MIT 14.582: PhD International Economics II
Sp 2026, Lecture 20: Economic Geography and Urban Economics (Dynamics)

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Dynamics (not just Space!): The Final Frontier?

- Fields of trade/geography have traditionally emphasized static (or perhaps long-run) responses of an economy to shocks
  - Big "N": models feature rich cross-sectional heterogeneity—many countries, regions, sectors, firms, factors
  - But $T = 1$

- That is slowly changing. Possible reasons (?):
  - Longer panel data on regions, sectors, firms, people
  - Empirical studies have increasingly documented just how slow (and interesting, and unequal) factor market adjustment (especially spatial adjustment) can be
  - Improved computational tools for big (N, T) models

- This lecture will cover a quick introduction to the migration side of dynamic modeling via two important papers:
  1. Artuc, Chaudhuri and McLaren (AER, 2010)
  2. Caliendo, Dvorkin and Parro (ECMA, 2019)

Artuc, Chaudhuri and McLaren (AER, 2010)

Basic Ingredients

- We start with a simple model of labor market dynamics
- We model the agent's decision of where to supply labor across markets as a *dynamic discrete choice problem*:
  - In response to shocks, the worker chooses whether to remain where she is or to move to another location
  - If the worker moves, she will pay a migration cost, which has two components:
    - A portion that is the same for all workers making the same move (moving costs, learning costs, etc.)
    - A time-varying idiosyncratic shock (e.g. personal situation)
  - Will refer to it as a "migration cost" but could be inter-sectoral/occupation adjustment cost (as in ACM's application) or even some combination of both spatial and sector/occupation adjustment (as in CDP's application)

Basic Ingredients

- Empirical motivation for idiosyncratic migration shocks:
  - First, gross flows are an order of magnitude larger than net flows, implying large numbers of workers moving in opposite directions at the same time
  - Second, a significant fraction of workers who change jobs voluntarily move to jobs which pay less than the job the worker left behind
- Idiosyncratic shocks imply transitional dynamics in response to location-specific shocks

Agent's Decision Problem

- Agent supplies 1 unit of labor to $n = 1, ..., N$ locations
  - Receives the competitive market wage w_t^n if employed in n
- Assume that the value of an agent in location n at time t given by

$$\mathrm{v}_t^n = \log(w_t^n) + \max_i \left\{ \beta E \left[\mathrm{v}_{t+1}^i \right] - \tau^{n,i} + \epsilon_t^i \right\},$$

  - $\beta \in (0, 1)$ discount factor
  - $\tau^{n,i}$ additive, *time invariant* migration costs to n from i ($\tau^{i,i} = 0$ by normalization)
  - ϵ_t^i are stochastic *i.i.d idiosyncratic* taste shocks
- $E \left[\mathrm{v}_{t+1}^i \right]$ = expectation over ϵ (no other source of uncertainty—i.e. we have idiosyncratic uncertainty but not aggregate uncertainty)

Agents' Decision Problem (Continued)

- Let $V_t^n \equiv E[\mathrm{v}_t^n]$ denote the expected lifetime utility of a household located in n at time t
- Taking expectations across ϵ_t^i in previous expression implies

$$V_t^n = \log(w_t^n) + E\left[\max_i \left\{\beta V_{t+1}^i - \tau^{n,i} + \epsilon_t^i\right\}\right],$$

- Next we impose distributional assumptions on ϵ_t^i to solve for

$$\Phi_t^n \equiv E\left[\max_i \left\{\beta V_{t+1}^i - \tau^{n,i} + \epsilon_t^i\right\}\right]$$

Multinomial Logit

- Idiosyncratic shocks are drawn from Gumbel distribution

$$F(\epsilon) = \exp\left(-\exp\left(-\frac{\epsilon - \bar{\gamma}}{\nu}\right)\right)$$

where $\bar{\gamma}$ is Euler's constant

- Then

$$\Phi_t^n = \nu \log\left[\sum_{i=1}^N \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}\right]$$

- See 14.581 EK lecture for extreme value algebra + relationship between Gumbel and Frechet

Choice Probabilities

- Let $\mu_t^{n,i}$ denote the fraction of workers from location n who choose to move to location i at date t
- This fraction is equal to the probability that a given worker moves from n to i at time t. Formally,

$$\mu_t^{n,i} = \Pr\left(\beta V_{t+1}^i - \tau^{n,i} + \epsilon_t^i \geqslant \max_{h \neq i}\{\beta V_{t+1}^h - \tau^{n,h} + \epsilon_t^h\}\right)$$

- For the case of Gumbel distributed idiosyncratic costs, this is:

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

- See again 14.581 EK lecture for extreme value algebra

Dynamic labor supply system

For a given sequence of wage vectors $\{w_t\}_{t=0}^{\infty}$ (one wage per location at t), and given starting population $\{L_0\}$, the dynamic labor supply system in this model is the sequence of populations in each location $\{L_t\}_{t=0}^{\infty}$ such that $\{L_t, \mu_t, V_t, \}_{t=0}^{\infty}$ are a solution to:

- Expected lifetime utilities satisfy

$$V_t^n = \log w_t^n + \nu \log \left[\sum_{i=1}^N \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

- Fractions of workers reallocating from market n to i satisfy

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

- Population in market n satisfies

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

Welfare

- First, note that the expected value of being in market n can be decomposed into

$$V_t^n = \underbrace{\log w_t^n}_{\text{current wage}} + \underbrace{\beta V_{t+1}^n}_{\text{value of staying}} + \underbrace{E\left[\max_{\{i\}_{i=1}^N}\left\{\beta\left(V_{t+1}^i - V_{t+1}^n\right) - \tau^{n,i} + \nu\,\epsilon_t^i\right\}\right]}_{\text{option value of migration}}$$

- Previous algebra shows that option value of migration is equal to

$$\nu \log\left[\sum_{i=1}^N \exp\left(\beta\left(V_{t+1}^i - V_{t+1}^n\right) - \tau^{n,i}\right)^{1/\nu}\right]$$

- This is revealed by agents' decisions to migrate or not

$$\mu_t^{n,n} = \frac{\exp\left(\beta V_{t+1}^n\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

which we can manipulate to obtain

$$\nu \log \sum_{h=1}^N \exp\left(\beta\left(V_{t+1}^h - V_{t+1}^n\right) - \tau^{n,h}\right)^{1/\nu} = -\nu \log \mu_t^{n,n}$$

Welfare

- Plugging previous expression into the value function, we get

$$V_t^n = \log w_t^n + \beta V_{t+1}^n - \nu \log \mu_t^{n,n} \tag{1}$$

- Iterating this equation forward we obtain

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{w_t^n}{\left(\mu_t^{n,n}\right)^\nu}$$

- Compared to static models, even a shock that reduces (the path of) wages in location n can increase the option value of migration and make workers starting out there better off

- Note (superficial) similarity to ACR formula for gains from trade in trade models (more below).
    - Both formulae apply same underlying math trick...
    - Use CES-like structure to replace hard-to-observe object (price index in ACR, option value of migration in ACM) with statistic based on share choosing the option whose price changes in simple way (intra-country trade cost in ACR, $\tau^{n,n} = 0$ here)

Estimation: Static Case

- How do we estimate ν and the structural residuals $\{\tau^{n,i}\}$?
- Consider first the static case:

$$\mu^{n,i} = \frac{[\exp(\ln w_i - \tau^{n,i})]^{1/\nu}}{\sum_h [\exp(\ln w_h - \tau^{n,h})]^{1/\nu}} \tag{2}$$

- Taking logs, we obtain a "gravity-like" equation for bilateral migration flows:

$$\ln \mu^{n,i} = \delta_n + \frac{1}{\nu} \ln w_i - \frac{1}{\nu} \tau^{n,i}$$

  with $\delta_n = (1/\nu) \ln \left(\sum_h [\exp(\ln w_h - \tau^{n,h})] \right)$
- In a trade context, we would typically "fixed-effect out" both δ_n and $\frac{1}{\nu} \ln w_i$ and use observable shifters of $\tau^{n,i}$, e.g. tariffs
- In a migration context:
  - harder to find bilateral shifters of migration costs...
  - so we need to use variation in wages across destinations
  - and we need a (labor) demand-side instrument for the wage

Estimation: Dynamic Case

- Now let's go back to the dynamic case:

$$\mu_t^{n,i} = \frac{[\exp(\beta V_{i,t+1} - \tau^{n,i})]^{1/\nu}}{\sum_h [\exp(\beta V_{h,t+1} - \tau^{n,h})]^{1/\nu}} \quad (3)$$

$$V_{i,t} = \ln w_{i,t} + \nu \ln \left(\sum_h [\exp(\beta V_{h,t+1} - \tau^{i,h})]^{1/\nu} \right) \quad (4)$$

- Problem: Where to get data on $V_{i,t+1}$? Depends on wages (perhaps observable), but also infinite stream of future wages (less so!)

- Solution: Use the fact that relative expected lifetime utilities at date $t+1$ are revealed by choices of the agent at date t (take ratio of 3 across *ni* and *nn*):

$$\beta(V_{i,t+1} - V_{n,t+1}) - \tau^{n,i} = \nu(\ln \mu_t^{n,i} - \ln \mu_t^{n,n}) \quad (5)$$

Estimation: Dynamic Case

- Lagged version of (5) is:

$$\nu \ln \left(\frac{\mu_{t-1}^{n,i}}{\mu_{t-1}^{n,n}} \right) = \beta (V_{i,t} - V_{n,t}) - \tau^{n,i} \tag{6}$$

- Using (1) this becomes:

$$\begin{aligned} \nu \ln \left(\frac{\mu_{t-1}^{n,i}}{\mu_{t-1}^{n,n}} \right) =& \beta (\ln w_{i,t} - \ln w_{n,t}) + \beta^2 (V_{i,t+1} - V_{n,t+1}) \\ & - \nu\beta (\ln \mu_t^{i,i} - \ln \mu_t^{n,n}) - \tau^{n,i} \end{aligned}$$

- And using (5) to get rid of V this is:

$$\nu \ln \left(\frac{\mu_{t-1}^{n,i}}{\mu_{t-1}^{n,n}} \right) = \beta (\ln w_{i,t} - \ln w_{n,t}) + (\beta - 1)\tau^{n,i} + \nu\beta \ln \left(\frac{\mu_t^{n,i}}{\mu_t^{i,i}} \right) \tag{7}$$

Estimation: Dynamic Case

- Then (7) can be written as

$$\ln \mu_{t-1}^{n,i} = \delta_{n,t} + \frac{\beta}{\nu} \ln w_{i,t} + \beta \ln \left(\frac{\mu_t^{n,i}}{\mu_t^{i,i}} \right) - \frac{1}{\nu}(1-\beta)\tau^{n,i}$$

with $\delta_{n,t} \equiv -\frac{\beta}{\nu} \ln w_{n,t} + \beta \ln \mu_{t-1}^{n,n}$

- This is a lot like our earlier static regression equation, but with the addition of the dynamic correction of $\beta \ln \left(\frac{\mu_t^{n,i}}{\mu_t^{i,i}} \right)$
- ACM (2010) take this to annual CPS data:
  - Assume $\tau^{n,i} = \tau$ and estimate τ (assume that agent's uncertainty about wages creates extra structural residual)
  - Assume $\beta = 0.97$
  - Instruments: lagged endogenous variables (wages and migration shares)
  - Estimate $\widehat{\tau} = 13.2$ and $\widehat{\nu} = 2.9$: mean moving cost is $13\times$ avg. w and SD of idiosyncratic moving cost is $7\times$ avg. w

Caliendo, Dvorkin and Parro (2019)

Caliendo, Dvorkin and Parro (2019)

- CDP take the ACM model and add a full (though static) GE structure—regions/sectors/countries that interact with one another
- GE structure is that of Caliendo-Parro (REStud, 2015):
  - Eaton and Kortum (2002) but with multiple sectors
  - Cobb-Douglas inter-sectoral input-output linkages
  - But static: capital can exist and even change, but any changes must be exogenous ones (endowment shocks)
- Main changes relative to model so far will be that:
  - Agents pay different consumer prices $P_{n,t}$ by location, so value the real wage $\omega_{n,t} \equiv w_{n,t}/P_{n,t}$
  - The real wage is endogenously determined by both labor demand and labor supply (where LS is the result of the above model)
- For ease, imagine that real wage $\omega_{n,t}$ is determined by a function $\omega(\cdot)$ of the labor supply vector L_t and the fundamentals (productivity, capital endowment, trade costs, etc) in the world economy (Θ_t). Denote this $\omega_{n,t} = \omega_n(L_t, \Theta_t)$

Aside: Welfare Revisited

- In one-sector EK (or any ACR-class) model, 14.581 covered how can express welfare as $\omega_{n,t} = (\pi_t^{n,n}/T_t^n)^{-1/\theta}$, where T_t^n is the productivity of region n at t and θ is the trade elasticity
- So using earlier expression for lifetime welfare have

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{(\pi_t^{n,n}/T_t^n)^{-\frac{1}{\theta}}}{(\mu_t^{n,n})^{\nu}}$$

- Summarizes welfare equations in static trade models as in ACR (2010) and dynamic models with exogenous trade as in ACM (2010)
- Sufficient statistic (if observe whole path of $\pi_t^{n,n}$ and $\mu_t^{n,n}$—not easy!) to measure welfare gains from trade and migration relative to a counterfactual scenario of trade autarky $\pi_t^{n,n} = 1$ and no migration $\mu_t^{n,n} = 1$
- See Caliendo, Opromolla, Parro and Sforza (JPE, 2020) for application of these ideas to a study of EU enlargement (reduced barriers to both trade and factor mobility)

Counterfactuals in this model

- Helpful to let $\tilde{\tau}^{n,i} \equiv e^{\tau^{n,i}}$, $u_t^n \equiv e^{V_t^n}$.
- Then can re-write previous system as

$$u_t^n = \omega_{n,t} \left[\sum_i \left(u_{t+1}^i \right)^{\beta/\nu} \left(\tilde{\tau}^{n,i} \right)^{-1/\nu} \right]^{\nu}$$

$$\mu_t^{n,i} = \frac{\left(u_{t+1}^i \right)^{\beta/\nu} \left(\tilde{\tau}^{n,i} \right)^{-1/\nu}}{\sum_{h=1}^N \left(u_{t+1}^h \right)^{\beta/\nu} \left(\tilde{\tau}^{n,h} \right)^{-1/\nu}}$$

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

- Or in proportional changes (denoting $\dot{y}_{t+1} \equiv y_{t+1}/y_t$, etc):

$$\dot{u}_{t+1}^n = \dot{\omega}_{n,t+1} \left[\sum_i \mu_t^{n,i} \left(\dot{u}_{t+2}^i \right)^{\beta/\nu} \right]^{\nu}$$

$$\mu_{t+1}^{n,i} = \frac{\mu_t^{n,i} \left(\dot{u}_{t+2}^i \right)^{\beta/\nu}}{\sum_{h=1}^N \mu_t^{n,h} \left(\dot{u}_{t+2}^h \right)^{\beta/\nu}}$$

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

"Exact hat" counterfactuals in this model

- Previous system is reminiscent of the Dekel-Eaton-Kortum (2008) system, but dynamic
- Indeed, the sub-problem of solving for $\dot{\omega}_{n,t+1} \equiv \omega_n(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$ is exactly covered by DEK—would result from one comparative static calculation in their model, from feeding in an exogenous shock given by $(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$
- Recall from 14.581 that the DEK method is to write all endogenous proportional changes as a function of initial trade and income shares, and the exogenous proportional change shocks $(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$
- CDP show how one can do something similar dynamically in their model

"Exact hat" counterfactuals in this model: 2 steps

- First solve for the ("data from the") baseline equilibrium (since this probably involves the future, so we have to solve for this future "data")
  - Need set of initial (time 0) trade flow and income share data, like in DEK
  - Now also need initial distribution of workers (L_0) and initial change in distribution of workers (μ_{-1})
  - And do need to know set of future exogenous changes that you think will factually happen (all $\dot{\Theta}_{t+1}$ for future t), which needs to be the same set of changes that the agents at 0 know when they are deciding on μ_{-1}
  - Equilibrium is system of nonlinear equations to solve for (just as DEK is for a static model), but it turns out to be a particularly tractable one, despite its size with large N and large T. This seems to be especially true when $\dot{\Theta}_{t+1}$ is a convergent sequence, so the the system will (probably) eventually be in steady-state; hence final period's changes are known to be zero.

- Then can do the counterfactuals.
  - Previous baseline equilibrium solution is the 'initial' data (sequence) that we are then shocking with a genuinely counterfactual path of $\dot{\Theta}_{t+1}$ events

Application: Effects (on U.S.) of productivity boom in China

- U.S. imports from China almost doubled from 2000 to 2007
  - At the same time, manufacturing employment fell while employment in other sectors, such as construction and services, grew

- Several studies document that an important part of the employment loss in manufactures was a consequence of China's trade expansion
  - e.g., Autor, Dorn, and Hanson (2013), Acemoglu, Autor, Dorn, and Hanson (2014), Pierce and Schott (2016)

- CDP use their model to quantify and understand the effects of the rise of China's trade expansion—the "China shock"
  - Initial period is the year 2000
  - Calculate the sectoral, regional, and aggregate employment and welfare effects of the China shock

Identifying the China "shock"

- Follow Autor, Dorn, and Hanson (2013)
  - Estimate

    $$\Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j,$$

    where j is a NAICS sector, $\Delta M_{USA,j}$ and $\Delta M_{other,j}$ are changes in U.S. and other adv. countries, imports from China from 2000 to 2007
  - Find $a_2 = 1.27$

- Obtain predicted changes in U.S. imports with this specification

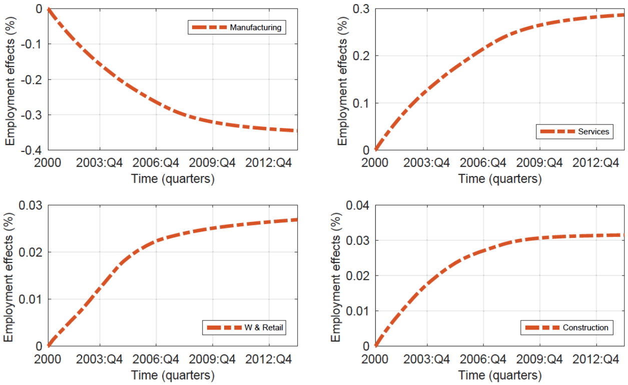
- Use the model to solve for the change in China's 12 manufacturing industries TFP $\left\{\hat{A}^{China,j}\right\}_{j=1}^{12}$ such that model's imports match predicted imports from China from 2000 to 2007
  - With model's generated data obtain $a_2 = 1.52$
  - Feed this into the model $\left\{\hat{A}^{China,j}\right\}_{j=1}^{12}$ by quarter from 2000 to 2007 to study the effects of the shock

Taking the Model to the Data

- 50 U.S. states, 22 sectors + non-empl. and 38 countries
- Need data for $(L_0, \mu_{-1}, \pi_0, VA_0, GO_0)$
  - L_0 : PUMS of the U.S. Census for the year 2000
    - Exclude empl. in farming, mining, utilities, and public sect.
  - μ_{-1} : Use CPS to compute intersectoral mobility and ACS to compute interstate mobility
  - π_0 : CFS and WIOD year 2000
  - VA_0 and GO_0 : BEA VA shares and U.S. IO, WIOD for other countries
- Need values for parameters (ν, θ, β)
  - θ : Use Caliendo and Parro (2015)
  - $\beta = 0.99$
  - $\nu = 5.34$ (implied elasticity of 0.2) using ACM's data and specification, adapted to CDP's model

Employment Effects

Figure: The effect of the China shock on employment shares



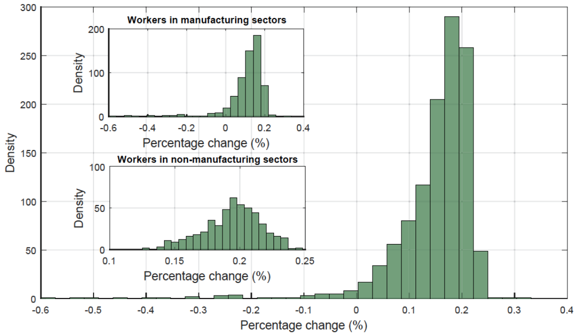
- Chinese competition reduced the share of manufacturing employment by 0.36% in the long run, ∼0.55 million employment loss

Employment Effects: Manufacturing

- Unequal sectoral effects of Chinese import competition
  - 1/2 of the decline in manufacturing employment originated in the Computer & Electronics and Furniture sectors
    - 1/4 of the decline comes from the Metal and Textiles sectors
  - Food, Beverage and Tobacco, gained employment
    - Less exposed to China, benefited from cheaper intermediate goods, other sectors, like Services, demanded more of them
- Unequal regional effects
  - Regions with a larger concentration of sectors that are more exposed to China lose more jobs
    - California, the region with largest share of employment in Computer & Electronics, contributed to 12% of the decline

Welfare Effects across Labor Markets

Figure: Welfare effects of the China shock across U.S. labor markets



- Very heterogeneous response to the same aggregate shock
- But most labor markets gain as a consequence of cheaper imports