Today’s Plan

1. General Comments
2. Endogenous growth models
3. Semi-endogenous growth models
1. General Comments
• In 14.581, we have considered neoclassical models of growth:
  • Focus on capital accumulation under perfect competition
• In this context, we have emphasized how trade may affect the predictions of close-economy models
  • If there are diminishing marginal returns to capital accumulation in closed economy, opening up to trade may allow countries to escape those through factor price equalitization (Ventura 1997’s growth miracles)
  • If there are no diminishing marginal returns to capital accumulation (AK model), opening up to trade may lead to diminishing marginal returns through terms-of-trade effects (Acemoglu and Ventura 2002)
Today, we will focus on imperfectly competitive models

- Firms earn profits
- Technological progress is profit-motivated

Two basic reasons why globalization may affect growth:

1. Changes the marginal benefit from innovating (if market size goes up, profits from innovating go up as well?)
2. Changes the marginal cost from innovating (if there are more countries, there are more ideas?)
Many economists have a gut instinct that the gains from trade are much larger than those predicted by static models.

- Do static models miss important sources of dynamic gains?

Previous discussion already highlights that key issue will be:

1. Is there too much innovation or too little innovation in the closed economy equilibrium?
2. Does opening up to trade alleviates or worsens the underlying distortion?

Nothing special about dynamics per se.

- Key issue = presence of distortions (similar to markup case)
We will split our discussion of such models into two groups:

1. Endogenous growth models (e.g. Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992))
2. Semi-endogenous growth models (e.g. Jones (1995), Kortum (1997), Segerstrom (1998))

Two key distinctions between the two groups:

1. Market size affects growth rates (endogenous growth) or income levels (semi-endogenous)
2. Policy affects growth rates (endogenous growth) or not (semi-endogenous)
2. Endogenous Growth Models
Two canonical endogenous growth models are:

   - Externality in R&D = the more varieties there are, the cheaper it becomes to create new varieties

   - Externality in R&D = the higher the quality of incumbent firm, the higher the quality of new competitor

Very similar aggregate implications
- CPI may be going down either because consumers get access to more varieties (which they love) or because a fixed set of varieties is becoming of higher quality (which they love)

Here we will focus on expanding variety model
- closer relationship to Dixit Stiglitz (1977) and Krugman (1980)
Assumptions

- Representative household at $t = 0$ has log-preferences
  
  $$ U = \int_0^{+\infty} \exp (-\rho t) \ln c(t) \, dt $$

- Labor is the only factor of production. Labor supply = $l$

- Final good is produced under perfect competition according to:
  
  $$ c(t) = \left( \int_0^{n(t)} x(\omega, t) \frac{\sigma - 1}{\sigma} \, d\omega \right)^{\frac{\sigma}{\sigma - 1}}, \sigma > 1. $$

- Inputs $\omega$ are produced under monopolistic competition according to:
  
  $$ x(\omega, t) = l(\omega, t). $$

- New inputs can be invented with the production function given by:
  
  $$ \frac{\dot{n}(t)}{n(t)} = \eta l^r(t). \quad (1) $$
Closed Economy Equilibrium

- Euler equation implies:
  \[
  \frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \tag{2}
  \]

- Monopolistic competition implies:
  \[
  p(\omega, t) = \frac{\sigma w(t)}{\sigma - 1}. \]

- Accordingly, instantaneous profits are equal to:
  \[
  \pi(\omega, t) = [p(\omega, t) - w(t)] l(\omega, t) = \frac{1}{\sigma - 1} \frac{w(t) l^e(t)}{n(t)}. \tag{3}
  \]

where \( l^e(t) \equiv \int_0^{n(t)} l(\omega, t) \, d\omega \) is total employment in production

- Because of symmetry, we drop index \( \omega \) from now on.
Closed Economy Equilibrium (Continued)

• Asset market equilibrium requires:

\[ r(t) v(t) = \pi(t) + \dot{v}(t). \]  

(4)

The value of a typical input producer at date \( t \) is:

\[ v(t) = \int_{t}^{+\infty} \exp \left( - \int_{t}^{s} r(s') ds' \right) \pi(s) ds. \]

• Free entry of input producers requires:

\[ \eta n(t) v(t) = w(t). \]  

(5)

• Finally, labor market clearing requires:

\[ l^r(t) + l^e(t) = l. \]  

(6)
Proposition In BGP equilibrium, aggregate consumption grows at a constant rate \( g^* = \frac{\eta l}{(\sigma - 1)\sigma} - \frac{\rho}{\sigma}. \)

Proof:

1. In BGP equilibrium: \( r(t) = r^*, l^e(t) = l^e^*, \) and \( l^r(t) = l^r^*. \)
2. From Euler equation, (2), we know that \( g^* \equiv \frac{\dot{c}(t)}{c(t)} = r^* - \rho. \)
3. From asset market clearing, (4), we also know that

\[
    r^* = \frac{\pi(t)}{v(t)} + \frac{\dot{v}(t)}{v(t)} = \frac{\eta(l - l^r^*)}{\sigma - 1} + \frac{\dot{w}(t)}{w(t)} - \frac{\dot{n}(t)}{n(t)}
\]

where the second equality derives from (3), (16), and (6).
4. By our choice of numeraire, \( \frac{\dot{w}(t)}{w(t)} = \frac{\dot{c}(t)}{c(t)} = g^*. \) Step 3 + eq (1) imply:

\[
    r^* = \frac{\eta(l - l^r^*)}{\sigma - 1} + g^* - \eta l^r^*.
\]
Proof (Continued):

5. Aggregate consumption is given by:

\[ c(t) = n^{\frac{\sigma}{\sigma-1}}(t) x(t) = n^{\frac{1}{\sigma-1}}(t) l^e(t). \]

6. In BGP equilibrium, we therefore have:

\[ \frac{\dot{c}(t)}{c(t)} = \left( \frac{1}{\sigma - 1} \right) \times \left( \frac{\dot{n}(t)}{n(t)} \right). \]

7. Using eq (1) and 6, we get:

\[ l^r^* = \frac{(\sigma - 1) g^*}{\eta} \]

8. Using Steps 4 and 7, we get:

\[ r^* = \frac{\eta l}{\sigma - 1} + (1 - \sigma) g^* \]

9. Using Steps 2 and 8, we finally get: \[ g^* = \frac{\eta l}{(\sigma - 1)\sigma} - \frac{\rho}{\sigma} \]
Graphically
A Note on Policy

• Suppose that government subsidizes R&D at rate $s$

• Same equilibrium conditions as before, except free entry given by:

\[ \eta n(t) v(t) = w(t) (1 - s). \]

• Following Steps 1-4, we now have:

\[ r^* = \frac{\eta (l - lr^*)}{\sigma - 1} + g^* - \eta lr^*. \]

• Following Steps 5-9, this leads to new equilibrium growth rate:

\[ g^* = \frac{\eta l}{(\sigma - 1)(\sigma + s(1 - \sigma))} - \frac{\rho}{\sigma + \frac{s}{1-s}} \]

• Positive subsidy to R&D increases long-run growth!
• In 14.581, we have already seen models with learning by doing
• Knowledge spillovers in a country and sector were given by:

\[
\frac{\dot{a}^i_j(t)}{a^i_j(t)} = \eta^i_j l^i_j(t).
\]

• Here equation (1) plays the exact same role
• Positive externality in one sector (here innovation) calls for a subsidy that reallocates workers towards that sector
• Although innovation is now profit-motivated rather than accidental, central economic mechanism remains unchanged
• Suppose that there are two identical countries indexed by \( j = 1, 2 \).

• In order to distinguish the effects of trade from those of technological diffusion, we start from a situation in which:
  
  1. There is no trade in intermediate inputs.
  2. There are knowledge spillovers across countries:

\[
\frac{\dot{n}_j(t)}{n_j(t) + \Psi n_{-j}(t)} = \eta l^r_j(t)
\]

where \( 1 - \Psi \in [0, 1] \equiv \) share of inputs produced in both countries.
In symmetric equilibrium with 2 countries, equation (19) becomes:

\[ \frac{\dot{n}(t)}{n(t)} = \eta (1 + \Psi) l^r (t). \]

All other equilibrium conditions are unchanged. New growth rate:

\[ g_{\text{open}}^* = \frac{\eta (1 + \Psi)}{\sigma (\sigma - 1)} - \frac{\rho}{\sigma} > g_{\text{autarky}}^* \]

Cost of innovation goes down, which raises growth rate
Open economy (II): Flow of Goods

- **Question:**
  What happens when two countries start trading intermediate inputs?

- **Answer:**
  1. Trade eliminates redundancy in R&D ($\Psi \rightarrow 1$), which $\uparrow$ growth rates. Producers now have incentive to not duplicate effort.
  2. However, trade has no further effect on growth rates.

- Formally, all equilibrium conditions remain unchanged
- Intuitively, when the two countries start trading:
  1. Spending $\uparrow$, which $\uparrow$ profits, and so, incentives to invest in R&D. (Market size effect)
  2. But entry of foreign suppliers $\downarrow$ lowers CES price index, which $\downarrow$ profits, and so, incentives to invest in R&D (competition effect).
  3. Under monopolistic competition + CES, 1 and 2 exactly cancel out.
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Answer:

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  3. Under monopolistic competition $+$ CES, 1 and 2 exactly cancel out.
• Previous neutrality result is obviously knife-edge
• Not hard to design endogenous growth models in which trade has a positive impact on growth rates (beyond R&D redundancy):
  1. Start from same expanding variety model, but drop CES, and assume

\[ c(t) = n^\alpha \left( \int_0^{n(t)} x(\omega, t) \frac{\sigma-1}{\sigma} d\omega \right)^{\frac{\sigma}{\sigma-1}} \]

If \( \alpha > 0 \), market size effect dominates. (If \( \alpha < 0 \), it’s the contrary.)

2. Start from a lab-equipment model in which final good rather than labor is used to produce new inputs (so that trade lowers innovation cost).

• But useful benchmark:
  • No systematic relationship between market size and innovation.
  • No presumption that trade will necessarily increase growth
3. Semi-Endogenous Growth Models
Two key relationships in endogenous growth models:

\[
\frac{\dot{c}(t)}{c(t)} = \left( \frac{1}{\sigma - 1} \right) \times \left( \frac{\dot{n}(t)}{n(t)} \right)
\]

\[
\frac{\dot{n}(t)}{n(t)} = \eta l^r(t)
\]

where \( n(t) \) stands more generally for stock of ideas at date \( t \)

Given these two relationships, endogenous growth models imply:

1. Growth increases with R&D employment:

\[
g^* = \frac{\eta}{\sigma - 1} l^r
\]

2. Growth increases with country size (strong scale effect)

\[
g^* = \frac{\eta}{\sigma - 1} \beta l(t)
\]

where \( \beta \) is share of population employed in R&D along BGP
In the data:

- No systematic relationship between growth and country size
- No systematic relationship between growth and R&D employment
- Motivation for semi-endogenous growth models (Jones 1995, 1999)
Basic Idea:

\[
\frac{\dot{c}(t)}{c(t)} = \left( \frac{1}{\sigma - 1} \right) \times \left( \frac{\dot{n}(t)}{n(t)} \right)
\]

\[
\dot{n}(t) = \eta l^r (t) n(t)^\phi
\]

where the parameter governing externality in R&D: \( \phi < 1 \)

Along BGP, two previous relationships imply:

\[
g^* = \frac{\nu}{(\sigma - 1)(1 - \phi)}
\]

where \( \nu \) denotes the growth rate of population

Country size does not affect growth rates, but it still affects income levels (weak scale effects):

\[
c^*(t) = (1 - \beta) \left( \frac{\eta (1 - \phi)}{\nu} \beta l(t) \right)^{\frac{1}{(\sigma - 1)(1 - \phi)}}
\]
What About Country Size - Income Relationship?

Rose (2006): Size really doesn’t matter
Could Trade Help Explain (Lack of) Correlation between Country Size and Income Levels?

Ramondo et al. (2016): International trade = No; Intranational trade = Yes

**Figure 3. Scale Effects, Trade Openness, and Domestic Trade Costs (Relative to US in logs)**

*Note:* R&D-adjusted country size refers to $\phi_n L_n$, where $\phi_n$ is the share of R&D employment and $L_n$ is equipped labor.
Concluding Remarks

• As already mentioned in 14.581: *Ultimately, whether trade has positive or negative effects on growth is an empirical question.*

• In this lecture, we have abstracted from issues related to firm-level heterogeneity and growth (e.g. learning by exporting, technology adoption at the firm-level, technology diffusion).
  
  • We’ll tackle some of those in the next lecture.