

MIT 14.582 PhD International Economics II  
— Lecture 18: Economic Geography and Urban  
Economics (Dynamics) —

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# Dynamics (not just Space!): The Final Frontier?

- Fields of trade/geography have traditionally emphasized static (or perhaps long-run) responses of an economy to shocks
  - Big “ $N$ ”: models feature rich cross-sectional heterogeneity—many countries, regions, sectors, firms, factors
  - But  $T = 1$
- That is slowly changing. Possible reasons (?):
  - Longer panel data on regions, sectors, firms, people
  - Empirical studies have increasingly documented just how slow (and interesting, and unequal) factor market adjustment (especially spatial adjustment) can be
  - Improved computational tools for big  $(N, T)$  models
- This lecture: a quick introduction to a few of these tools

# Quick overview of literature on big $(N, T)$ models

- Partial equilibrium, with focus on labor market adjustment:
  - Lee and Wolpin (2006), Kennan and Walker (2011)
  - Artuc, Chaudhuri, McLaren (2010)
  - Dix-Carneiro (2014)
- General equilibrium counterfactuals:
  - Desmet and Rossi-Hansberg (2014), etc.
  - Eaton, Kortum, Neiman, Romalis (2016); Reyes-Heroles (2016)
  - Ravikumar, Santacreu, Sposi (2019)
  - Anderson, Larch, Yotov (2020)
  - Caliendo, Dvorkin, Parro (2019)
  - Traiberman (2019)
  - Dix-Carneiro, Pessoa, Reyes-Heroles, Traiberman (2020)
- So far, GE side hasn't been able (to my knowledge) to handle **aggregate** uncertainty...

# Today's Plan

- ① Artuc, Chaudhuri and McLaren (AER, 2010)
- ② Caliendo, Dvorkin and Parro (ECMA, 2019)

Artuc, Chaudhuri and McLaren (AER, 2010)

# Basic Ingredients

- We start with a simple model of labor market dynamics
- We model the agent's decision of where to supply labor across markets as a dynamic discrete choice problem
  - In response to shocks, the worker chooses whether to remain where she is or to move to another location
  - If the worker moves, she will pay a migration cost, which has two components:
    - A portion that is the same for all workers making the same move (moving costs, learning costs, etc.)
    - A time-varying idiosyncratic shock (e.g. personal situation)
  - Will refer to it as a “migration cost” but could be inter-sectoral/occupation adjustment cost (as in ACM's application) or even some combination of both spatial and sector/occupation adjustment (as in CDP's application)

- Empirical motivation for idiosyncratic migration shocks:
  - First, gross flows are an order of magnitude larger than net flows, implying large numbers of workers moving in opposite directions at the same time
  - Second, a significant fraction of workers who change jobs voluntarily move to jobs which pay less than the job the worker left behind
- Idiosyncratic shocks imply transitional dynamics in response to location-specific shocks

# Agent's Decision Problem

- Agent supplies 1 unit of labor to  $n = 1, \dots, N$  locations
  - Receives the competitive market wage  $w_t^n$  if employed in  $n$
- Assume that the value of an agent in location  $n$  at time  $t$  given by

$$v_t^n = \log(w_t^n) + \max_{\{i\}_{i=1}^N} \{ \beta E [v_{t+1}^i] - \tau^{n,i} + \epsilon_t^i \},$$

- $\beta \in (0, 1)$  discount factor
- $\tau^{n,i}$  additive, *time invariant* migration costs to  $n$  from  $i$  ( $\tau^{i,i} = 0$  by normalization)
- $\epsilon_t^i$  are stochastic *i.i.d idiosyncratic* taste shocks
- $E [v_{t+1}^i]$  = expectation over  $\epsilon$  (no other source of uncertainty—i.e. we have idiosyncratic uncertainty but not aggregate uncertainty)

# Agents' Decision Problem (Continued)

- Let  $V_t^n \equiv E[v_t^n]$  denote the expected lifetime utility of a household located in  $n$  at time  $t$
- Taking expectations across  $\epsilon_t^i$  in previous expression implies

$$V_t^n = \log(w_t^n) + E \left[ \max_{\{i\}_{i=1}^N} \{ \beta V_{t+1}^i - \tau^{n,i} + \epsilon_t^i \} \right],$$

- Next we impose distributional assumptions on  $\epsilon_t^i$  to solve for

$$\Phi_t^n \equiv E \left[ \max_{\{i\}_{i=1}^N} \{ \beta V_{t+1}^i - \tau^{n,i} + \epsilon_t^i \} \right]$$

- Idiosyncratic shocks are drawn from Gumbel distribution

$$F(\epsilon) = \exp \left( - \exp \left( - \frac{\epsilon - \bar{\gamma}}{\nu} \right) \right)$$

where  $\bar{\gamma}$  is Euler's constant

- Then

$$\Phi_t^n = \nu \log \left[ \sum_{i=1}^N \exp \left( \beta V_{t+1}^i - \tau^{n,i} \right)^{1/\nu} \right]$$

- See 14.581 EK lecture for extreme value algebra + relationship between Gumbel and Frechet

# Choice Probabilities

- Let  $\mu_t^{n,i}$  denote the fraction of workers from location  $n$  who choose to move to location  $i$  at date  $t$
- This fraction is equal to the probability that a given worker moves from  $n$  to  $i$  at time  $t$ . Formally,

$$\mu_t^{n,i} = \Pr \left( \beta V_{t+1}^i - \tau^{n,i} + \epsilon_t^i \geq \max_{h \neq i} \{ \beta V_{t+1}^h - \tau^{n,h} + \epsilon_t^h \} \right).$$

- For Gumbel distribution, fraction of workers that reallocate from location  $n$  to  $i$  is:

$$\mu_t^{n,i} = \frac{\exp(\beta V_{t+1}^i - \tau^{n,i})^{1/\nu}}{\sum_{h=1}^N \exp(\beta V_{t+1}^h - \tau^{n,h})^{1/\nu}}$$

- See again 14.581 EK lecture for extreme value algebra

# (Partial) Equilibrium

An equilibrium (for a given sequence of wages,  $w_t^n$ , in each location) corresponds to  $\{L_t, \mu_t, V_t, \}_{t=0}^{\infty}$  such that:

- Expected lifetime utilities satisfy

$$V_t^n = \log w_t^n + \nu \log \left[ \sum_{i=1}^N \exp(\beta V_{t+1}^i - \tau^{n,i})^{1/\nu} \right]$$

- Fractions of workers reallocating from market  $n$  to  $i$  satisfy

$$\mu_t^{n,i} = \frac{\exp(\beta V_{t+1}^i - \tau^{n,i})^{1/\nu}}{\sum_{h=1}^N \exp(\beta V_{t+1}^h - \tau^{n,h})^{1/\nu}}$$

- Population in market  $n$  satisfies

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

- Since model satisfies same functional forms as EK, we should be able to deal with welfare in a similar manner
- First, note that the expected value of being in market  $n$  can be decomposed into

$$V_t^n = \underbrace{\log w_t^n}_{\text{current wage}} + \underbrace{\beta V_{t+1}^n}_{\text{value of staying}} + \underbrace{E \left[ \max_{\{i\}_{i=1}^N} \left\{ \beta \left( V_{t+1}^i - V_{t+1}^n \right) - \tau^{n,i} + v \epsilon_t^i \right\} \right]}_{\text{option value of migration}}$$

- Previous algebra shows that option value of migration is equal to

$$v \log \left[ \sum_{i=1}^N \exp \left( \beta \left( V_{t+1}^i - V_{t+1}^n \right) - \tau^{n,i} \right)^{1/v} \right]$$

- This is revealed by agents' decisions to migrate or not

$$\mu_t^{n,n} = \frac{\exp \left( \beta V_{t+1}^n \right)^{1/v}}{\sum_{h=1}^N \exp \left( \beta V_{t+1}^h - \tau^{n,h} \right)^{1/v}}$$

which we can manipulate to obtain

$$v \log \sum_{h=1}^N \exp \left( \beta \left( V_{t+1}^h - V_{t+1}^n \right) - \tau^{n,h} \right)^{1/v} = -v \log \mu_t^{n,n}.$$

- Plugging previous expression into the value function, we get

$$V_t^n = \log w_t^n + \beta V_{t+1}^n - \nu \log \mu_t^{n,n} \quad (1)$$

- Iterating this equation forward we obtain

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{w_t^n}{(\mu_t^{n,n})^\nu}$$

- Compared to static models, even a shock that reduces (the path of) wages in location  $n$  can increase the option value of migration and make workers starting out there better off

# Estimation: Static Case

- How do we estimate  $\nu$  and the structural residuals  $\{\tau^{n,i}\}$ ?
- Consider first the static case:

$$\mu^{n,i} = \frac{[\exp(\ln w_i - \tau^{n,i})]^{1/\nu}}{\sum_h [\exp(\ln w_h - \tau^{n,h})]^{1/\nu}} \quad (2)$$

- Taking logs, we obtain a “gravity-like” equation for bilateral migration flows:

$$\ln \mu^{n,i} = \delta_n + \frac{1}{\nu} \ln w_i - \frac{1}{\nu} \tau^{n,i}$$

with  $\delta_n = (1/\nu) \ln (\sum_h [\exp(\ln w_h - \tau^{n,h})])$

- In a trade context, we would typically “fixed-effect out” both  $\delta_n$  and  $\frac{1}{\nu} \ln w_i$  and use observable shifters of  $\tau^{n,i}$ , e.g. tariffs
- In a migration context:
  - harder to find bilateral shifters of migration costs...
  - so we need to use variation in wages across destinations
  - and we need a (demand-side) instrument for the wage

# Estimation: Dynamic Case

- Now let's go back to the dynamic case:

$$\mu_t^{n,i} = \frac{[\exp(\beta V_{i,t+1} - \tau^{n,i})]^{1/\nu}}{\sum_h [\exp(\beta V_{h,t+1} - \tau^{n,h})]^{1/\nu}} \quad (3)$$

$$V_{i,t} = \ln w_{i,t} + \nu \ln \left( \sum_h [\exp(\beta V_{h,t+1} - \tau^{i,h})]^{1/\nu} \right) \quad (4)$$

- Problem: Where to get data on  $V_{i,t+1}$ ? Depends on wages (perhaps observable), but also infinite stream of future wages (less so!)
- Solution: Use the fact that relative expected lifetime utilities at date  $t + 1$  are revealed by choices of the agent at date  $t$ :

$$\beta(V_{i,t+1} - V_{n,t+1}) - \tau^{n,i} = \nu(\ln \mu_t^{n,i} - \ln \mu_t^{n,n}) \quad (5)$$

# Estimation: Dynamic Case

- Using equation (1), we can rearrange equations (3) and (4) as

$$\mu_t^{n,i} = \frac{[\exp(\beta V_{i,t+1} - \tau^{n,i})]^{1/\nu}}{\sum_h [\exp(\beta V_{h,t+1} - \tau^{n,h})]^{1/\nu}} \quad (6)$$

$$V_{i,t} = \ln w_{i,t} + \beta V_{i,t+1} - \nu \ln \mu_t^{i,i} \quad (7)$$

- Now consider two consecutive periods,  $t - 1$  and  $t$ . Substitute (7) at  $t$  into (6) at  $t - 1$ :

$$\mu_{t-1}^{n,i} = \frac{[\exp(\beta \{\ln w_{i,t} + \beta V_{i,t+1} - \nu \ln \mu_t^{i,i}\} - \tau^{n,i})]^{1/\nu}}{\sum_h [\exp(\beta \{\ln w_{h,t} + \beta V_{h,t+1} - \nu \ln \mu_t^{h,h}\} - \tau^{n,h})]^{1/\nu}}$$

- Taking the ratio of the previous expression for destination  $i$  and  $n$  and rearranging implies

$$\begin{aligned} \nu \ln \left( \frac{\mu_{t-1}^{n,i}}{\mu_{t-1}^{n,n}} \right) &= \beta (\ln w_{i,t} - \ln w_{n,t}) + \beta^2 (V_{i,t+1} - V_{n,t+1}) \\ &\quad - \nu \beta (\ln \mu_t^{i,i} - \ln \mu_t^{n,n}) - \tau^{n,i} \end{aligned}$$

# Estimation: Dynamic Case

- Substituting (5) and rearranging

$$\ln \mu_{t-1}^{n,i} = \delta_{n,t} + \frac{\beta}{\nu} \ln w_{i,t} + \beta \ln \left( \frac{\mu_t^{n,i}}{\mu_t^{i,i}} \right) - \frac{1}{\nu} (1 - \beta) \tau^{n,i}$$

with  $\delta_{n,t} \equiv -\frac{\beta}{\nu} \ln w_{n,t} + \beta \ln \mu_{t-1}^{n,n}$

- This is a lot like our earlier static regression equation, but with the dynamic correction of  $\beta \ln \left( \frac{\mu_t^{n,i}}{\mu_t^{i,i}} \right)$
- ACM (2010) and CDP (2019) take this to the data. Not our focus, but a few comments:
  - $\tau^{n,i}$  not (fully) observed by econometrician
  - In ACM (2010):  $\tau^{n,i} = \tau$  but agents uncertain about wages/states, which creates extra structural residual
  - In CDP (2019): no uncertainty about wages/states
  - Clearly need demand-side instruments (for both  $\ln w_{i,t}$  and  $\ln \left( \frac{\mu_t^{n,i}}{\mu_t^{i,i}} \right)$ ) in either case

## 2. Caliendo, Dvorkin and Parro (2019)

- CDP take the ACM model and add a full (though static) GE structure—regions/sectors/countries that interact with one another
- GE structure is that of Caliendo-Parro (REStud, 2015):
  - Eaton and Kortum (2002) but with multiple sectors
  - Cobb-Douglas inter-sectoral input-output linkages
  - But static: capital can exist and even change, but any changes must be exogenous ones (endowment shocks)
- Main changes relative to model so far will be that:
  - Agents pay different consumer prices  $P_{n,t}$  by location, so value the real wage  $\omega_{n,t} \equiv w_{n,t}/P_{n,t}$
  - The real wage is endogenously determined by both labor demand and labor supply (where LS is the result of the above model)
- For ease, imagine that real wage  $\omega_{n,t}$  is determined by a function  $\omega(\cdot)$  of the labor supply vector  $L_t$  and the fundamentals (productivity, capital endowment, trade costs, etc) in the world economy ( $\Theta_t$ ). Denote this  $\omega_{n,t} = \omega_n(L_t, \Theta_t)$

## Aside: Welfare Revisited

- In one-sector EK (or any ACR-class) model, 14.581 covered how can express welfare as  $\omega_{n,t} = (\pi_t^{n,n} / T_t^n)^{-1/\theta}$ , where  $T_t^n$  is the productivity of region  $n$  at  $t$  and  $\theta$  is the trade elasticity
- So using earlier expression for lifetime welfare have

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{(\pi_t^{n,n} / T_t^n)^{-1/\theta}}{(\mu_t^{n,n})^v}$$

- Summarizes welfare equations in static trade models as in ACR (2010) and dynamic models with exogenous trade as in ACM (2010)
- Sufficient statistic (if observe whole path of  $\pi_t^{n,n}$  and  $\mu_t^{n,n}$ —not easy!) to measure welfare gains from trade and migration relative to autarky  $\pi_t^{n,n} = 1$  and no migration  $\mu_t^{n,n} = 1$
- See Caliendo, Oromolla, Parro and Sforza (JPE, 2020) for application of these ideas to a study of EU enlargement (reduced barriers to both trade and factor mobility)

# Counterfactuals in this model

- Helpful to let  $\tilde{\tau}^{n,i} \equiv e^{\tau^{n,i}}$ ,  $u_t^n \equiv e^{V_t^n}$ .
- Then can re-write previous system as

$$u_t^n = \omega_{n,t} \left[ \sum_i (u_{t+1}^i)^{\beta/v} (\tilde{\tau}^{n,i})^{-1/v} \right]^v$$

$$\mu_t^{n,i} = \frac{(u_{t+1}^i)^{\beta/v} (\tilde{\tau}^{n,i})^{-1/v}}{\sum_{h=1}^N (u_{t+1}^h)^{\beta/v} (\tilde{\tau}^{n,h})^{-1/v}}$$

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

- Or in proportional changes (denoting  $\dot{y}_{t+1} \equiv y_{t+1}/y_t$ , etc):

$$\dot{u}_{t+1}^n = \dot{\omega}_{n,t+1} \left[ \sum_i \mu_t^{n,i} (\dot{u}_{t+2}^i)^{\beta/v} \right]^v$$

$$\mu_{t+1}^{n,i} = \frac{\mu_t^{n,i} (\dot{u}_{t+2}^i)^{\beta/v}}{\sum_{h=1}^N \mu_t^{n,h} (\dot{u}_{t+2}^h)^{\beta/v}}$$

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

## “Exact hat” counterfactuals in this model

- Previous system is reminiscent of the DEK (2008) system, but dynamic
- Indeed, the sub-problem of solving for  $\dot{\omega}_{n,t+1} \equiv \omega_n(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$  is exactly covered by DEK—would result from one comparative static calculation in their model, from feeding in an exogenous shock given by  $(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$
- Recall that the DEK method is to write all endogenous proportional changes as a function of initial trade and income shares, and the exogenous proportional change shocks  $(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$
- CDP show how one could do something similar dynamically in their model

## “Exact hat” counterfactuals in this model: 2 steps

- First solve for the (“data from the”) baseline equilibrium (since this probably involves the future, so we have to solve for this future “data”)
  - Need set of initial (time 0) trade flow and income share data, like in DEK
  - Now also need initial distribution of workers ( $L_0$ ) and initial change in distribution of workers ( $\mu_{-1}$ )
  - And do need to know set of future exogenous changes that you think will factually happen ( $\dot{\Theta}_{t+1}$ ), which needs to be the same set of changes that the agents at 0 know when they are deciding on  $\mu_{-1}$
  - Nothing magical here. Equilibrium is system of nonlinear equations to solve for, but it is (evidently) a particularly tractable one, despite its size, since we know the system will (as long as  $\dot{\Theta}_{t+1}$  is a convergent sequence) eventually be in SS.
- Then can do the counterfactuals.
  - Previous baseline equilibrium solution is the ‘initial’ data (sequence) that we are then shocking with a genuinely counterfactual path of  $\dot{\Theta}_{t+1}$  events

# Different interpretation

- Recall that for normal/static DEK, there are 3 interpretations of what is going on:
  - ① EK (2002): can estimate the unknown “shifter” parameters (trade costs, productivity) from fixed effects and residuals in gravity equation. Once that is done we can just solve the model in the usual way (i.e. as function of primitive parameters).
  - ② DEK (2008): happen to be able to write the CES model’s system as function of initial levels and proportional changes (of shocks and endog vars)—more elegant and less coding than EK (2002), but nothing magical (and no weaker maintained assumptions).
  - ③ ACD (2017): nothing special about CES/gravity for these ideas.
- CDP method is explicated like #2 in the paper. But an alternative interpretation of what they are doing is that it is has an analog to #1 (or #3).

# Different interpretation

- In particular, can show that the method amounts to an elaborate method for “inverting” the model in “initial period” (trade and income shares at time 0, plus  $\mu_{-1}$ ) to recover the set of unknown migration costs ( $\tau^{i,n}$ ).
- Further, there are potentially simpler methods if you had more than one period of migration data (e.g. both  $\mu_0$  and  $\mu_{-1}$ ).
  - E.g residuals in ACM's estimating equation, under CDP's assumption that researcher knows all aggregate shocks ( $\hat{\Theta}_{t+1}$ ), would allow one to estimate migration costs too.

# Application: Effects (on U.S.) of the rise of China

- U.S. imports from China almost doubled from 2000 to 2007
  - At the same time, manufacturing employment fell while employment in other sectors, such as construction and services, grew
- Several studies document that an important part of the employment loss in manufactures was a consequence of China's trade expansion
  - e.g., Autor, Dorn, and Hanson (2013), Acemoglu, Autor, Dorn, and Hanson (2014), Pierce and Schott (2016)
- CDP use their model to quantify and understand the effects of the rise of China's trade expansion, "China shock"
  - Initial period is the year 2000
  - Calculate the sectoral, regional, and aggregate employment and welfare effects of the China shock

# Identifying the China “shock”

- Follow Autor, Dorn, and Hanson (2013)

- Estimate

$$\Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j,$$

where  $j$  is a NAICS sector,  $\Delta M_{USA,j}$  and  $\Delta M_{other,j}$  are changes in U.S. and other adv. countries, imports from China from 2000 to 2007

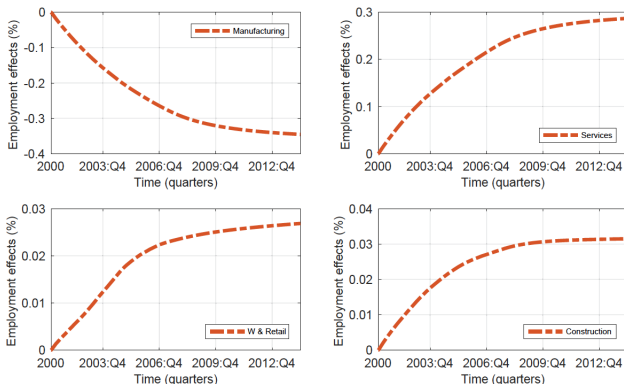
- Find  $a_2 = 1.27$
- Obtain predicted changes in U.S. imports with this specification
- Use the model to solve for the change in China’s 12 manufacturing industries TFP  $\{\hat{A}^{China,j}\}_{j=1}^{12}$  such that model’s imports match predicted imports from China from 2000 to 2007
  - With model’s generated data obtain  $a_2 = 1.52$
  - Feed this into the model  $\{\hat{A}^{China,j}\}_{j=1}^{12}$  by quarter from 2000 to 2007 to study the effects of the shock

# Taking the Model to the Data

- 50 U.S. states, 22 sectors + non-empl. and 38 countries
- Need data for  $(L_0, \mu_{-1}, \pi_0, VA_0, GO_0)$ 
  - $L_0$  : PUMS of the U.S. Census for the year 2000
    - Exclude empl. in farming, mining, utilities, and public sect.
  - $\mu_{-1}$  : Use CPS to compute intersectoral mobility and ACS to compute interstate mobility
  - $\pi_0$  : CFS and WIOD year 2000
  - $VA_0$  and  $GO_0$  : BEA VA shares and U.S. IO, WIOD for other countries
- Need values for parameters  $(\nu, \theta, \beta)$ 
  - $\theta$  : Use Caliendo and Parro (2015)
  - $\beta = 0.99$
  - $\nu = 5.34$  (implied elasticity of 0.2) Using ACM's data and specification, adapted to their model

# Employment Effects

Figure: The effect of the China shock on employment shares



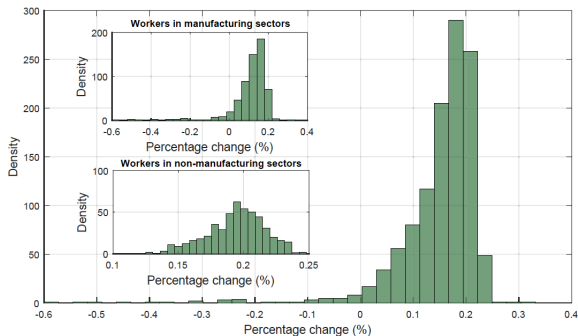
- Chinese competition reduced the share of manufacturing employment by 0.36% in the long run,  $\sim 0.55$  million employment loss
  - About 36% of the change not explained by a secular trend

# Employment Effects: Manufacturing

- Unequal sectoral effects of Chinese import competition
  - 1/2 of the decline in manufacturing employment originated in the Computer & Electronics and Furniture sectors
    - 1/4 of the decline comes from the Metal and Textiles sectors
  - Food, Beverage and Tobacco, gained employment
    - Less exposed to China, benefited from cheaper intermediate goods, other sectors, like Services, demanded more of them
- Unequal regional effects
  - Regions with a larger concentration of sectors that are more exposed to China lose more jobs
    - California, the region with largest share of employment in Computer & Electronics, contributed to 12% of the decline

# Welfare Effects across Labor Markets

Figure: Welfare effects of the China shock across U.S. labor markets



- Very heterogeneous response to the same aggregate shock
- But most labor markets gain as a consequence of cheaper imports