14.581 International Trade
— Lecture 18: Aggregate Fluctuations —

Today's Plan

1. A Refresher on Growth Accounting
2. Asymptotic Results
   - Gabaix (ECTA, 2011)
   - Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (ECTA, 2012)
3. Micro to Macro Accounting
   - di Giovanni, Levchenko and Mejean (ECTA, 2014)

1. A Refresher on Growth Accounting

What is an Aggregate Productivity Shock?

- Consider an efficient economy with real GDP given by:

$$
\begin{aligned}
Y(L, K, z) = \max_{c,l,x} & \; U(c_1, ..., c_n) \\
s.t. : & c_i + \sum_j x_{ij} \leq z_i f_i(l_i, k_i, x_{1i}, ..., x_{ni}) \\
& \sum_i l_i \leq L \\
& \sum_i k_i \leq K
\end{aligned}
$$

  where L and K are labor and capital stocks
- The Envelope Theorem implies:

$$
dY = \sum_i \mu_i f_i(l_i, k_i, x_{1i}, ..., x_{ni}) dz_i + \lambda_L dL + \lambda_K dK
$$

  where:
  - μ_i = lagrange multiplier associated with good constraint (price of good i)
  - λ_L = lagrange multiplier associated with labor constraint (price of labor)
  - λ_K = lagrange multiplier associated with capital constraint (price of capital)

What is an Aggregate Productivity Shock?

- Taking logs:

$$d \ln Y = s_L d \ln L + s_K d \ln K + \sum_i v_i d \ln z_i$$

  where:
  - $s_L = \frac{\lambda_L L}{Y}$ = labor share
  - $s_K = \frac{\lambda_K K}{Y}$ = capital share
  - $v_i = \frac{\mu_i z_i f_i(l_i, k_i, x_{1i}, \ldots, x_{ni})}{Y}$ = sector i "share" (numerator is *gross output*, not value added. So $\sum_i v_i$ may not be equal to one.)
- Let us now define an aggregate productivity shock, $d \ln Z$, as the percentage change in real GDP, holding primary factors fixed:

$$d \ln Z = d \ln Y - s_L d \ln L - s_K d \ln K$$

What is an Aggregate Productivity Shock?

- By definition, $d \ln Z$ is the Solow residual
- And from our previous algebra, we immediately get:

$$d \ln Z = \sum_i v_i d \ln z_i$$

- Up to a first-order approximation, changes in aggregate productivity are equal to the average of "good-specific" productivity shocks
- This specific application of the Envelope Theorem is often referred to as Hulten's (RES, 1978) Theorem
  - One can also relax no-joint productiion and Hicks-neutral technical change
  - Restriction to two primary factors plays no role. By reinterpreting goods and factors, one can also study trade costs shocks in a world economy. See e.g. Burstein and Cravino (AEJ, 2015)

2. Asymptotic Results

Gabaix (2011)

The Standard Case for the Irrelevance of Idiosyncratic Shocks

- From now on, let us ignore variation in primary factors so that aggregate volatility—meaning the volatility of log GDP—is equal to the volatility of aggregate productivity—meaning the volatility of $d \ln Z$:

$$\sigma_{GDP} = \sigma_Z$$

- Suppose that:
  - all sectors/firms are initially of identical size $v_i = S_i/S = 1/n$
  - all sector/firm shocks $d \ln z_i$ are independent with standard deviation $\sigma_{z_i} = \sigma$
- Then from Hulten's Theorem, we get:

$$\sigma_{GDP} = \sqrt{\sum_i (v_i)^2 \sigma_{z_i}^2} = \frac{\sigma}{\sqrt{n}}$$

Gabaix (2011)

The Standard Case for the Irrelevance of Idiosyncratic Shocks

- Gabaix (2011) notes that in the US data:
  - $\sigma = 12\%$
  - $n = 10^6$
- This simple calibration leads to:

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{n}} = 0.012\%$$

- From this expression, the "conventional wisdom"—e.g., as articulated by Lucas (*Theories of Business Cycles*, 1984)—can be understood: σ_{GDP} is trivial—*no aggregate fluctuations without aggregate shocks.*

- More generally, as long as the law of large number applies, we should expect the σ_{GDP} to decrease at the same rate as $1/\sqrt{n}$
- **Proposition**: *Consider an economy with n firms whose sizes S_i are drawn from a distribution with finite variance. Suppose that they all have the same volatility σ. Then, as $n \to \infty$, σ_{GDP} follows:*

$$\sigma_{GDP} \sim \frac{E[S^2]^{1/2}}{E[S]} \frac{\sigma}{\sqrt{n}}$$

- **Proof**:
  1. From Hulten's Theorem, we know that:
  $$\sigma_{GDP} = \sigma h$$
  with $h = \sqrt{\sum_i (v_i)^2}$ and $v_i = S_i / \sum_i (S_i)$ in Gabaix's endowment economy
  2. Let us rearrange the herfindahl index h as
  $$n^{1/2} h = \frac{(n^{-1} \sum_i (S_i)^2)^{1/2}}{n^{-1} \sum_i (S_i)}$$
  3. Applying the LLN to the numerator and denominator, we get
  $$n^{1/2} h \rightarrow \frac{E[S^2]^{1/2}}{E[S]}$$
  4. Proposition follows from 1 and 3.

Gabaix (2011)

But firm size distribution has fat tails.

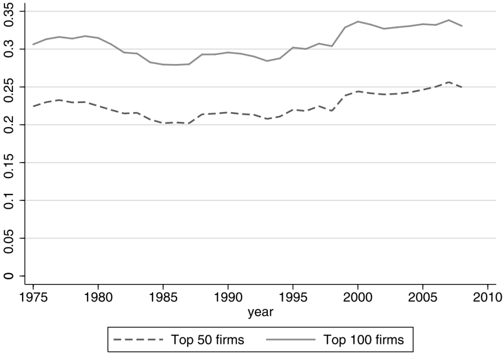


FIGURE 1.—Sum of the sales of the top 50 and 100 non-oil firms in Compustat, as a fraction of GDP. Hulten's theorem (Appendix B) motivates the use of sales rather than value added.

- **Proposition**: *Consider an economy with n firms whose sizes are drawn from a power law distribution*

$$P(S > x) = ax^{-\zeta}$$

with exponent $\zeta \geq 1$. Suppose that firms all have the same volatility σ. Then, as $n \to \infty$ goes to infinity, σ_{GDP} follows:

$$\sigma_{GDP} \sim \frac{v_\zeta}{\ln n}\sigma \text{ for } \zeta = 1$$
$$\sigma_{GDP} \sim \frac{v_\zeta}{n^{1-1/\zeta}}\sigma \text{ for } 1 < \zeta < 2$$
$$\sigma_{GDP} \sim \frac{v_\zeta}{n^{1/2}}\sigma \text{ for } \zeta \geq 2$$

Comments

- One can think of previous asymptotic results as providing a simple theory of distribution of firm size, i.e. a theory of $\{v_i\}$
- But alternatively, one could simply look at $\{v_i\}$ in the data
- Recall that from Hulten's Theorem, we get:

$$\sigma_{GDP} = \sigma h$$

  where h is the herfindahl of the economy
- In the US data, $h = 5.3\%$. With $\sigma = 12\%$, we therefore immediately get:

$$\sigma_{GDP} = \sigma h = 0.63\% \gg 0.012\%$$

Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012)

- Simple model of input-output linkages.
- Static, perfectly competitive economy with n industries.
- Cobb-Douglas production function for each industry:

$$y_i = z_i l_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{a_{ij}}, \quad i \in \{1, .., n\}$$

- Cobb-Douglas utility:

$$u(c_1, c_2, ..., c_n) = \prod_{i=1}^{n} c_i^{1/n},$$

Aggregate Implications

- Since the model is log-linear, first-order approximation is exact.
- Log GDP satisfies:

$$y \equiv \log(GDP) = \mathbf{v}' \ln \mathbf{z},$$

where v_i still represents the "share" of sector i

$$v_i = \frac{p_i x_i}{\alpha \sum_{j=1}^{n} p_j x_j}.$$

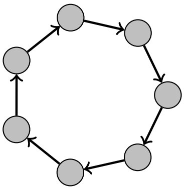
- $\mathbf{v}$ also corresponds to the **influence vector** or the vector of **Bonacich centrality** indices defined as

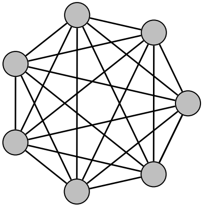
$$v_i \equiv \sum_{j=1}^{n} \frac{1}{n} h_{ji},$$

where h_{ij} denotes entries of the Leontief inverse.

Irrelevance of Micro Shocks Redux

- We say that the network is **regular** if $d_i = d$ for each i, where $d_i = \sum_{j=1}^{n} a_{ji}$.
  - Each sector has a similar degree of importance as a supplier to other sectors.
- Examples of regular networks:
  - **rings:** the most "*sparse*" input-output matrix, where each sector grows all of its inputs from a single other sector.
  - **complete graphs**: where each sector equally draws inputs from all other sectors.





Irrelevance of Micro Shocks Redux

- Suppose again that $\sigma_i = \sigma$ for each i.
- Then we have that for all regular networks:

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{n}}$$

(see also Dupor, *Journal of Monetary Economics*,1999).

- Conditional on sales, the shape of the network does not matter
  - In all regular networks, sales are equal across firms
  - Rings—which one might have conjectured to be prone to "domino effects"–do not lead to higher aggregate volatility
- For the reasons discussed before, this implies that idiosyncratic shocks have a negligible effect on aggregate volatility in regular networks

Generalization

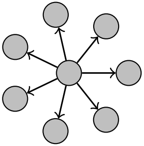
- We say that a sequence of economies is **balanced** if $\max_i d_i < c$ for some c.
- This is clearly much weaker than regularity.
- It can be shown that, for any sequence of balanced economies,

$$\sigma_{agg} \sim \frac{1}{\sqrt{n}}.$$

- Once again rings and complete networks are equally stable
  - sparseness of the input-output matrix has little to do with aggregate volatility
  - sales are the sufficient statistics

Asymmetric Networks Are Fragile

- However, network irrelevance is not generally correct, provided that network leads to very asymmetric distribution of sales.
- The extreme example is the **star network**, with degrees summing to $1 - \alpha$:



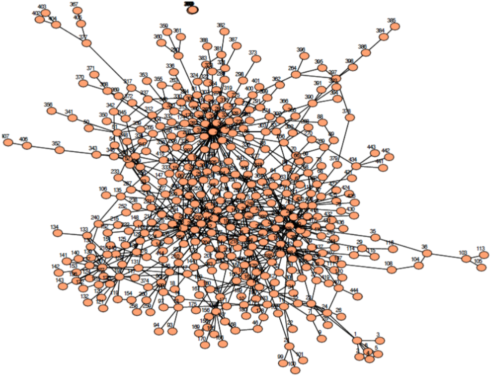
Asymmetric Networks Are Fragile (continued)

- In fact, it can be shown that the highest level of aggregate volatility is generated by the **star network** and is equal to

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{1 - \left(\frac{n-1}{n}\right)\alpha\,(1-\alpha)}},$$

  which is much greater than $\sigma/\sqrt{n}$ when n is large.
- In fact, this is not just high volatility, but **systemic volatility** ($\approx$ "system-wide" volatility: shocks to the central sector spread to the rest, creating system-wide co-movement—we return to systemic volatility below.

What Does the US Input-Output Network Look Like?



- Intersectoral network corresponding to the US input-output matrix in 1997. For every input transaction above 5% of the total input purchases of the destination sector, a link between two vertices is drawn.

More Asymptotic Results

- To obtain sharper theoretical results, consider a sequence of economies with input-output matrix A_n and $n \to \infty$.
- So we will be looking at "*law of large numbers*"-type results.
- Suppose that $\sigma_i \in (\underline{\sigma}, \overline{\sigma})$.
- Then the greatest degree of "*stability*" or "*robustness*" (least systemic risk) corresponds to

$$\sigma_{GDP} \sim 1/\sqrt{n}$$

  (as in standard law of large numbers for independent variables).
- Define the **coefficient of variation of degrees** (of an economy with n sectors) as

$$\mathrm{CV}_n \equiv \frac{1}{d_{\mathrm{avg}}} \left[\frac{1}{n-1} \sum_{i=1}^{n} (d_i - d_{\mathrm{avg}}) \right]^{1/2},$$

  where $d_{\mathrm{avg}} = \frac{1}{n} \sum_i d_i$ is the average degree.

First-Order Results

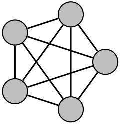
- Just considering the first-order downstream impacts,

$$\sigma_{GDP} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\mathrm{CV}_n}{\sqrt{n}}\right).$$

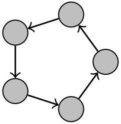
  The Ω notation implies $\sigma_{GDP} \to 0$ as $n \to 0$ no faster than $\frac{1+\mathrm{CV}_n}{\sqrt{n}}$.

- For regular networks, $\mathrm{CV}_n = 0$, so $\sigma_{GDP} \to 0$ could (should) go to zero at the rate $\frac{1}{\sqrt{n}}$.
- For the star network, $\mathrm{CV}_n \not\to 0$ as $n \to 0$, so $\sigma_{GDP} \not\to 0$ and the law of large numbers fails.

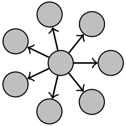
$c_n = \Omega(b_n) \iff \liminf_{n\to\infty} c_n / b_n > 0$



$\mathrm{CV}_n = 0$



$\mathrm{CV}_n = 0$



$\mathrm{CV}_n \sim \sqrt{n}$

First-Order Results (continued)

- We can also make these results easier to apply.
- We say that the degree distribution for a sequence of economies has **power law tail** if, there exists $\beta > 1$ such that for each n and for large k,

$$P_n(k) \propto k^{-\beta},$$

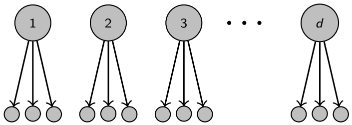
  where $P_n(k)$ is the counter-cumulative distribution of degrees and β is the shape parameter.
- It can be shown that if a sequence of economies has power law tail with shape parameter $\beta \in (1, 2)$, then

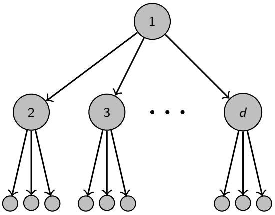
$$\sigma_{GDP} = \Omega\left(n^{-\frac{\beta-1}{\beta}-\varepsilon}\right)$$

  where $\varepsilon > 0$ is arbitrary.
- A smaller β corresponds to a "thicker" tail and thus higher coefficient of variation, and greater fragility.

Higher-Order Results

- In the same way that first-order downstream effects do not capture the full implications of negative shocks to a sector, the degree distribution does not capture the full extent of asymmetry/inequality of "connections".
- Two economies with the same degree distribution can have very different structures of connections and very different nature of volatility:



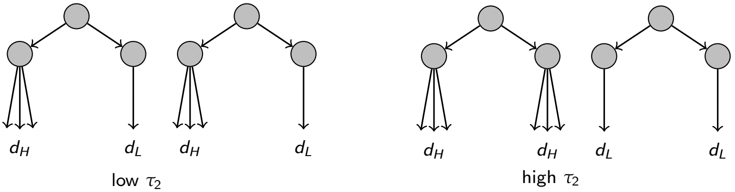


Higher-Order Results (continued)

- We define the **second-order interconnectivity coefficient** as

$$\tau_2(A_n) \equiv \sum_{i=1}^{n} \sum_{j \neq i} \sum_{k \neq i,j} a_{ji} a_{ki} d_j d_k .$$

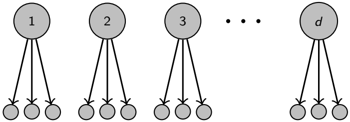
- This will be higher when high degree sectors share “upstream parents”:



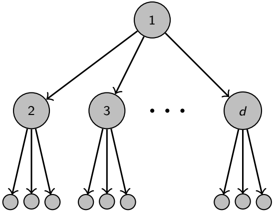
Higher-Order Results (continued)

- It can be shown that

$$\sigma_{GDP} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\mathrm{CV}_n}{\sqrt{n}} + \frac{\sqrt{\tau_2(A_n)}}{n}\right).$$



$\tau_2 = 0$



$\tau_2 \sim n^2$

Higher-Order Results (continued)

- Define **second-order degree** as

$$q_i \equiv \sum_{j=1}^{n} d_j a_{ji}.$$

- For a sequence of economies with a power law tail for the second-order degree with shape parameter $\zeta \in (1, 2)$, we have

$$\sigma_{GDP} = \Omega\left(n^{-\frac{\zeta-1}{\zeta}-\varepsilon}\right),$$

  for any $\varepsilon > 0$.

- If both first and second-order degrees have power laws, then

$$\sigma_{GDP} = \Omega\left(n^{-\frac{\zeta-1}{\zeta}-\varepsilon} + n^{-\frac{\beta-1}{\beta}}\right),$$

  i.e., dominant term: $\min\{\beta, \zeta\}$.

3. Micro to Macro Accounting

Di Giovanni, Levchenko, and Mejean (2014)

- **Question:** What is the role of individual firms in generating aggregate fluctuations of French sales growth over 1990–2007?
- **Strategy:**
  1. Decompose a *firm's* annual sales growth into: i) Firm and firm-destination shock ("micro"), ii) Sector-destination shocks ("macro")
  2. Use estimates to measure the contribution of the firm component to *aggregate* fluctuations (measured by variance of aggregate sales growth)
  3. Relate the contribution of the firm component to the firm size concentration and the interconnection between firms

Overview of Results

1. More than 90% of the variance in firm-level growth rates accounted by the firm-destination component
2. Around 40% of aggregate growth rate accounted by by firm component, both for the manufacturing sector and for the whole economy
3. Contribution of firm-component is larger for fluctuations in aggregate exports
4. The volatility of the firm-specific component is correlated with the distribution of firm size and the magnitude of IO linkages

Aggregate Growth

- Total aggregate sales by all French firms:

$$X_t = \sum_{f,n} x_{fnt},$$

where x_{fnt} is firm f's sales to destination n at time t

- Let $\gamma_{At} = \ln X_t - \ln X_{t-1}$. Up to a first-order approximation:

$$\gamma_{At} = \sum_{f,n} w_{fnt_0} \gamma_{fnt}$$

where:

  - w_{fnt_0} = share of firm f's sales in market n in aggregate sales in base period t_0
  - γ_{fnt} = growth rate between $t-1$ and t

Firm-Level Growth

- Decompose firm-level growth into:

$$\gamma_{fnt} = \delta_{jnt} + \varepsilon_{fnt}$$

  where:
  - δ_{jnt} = sector-destination-year specific shock ("macro")
  - ε_{fnt} is a firm-destination-year specific shock ("micro")
- Previous decomposition can be motivated by a multi-sector heterogeneous firms model in the spirit of Melitz (2003) and Eaton et al. (2011)
  - firm-level residuals then capture both productivity and demand shocks
- This can be estimated, year-by-year and destination-by- destination, using OLS with fixed effects to identify sector-destination shocks

Variance Decomposition

- The variance of aggregate growth is

$$\sigma^2_{At_0} = \underbrace{var(\sum_{j,n} w_{jnt_0}\delta_{jnt})}_{Macro\ Volatility} + \underbrace{var(\sum_{f,n} w_{fnt_0}\varepsilon_{fnt})}_{Firm\ Volatility} + COV_t$$

- Goal of DLM is to study to what extent the "Firm" component ''explains" aggregate fluctuations
  - Same strategy as in development accounting literature
  - Compute ratio between between firm component and variance of aggregate growth. If large, then firm is important

Data Description

- Firm-level domestic and export sales data for the universe of French firms over 1990-2007
- Merge two large datasets:
  - Fiscal administration: firm tax forms from BRN and RSI (small firms). BRN covers 1.6 million firms and 52 NAF sectors. Manufacturing has 209 thousand firms and 22 NAF industries, representing 30% of total sales
  - Customs: firm-destination exports

Firm-Level Accounting: Whole Economy

98.7% of the observed variance is explained by the firm-level component.

	(1)	(2)	(3)	(4)
I. Total Sales				
	Obs.	Mean	St. Dev.	Correlation
Actual	9,856,893	0.0467	0.2601	1.0000
Firm-Specific	9,856,893	0.0000	0.2583	0.9934
Sector-Destination	16,238	0.0763	0.1260	0.1146
II. Domestic Sales				
	(1)	(2)	(3)	(4)
	Obs.	Mean	St. Dev.	Correlation
Actual	8,031,452	0.0410	0.2266	1.0000
Firm-Specific	8,031,452	0.0000	0.2255	0.9954
Sector-Destination	595	0.0453	0.0.04	0.0957
III. Export Sales				
	(1)	(2)	(3)	(4)
	Obs.	Mean	St. Dev.	Correlation
Actual	1,825,441	0.0718	0.3723	1.0000
Firm-Specific	1,825,441	0.0000	0.3697	0.9930
Sector-Destination	15,643	0.0775	0.1281	0.1185

Firm-Level Accounting: Manufacturing

98.2% of the observed variance is explained by the firm-level component.

	(1) Obs.	(2) Mean	(3) St. Dev.	(4) Correlation
I. Total Sales				
Actual	2,436,017	0.0542	0.3038	1.0000
Firm-Specific	2,436,017	0.0000	0.3010	0.9908
Sector-Destination	10,269	0.0741	0.0968	0.1357
II. Domestic Sales				
	(1) Obs.	(2) Mean	(3) St. Dev.	(4) Correlation
Actual	1,233,903	0.0378	0.2233	1.0000
Firm-Specific	1,233,903	0.0000	0.2214	0.9917
Sector-Destination	306	0.0414	0.0322	0.1285
III. Export Sales				
	(1) Obs.	(2) Mean	(3) St. Dev.	(4) Correlation
Actual	1,202,114	0.0709	0.3679	1.0000
Firm-Specific	1,202,114	0.0000	0.3651	0.9924
Sector-Destination	9,963	0.0752	0.0980	0.1228

Aggregate Volatility Accounting

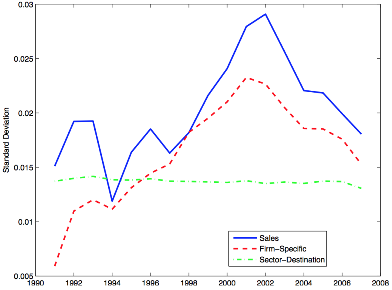
Contribution of firm-component is equivalent to the contribution of all sectoral and macro shocks. Firm-component contribution is larger for exports

	Whole Economy (1) St. Dev.	Whole Economy (2) Relative SD	Manufacturing Sector (3) St. Dev.	Manufacturing Sector (4) Relative SD
I. Total Sales				
Actual	0.0214	1.0000	0.0261	1.0000
Firm-Specific	0.0164	0.7584	0.0165	0.6266
Sector-Destination	0.0137	0.6663	0.0189	0.7394
II. Domestic Sales				
Actual	0.0185	1.0000	0.0195	1.0000
Firm-Specific	0.0139	0.7441	0.0114	0.5778
Sector-Destination	0.0127	0.7148	0.0157	0.8186
III. Export Sales				
Actual	0.0037	1.0000	0.0086	1.0000
Firm-Specific	0.0029	0.7874	0.0062	0.7224
Sector-Destination	0.0016	0.4475	0.0041	0.4909

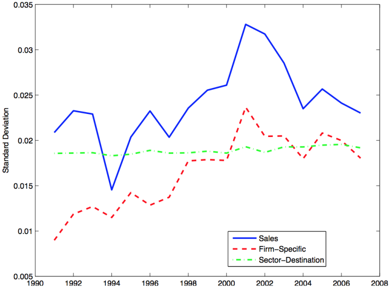
Aggregate Volatility Accounting

Contribution of firm-component is increasing over time (because weights vary across base periods), both for the manufacturing sector and for the whole economy

Whole Economy



Manufacturing Sector



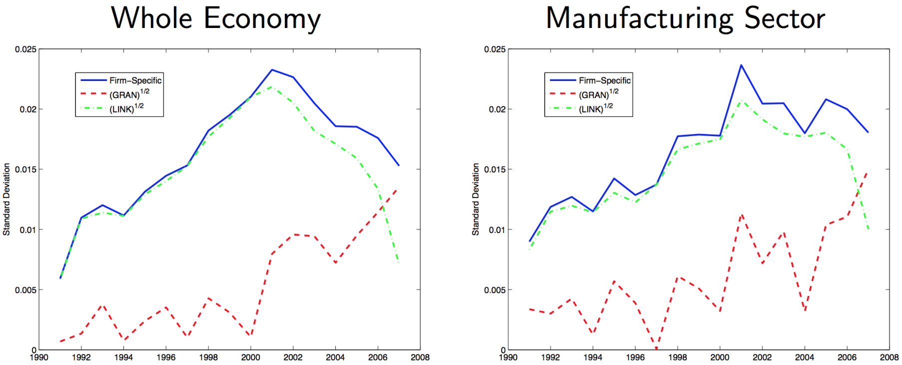
What Determines the Firm-Component?

- Recall the definition of the firm-specific volatility

$$\begin{aligned}\sigma^2_{Ft_0} &= var(\sum_{f,n} w_{fnt_0}\varepsilon_{fnt}) \\ &= \underbrace{\sum_{f,n} w^2_{fnt_0} var(\varepsilon_{fnt})}_{GRAN} + \underbrace{\sum_{g\neq f, m\neq n}\sum_{f,n} w_{gmt_0} w_{fnt_0} cov(\varepsilon_{gmt}, \varepsilon_{fnt})}_{LINK}\end{aligned}$$

- With i.i.d shocks and symmetric firms, we would expect GRAN = LINK = 0
- Which departure matters more in practice?

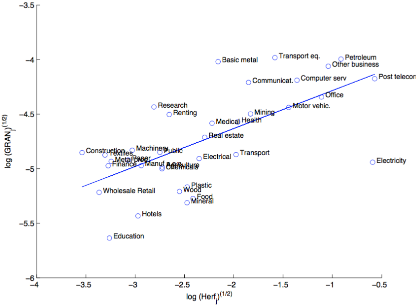
Granularity and Linkages



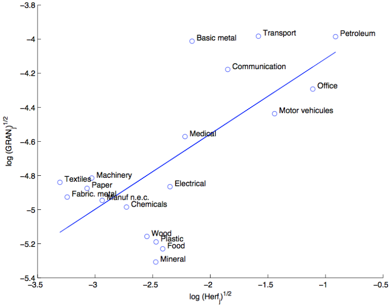
- *LINK* component explains the majority of total firm-level volatility
- Still true at the sectoral level with some heterogeneity (larger role of *GRAN* in the "Petroleum" sector)
- Contribution of *GRAN* increases over time

Granularity Across Sectors

Whole Economy



Manufacturing Sector



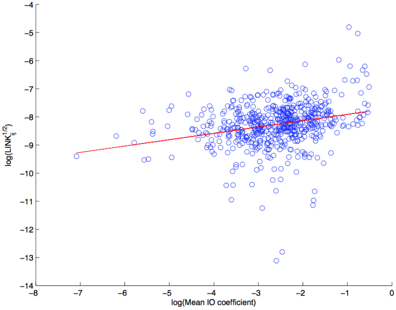
- At the sector level:

$$GRAN = \sum_j GRAN^j \quad and \quad GRAN^j = \sum_{(f,n)\in j} w^2_{fnt-1} var(\varepsilon_{fnt}) = \sigma^2 HERF^j_{t-1}$$

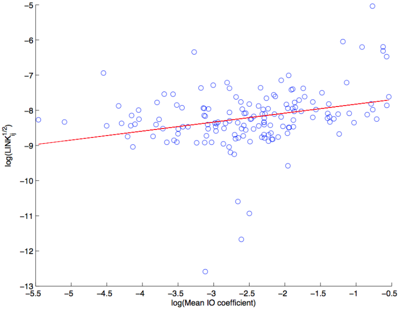
$\Rightarrow$ Sectors more concentrated should display more volatility

- Correlation less than perfect because, in the data, small firms tend to be more volatile

Whole Economy



Manufacturing Sector



- At the sector level:

$$LINK = \sum_{i=1}^{J} \sum_{j=1}^{J} LINK^{ij} \text{ and } LINK^{ij} \equiv \sum_{(f,n)\in i} \sum_{(g,m)\in j} w_{fnt-1} w_{gmt-1} cov(\varepsilon_{fnt}, \varepsilon_{gmt})$$

Concluding Remarks

- Hulten's Theorem $\Rightarrow$ link between "micro" shocks & aggregate volatility
  - Sector/firm sales are sufficient statistic for effect of a sector/firm shock
  - Primitive assumptions about firm growth and network structure only matter for aggregate volatility to the extent that they affect distribution of sales
- Hulten's Theorem $\Rightarrow$ link between globalization & aggregate volatility:
  - If globalization makes the distribution of firm sales "fatter" because only the largest firms export, then globalization should increase aggregate volatility
  - Di Giovanni and Levchenko (JPE, 2012) explore that idea using Melitz (2003)
- One can go beyond Hulten's Theorem by:
  - Relaxing efficiency (e.g. Baqaee 2016)
  - Focusing on other moments of the distribution (e.g. Acemoglu et al. 2017)
  - Studying second-order approximations (e.g. Baqaee and Farhi 2017)
  - Going full CGE (e.g. Costinot, Donaldson, Smith and Caliendo, Parro, Rossi-Hansberg, Sarte 2017)