the Ricardian model:
Where do productivity differences across countries come from?

For agricultural goods:
Weather conditions (Portuguese vs. English wine)

For manufacturing goods:
Why don’t the most productive firms reproduce their production process everywhere?

“Institutions and Trade” literature offers answer to this question
Institutions as a Source of Ricardian CA

**Basic Idea:**

1. Even if firms have access to same technological know-how around the world, institutional differences across countries may affect how firms will organize their production process, and, in turn, their productivity.
2. If institutional differences affect productivity relatively more in some sectors, than institutions become source of comparative advantage.

**General Theme:**

Countries with “better institutions” tend to be relatively more productive, and so to specialize, in sectors that are more “institutionally dependent.”
Examples of Institutional Trade Theories

1. **Contract Enforcement**

2. **Financial Institutions**

3. **Labor Market Institutions**
   (* denote papers explicitly building on DFS 1977)
A Simple Example
Costinot JIE (2009)

- **Starting point:**
  
  *Division of labor* $\equiv$ key determinant of productivity differences

- **Basic trade-off:**
  
  1. *Gains from specialization*
     
     $\Rightarrow$ vary with *complexity* of production process (sector-specific)
  
  2. *Transaction costs*
     
     $\Rightarrow$ vary with *quality of contract enforcement* (country-specific)

- **Two steps:**
  
  1. *Under autarky*, trade-off between these 2 forces pins down the extent of the division of labor across sectors in each country
  
  2. *Under free trade*, these endogenous differences in the efficient organization of production determine the pattern of trade
A Simple Example
Technological know-how

- 2 countries, one factor of production, and a continuum of goods
- Workers are endowed with 1 unit of labor in both countries
- **Technology (I): Complementarity.** In order to produce each good $z$, a continuum of tasks $t \in [0, z]$ must be performed:

$$q(z) = \min_{t \in [0, z]} [q_t(z)]$$

- **Technology (II): Increasing returns.** Before performing a task, workers must learn how to perform it:

$$l_t(z) = q_t(z) + f_t$$

- For simplicity, suppose that fixed training costs are s.t. $\int_0^z f_t dt = z$
- Sectors differ in terms of **complexity** $z$: the more complex a good is, the longer it takes to learn how to produce it
A Simple Example

Institutional constraints

- Crucial, function of institutions: **contract enforcement**
- Contracts assign tasks to workers
- Better institutions—either formal or informal—increase the probability that workers perform their contractual obligations
- $e^{-\frac{1}{\theta}}$ and $e^{-\frac{1}{\theta^*}}$ denote this probability at Home and Abroad
- Home has **better institutions**: $\theta > \theta^*$:
A Simple Example
Endogenous organization

- In each country and sector $z$, firms choose "division of labor" $N \equiv$ number of workers cooperating on each unit of good $z$
- Conditional on the extent of the division of labor, (expected) unit labor requirements at Home can be expressed as:

$$a(z, N) = \frac{ze^N}{\theta} \left(1 - \frac{z}{N}\right)$$

- In a competitive equilibrium, $N$ will be chosen optimally

$$a(z) = \min_N a(z, N)$$

- Similar expressions hold for $a^*(z, N)$ and $a^*(z)$ Abroad
A Simple Example
The Origins of Comparative Advantage

- **Proposition** If \( \theta > \theta^* \), then \( A(z) \equiv a^*(z) / a(z) \) is decreasing in \( z \)

- From that point on, we can use DFS 1977 to determine the pattern of trade and do comparative statics.

- One benefit of micro-foundations is that they impose some structure on \( A \) as a function of \( \theta \) and \( \theta^* \):
  - So we can ask what will be the welfare impact of institutional improvements at Home and Abroad?

- The same result easily generalizes to multiple countries by setting “\( \gamma^i \equiv \theta^i \)” and “\( \sigma^g \equiv z \)”
  - Key prediction is that \( a(\sigma, \gamma) \) is log-submodular.
Institutional trade theories differ in terms of content given to notions of institutional quality ($\gamma$) and institutional dependence ($\sigma$).

Examples:
1. Matsuyama (2005): $\gamma \equiv \text{“credit access”}; \sigma \equiv \text{“pledgeability”}$
2. Cunat and Melitz (2007): $\gamma \equiv \text{“rigidity labor market”}; \sigma \equiv \text{“volatility”}$

However, institutional trade theories share the same fundamental objective:
Providing micro-foundations for the log-submodularity of $a(\sigma, \gamma)$.

**Key theoretical question:**
Why are high-$\gamma$ countries relatively more productive in high-$\sigma$ sectors?
Non-homothetic preferences: Matsuyama (2000)
- Goods are indexed according to priority
- Home has a comparative advantage in the goods with lowest priority

External economies of scale: Grossman and Rossi-Hansberg (2009), Matsuyama (2011)
- Unit labor requirements depend on total output in a given country-industry
- Like institutional models, $a$ is endogenous, but there is a two-way relationship between trade on productivity