Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade

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What if?

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• Without access to (much) quasi-experimental variation, traditional approach in the field has been to model everything: demand-side, supply-side, market structure, trade costs
  • E.g. #1: Old CGE: GTAP model [13,000 structural parameters]
  • E.g. #2: New CGE: EK model [1 key parameter]
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  - E.g. #2: New CGE: EK model [1 key parameter]

- Question: Can we relax EK’s strong functional form assumptions without circling back to GTAP’s 13,000 parameters?
ACD: 4 Contributions

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   - Reduced factor demand system sufficient for counterfactual analysis
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   - Dekle, Eaton and Kortum (2008): exact hat algebra
   - Arkolakis, Costinot, and Rodriguez-Clare (2012): welfare gains
   - Head and Ries (2001): trade costs
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4. Empirical application: What was the impact of China’s integration into the world economy in the past two decades?
   - Departures from CES modeled in the spirit of BLP (1995)
This Lecture

- Focus on contributions 1 and 2
  - Dave discuss empirics in next class
- Revisit valuation of gains from trade using factor approach
Related Literature

• **GE Theory and Trade:**
  - Taylor (1938); Rader (1972); Mas-Colell (1991); Meade (1952); Helpman (1976); Wilson (1980); Neary and Schweinberger (1986)

• **IO and Trade:**
  - Berry, Levinsohn and Pakes (1995); Nevo (2011); Berry, Gandhi and Haile (2013); Berry and Haile (2014)

• **Bridge within Trade:**
  - *Gravity*: Eaton and Kortum (2002); Anderson and van Wincoop (2003); handbook chapters of Costinot and Rodriguez-Clare (2013) and Head and Mayer (2013)
Outline of Lecture

1. Introduction
2. Neoclassical trade models as factor exchange models
3. Counterfactual and welfare analysis
4. Gains from trade revisited
Neoclassical Trade Model

- \( i = 1, \ldots, I \) countries
- \( k = 1, \ldots, K \) goods
- \( n = 1, \ldots, N \) factors

- Goods consumed in country \( i \):
  \[
  q_i \equiv \{ q_{ji}^k \}
  \]

- Factors used in country \( i \) to produce good \( k \) for country \( j \):
  \[
  l_{ij}^k \equiv \{ l_{ji}^{nk} \}
  \]
Neoclassical Trade Model

- Preferences:
  \[ u_i = u_i(q_i) \]

- Technology:
  \[ q_{ij}^k = f_{ij}^k(l_{ij}^k) \]

- Factor endowments:
  \[ \nu_i^n > 0 \]
Competitive Equilibrium

A $q \equiv \{q_i\}$, $l \equiv \{l_i\}$, $p \equiv \{p_i\}$, and $w \equiv \{w_i\}$ such that:

1. Consumers maximize their utility:

$$q_i \in \arg\max_{\tilde{q}_i} u_i(\tilde{q}_i)$$

$$\sum_{j,k} p_{ji}^k \tilde{q}_{ji}^k \leq \sum_n w_i^n \nu_i^n \text{ for all } i;$$

2. Firms maximize their profits:

$$l_{ij}^k \in \arg\max_{\tilde{l}_{ij}^k} \{p_{ij}^k f_{ij}^k(\tilde{l}_{ij}^k) - \sum_n w_i^n \tilde{l}_{ij}^n \} \text{ for all } i, j, \text{ and } k;$$

3. Goods markets clear:

$$q_{ij}^k = f_{ij}^k(l_{ij}^k) \text{ for all } i, j, \text{ and } k;$$

4. Factors markets clear:

$$\sum_{j,k} l_{ij}^{nk} = \nu_i^n \text{ for all } i \text{ and } n.$$
Reduced Exchange Model

- Fictitious endowment economy in which consumers directly exchange factor services

- Reduced preferences over primary factors of production:

\[ U_i(L_i) \equiv \max_{\tilde{q}_i, \tilde{l}_i} u_i(\tilde{q}_i) \]
\[ \tilde{q}_{ji}^k \leq f_{ji}^k(\tilde{l}_{ji}^k) \text{ for all } j \text{ and } k, \]
\[ \sum_k \tilde{l}_{ji}^{nk} \leq L_{ji}^n \text{ for all } j \text{ and } n, \]
Reduced Equilibrium

Corresponds to \( L \equiv \{ L_i \} \) and \( w \equiv \{ w_i \} \) such that:

1. Consumers maximize their reduced utility:

\[
L_i \in \text{argmax}_{\tilde{L}_i} U_i(\tilde{L}_i) \leq \sum_{j,n} w_{j n}^n \tilde{L}_{ji} \leq \sum_{n} w_i^n \nu_i^n \text{ for all } i;
\]

2. Factor markets clear:

\[
\sum_{j} L_{ij}^n = \nu_i^n \text{ for all } i \text{ and } n.
\]
Equivalence

- **Proposition 1**: For any competitive equilibrium, \((q, l, p, w)\), there exists a reduced equilibrium, \((L, w)\), with:
  1. the same factor prices, \(w\);
  2. the same factor content of trade, \(L_{ji}^n = \sum_k l_{ji}^{nk}\) for all \(i, j,\) and \(n\);
  3. the same welfare levels, \(U_i(L_i) = u_i(q_i)\) for all \(i\).

Conversely, for any reduced equilibrium, \((L, w)\), there exists a competitive equilibrium, \((q, l, p, w)\), such that 1-3 hold.
Equivalence

Comments:

Proof is similar to First and Second Welfare Theorems. Key distinction is that standard Welfare Theorems go from CE to global planner’s problem, whereas RE remains a decentralized equilibrium (but one in which countries fictitiously trade factor services and budget is balanced country by country).

Key implication of Prop. 1: If one is interested in the factor content of trade, factor prices and/or welfare, then one can always study a RE instead of a CE. One doesn’t need direct knowledge of primitives $u$ and $f$ but only of how these indirectly shape $U$. 

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1. Introduction
2. Neoclassical trade models as exchange models
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Reduced Counterfactuals

• Suppose that the reduced utility function over primary factors in this economy can be parametrized as

\[ U_i(L_i) \equiv \bar{U}_i(\{L^n_{ji}/\tau^n_{ji}\}), \]

where \( \tau^n_{ji} > 0 \) are exogenous preference shocks

• **Counterfactual question:** What are the effects of a change from \((\tau, \nu)\) to \((\tau', \nu')\) on trade flows, factor prices, and welfare?
Reduced Factor Demand System

- Start from factor demand = solution of reduced UMP:

\[ L_i(w, y_i | \tau_i) \]

- Compute associated expenditure shares:

\[ \chi_i(w, y_i | \tau_i) \equiv \{ \{ x_{ji}^n \} | x_{ji}^n = w_j^n L_{ji}^n / y_i \text{ for some } L_i \in L_i(w, y_i | \tau_i) \} \]

- Rearrange in terms of *effective factor prices*, \( \omega_i \equiv \{ w_j^n \tau_{ji}^n \} \):

\[ \chi_i(w, y_i | \tau_i) \equiv \chi_i(\omega_i, y_i) \]
Reduced Equilibrium

• RE:

\[ x_i \in \chi_i(\omega_i, y_i), \text{ for all } i, \]
\[ \sum_{j} x^n_{ij} y_j = y^n_i, \text{ for all } i \text{ and } n \]
Reduced Equilibrium

• RE:

\[ x_i \in \chi_i(\omega_i, y_i), \text{ for all } i, \]
\[ \sum_j x_{ij}^n y_j = y_i^n, \text{ for all } i \text{ and } n \]

• **Gravity model:** Reduced factor demand system is CES

\[ \chi_{ji}(\omega_i, y_i) = \frac{(\omega_{ji})^\epsilon}{\sum_l (\omega_{li})^\epsilon}, \text{ for all } j \text{ and } i \]
Exact Hat Algebra

• Start from the counterfactual equilibrium:

\[ x'_i \in \chi_i(\omega'_i, y'_i) \text{ for all } i, \]

\[ \sum_j (x^n_{ij})' y'_j = (y^n_i)' \text{, for all } i \text{ and } n. \]
Exact Hat Algebra

- Start from the counterfactual equilibrium:

\[ x'_i \in \chi_i(\omega'_i, y'_i) \text{ for all } i, \]
\[ \sum_j (x'_{ij})' y'_j = (y'_n)' \text{, for all } i \text{ and } n. \]

- Rearrange in terms of proportional changes:

\[ \{ \hat{x}^n_{ji} x'^n_{ji} \} \in \chi_i(\{ \hat{\omega}^n_{ji} \}, \sum_n \hat{\omega}^n_{ji} \hat{y}^n_{ji} y^n_{ji}) \text{ for all } i, \]
\[ \sum_j \hat{x}^n_{ij} x'^n_{ij} (\sum_n \hat{\omega}^n_{ji} \hat{y}^n_{ji} y^n_{ji}) = \hat{\omega}^n_i \hat{\nu}^n_i y^n_i, \text{ for all } i \text{ and } n. \]
Counterfactual Trade Flows and Factor Prices

• Wlog, pick location of preference shocks so that effective factor prices in the initial equilibrium are equal to one in all countries,

\[ \omega^n_{ji} = 1, \text{ for all } i, j, \text{ and } n. \]
Counterfactual Trade Flows and Factor Prices

- **Proposition 2** Under A1, proportional changes in expenditure shares and factor prices, \( \hat{x} \) and \( \hat{w} \), caused by proportional changes in preferences and endowments, \( \hat{\tau} \) and \( \hat{\nu} \), solve

\[
\{ \hat{x}_{ji}^n \} \in \chi_i(\{ \hat{w}_j^n \hat{\tau}_{ji}^n \omega_{ji}^n \}, \sum_n \hat{w}_i^n \hat{\nu}_i^n y_i^n) \quad \forall \quad i,
\]

\[
\sum_j \hat{x}_{ij}^n \hat{x}_{ij}^n \left( \sum_n \hat{w}_j^n \hat{\nu}_j^n y_j^n \right) = \hat{w}_i^n \hat{\nu}_i^n y_i^n \quad \forall \quad i \text{ and } n.
\]
Welfare

- Equivalent variation for country $i$ associated with change from $(\tau, \nu)$ to $(\tau', \nu')$, expressed as fraction of initial income:

$$\Delta W_i = \frac{(e_i(\omega_i, U'_i)) - y_i}{y_i},$$

with $U'_i =$ counterfactual utility and $e_i =$ expenditure function,

$$e_i(\omega_i, U'_i) \equiv \min_{\tilde{L}_i} \sum \omega^n_{ji} L^n_{ji} \tilde{U}_i(\tilde{L}_i) \geq U'_i.$$
Integrating Below Factor Demand Curves

- To go from $\chi_i$ to $\Delta W_i$, solve system of ODEs
- For any selection $\{x^n_{ji}(\omega, y)\} \in \chi_i(\omega, y)$, Envelope Theorem:

  $$\frac{d \ln e_i(\omega, U'_i)}{d \ln \omega^n_j} = x^n_{ji}(\omega, e_i(\omega, U'_i)) \text{ for all } j \text{ and } n.$$  \hspace{1cm} (1)

- Budget balance in the counterfactual equilibrium

  $$e_i(\omega'_i, U'_i) = y'_i.$$  \hspace{1cm} (2)
• Proposition 3 Under A1, equivalent variation associated with change from \((\tau, \nu)\) to \((\tau', \nu')\) is

\[
\Delta W_i = \frac{(e(\omega_i, U'_i) - y_i)}{y_i},
\]

where \(e(\cdot, U'_i)\) is the unique solution of (1) and (2).
Application to Neoclassical Trade Models

- Suppose that technology in neoclassical trade model satisfies:
  \[ f_{ij}^k(l_{ij}^k) \equiv \bar{f}_{ij}^k(\{l_{nj}^k/\tau_{nj}^k\}), \text{ for all } i, j, \text{ and } k, \]

- Reduced utility function over primary factors of production:
  \[ U_i(L_i) \equiv \max_{\bar{q}_i, \bar{l}_i} u_i(\bar{q}_i) \]
  \[ q_{ji}^k \leq \bar{f}_{ji}^k(\{\bar{l}_{nj}^k/\tau_{nj}^k\}) \text{ for all } j \text{ and } k, \]
  \[ \sum_k \bar{l}_{nj}^k \leq L_{ji}^n \text{ for all } j \text{ and } n. \]

- Change of variable: \( U_i(L_i) \equiv \bar{U}_i(\{L_{ji}^n/\tau_{ji}^n\}) \Rightarrow \text{factor-augmenting productivity shocks in CE = preference shocks in RE} \)
Propositions 2 and 3 provide a system of equations that can be used for counterfactual and welfare analysis in RF economy.

- Proposition 1 $\Rightarrow$ same system can be used in neoclassical economy.

Gravity tools—developed for CES factor demands—extends nonparametrically to any factor demand system.

Given data on expenditure shares and factor payments, $\{x^n_i, y^n_i\}$, if one knows factor demand system, $\chi_i$, then one can compute counterfactual factor prices, aggregate trade flows, and welfare.
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Valuation of the Gains from Trade

- Two equilibria: Trade (T) and Autarky (A)
- Prices: $p_T$ and $p_A$
- Utility: $U_T$ and $U_A$
- Gains from Trade (GT) = welfare cost of autarky = money that country would be willing to pay to avoid going from T to A
- Expressed as a fraction of initial GDP:

$$GT = 1 - \frac{e(p_T, U_A)}{e(p_T, U_T)}$$
Back to The Textbook Approach

\[ C_A = Q_A \]

Diagram showing a graph with axes labeled "Bananas" and "Apples." Points labeled as \( C_T \), \( U_T \), \( U_A \), and \( Q_T \) are marked on the graph.
Can We Just Scale It Up?

- In practice, countries produce and consume MANY goods
  - US has positive exports in 8,500 HS-10 digit product categories
  - plenty of product differentiation even within these categories

- Potential strategy to estimate GT:
  - Estimate production sets and indifference curves around the world
  - Compute counterfactual autarky equilibrium
  - Solve for $p_A$ and $U_A$
  - Use previous formula

- Scaling up the textbook approach requires A LOT of information
  - Not just own-price and cross-price elasticities within a given industry
  - But also US smart phones vs. French red wine, Japanese hybrid cars vs. Costa-Rican coffee etc.
The Factor Approach

• We can apply ACD’s approach to valuation of GT
  • Instead of estimating production and demand functions around the world ...
  • ... we need to estimate reduced factor demand = demand for factor services embodied in goods purchased around the world
The Factor Approach
Parallel with New Good Problem

• Parallel between valuation of GT and “new good” problem in IO

• In order to evaluate the welfare gains from the introduction of a new product (e.g. Apple Cinnamon Cheerios, minivan), we can:
  • Estimate the demand for such products
  • Determine the reservation price at which demand would be zero
  • Measure consumer surplus by looking at the area under the (compensated) demand curve

• We can follow a similar strategy to measure GT:
  • foreign factor services are just like new products that appear when trade is free, but disappear under autarky
From Factor Demand to GT

- Recall definition of expenditure function:
  \[ e(p, U) = \min_{\{c_i\}} \left\{ \sum_i p_i c_i \, | \, u(\{c_i\}) \geq U \right\} \]

- Assume one domestic factor (numeraire) and one foreign factor (p)

- Envelope Theorem (Shepard’s Lemma in this context) implies:
  \[ de(p, U) = q_F dp \]
  \[ \iff d \ln e(p, U) = \frac{pq_F}{e(p, U)} d \ln p = \lambda_F(\ln p, U) d \ln p \]

- Integrating between \( \ln p_T \) and \( \ln p_A \) for \( U = U_A \):
  \[ \ln e(p_A, U_A) - \ln e(p_T, U_A) = \int_{\ln p_T}^{\ln p_A} \lambda_F(x, U_A) dx \equiv A \]

- Noting that \( e(p_A, U_A) = e(p_T, U_T) \)
  \[ GT = 1 - \exp(-A) \]
Integrating Below the (Compensated) Demand Curve

\[
\ln(p_T) - \ln(p_A) = \ln(\text{Expenditure Share of Foreign Factors})
\]

\[
\lambda_F
\]

\[
\mathcal{A}
\]

Expenditure Share of Foreign Factors
CES Example

• Suppose that factor demand is CES, as in ACR

\[ \lambda_F(\ln p, U) = \frac{\exp(-\varepsilon \ln p)}{1 + \exp(-\varepsilon \ln p)} \]

• This leads to

\[ \mathcal{A} = \int_{\ln p_T}^{\infty} \frac{\exp(-\varepsilon x)}{1 + \exp(-\varepsilon x)} dx = \frac{\ln(1 + p_T^{-\varepsilon})}{\varepsilon} \]

• Since CES demand system is invertible, we can also express relative price of foreign factor services as a function of initial expenditure share

\[ \lambda_F = \frac{p_T^{-\varepsilon}}{1 + p_T^{-\varepsilon}} \iff 1 + p_T^{-\varepsilon} = \frac{1}{1 - \lambda_F} \]

• Combining the previous expressions, we get

\[ G_T = 1 - \exp \left( \frac{\ln(1 - \lambda_F)}{\varepsilon} \right) = 1 - \lambda_D^{1/\varepsilon} \]
Take-Away From the Previous Formula

• CES is a very strong functional-form restriction
  • Popular in the trade literature because tractable
  • No reason why it should be the best guide to estimate GT in practice
• But CES/ACR formula captures the 2 key issues for valuation of GT:
  1. How large are imports of factor services in the current trade equilibrium?
  2. How elastic is the demand for these imported services along the path from trade to autarky?
• Basic idea: If we do not trade much or if the factor services that we import are close substitutes to domestic ones, then small GT
CES versus Mixed CES

The graph shows the U.S. gains from trade (in percent) from 1995 to 2010. The line graph compares CES and Mixed CES models. The CES model shows a steady increase in gains from trade throughout the period, while the Mixed CES model starts lower but also shows an upward trend with fluctuations.
Some Issues to Keep in Mind

- **Aggregation:**
  - There may not be a single “domestic” and a single “foreign” factor
    - True under CES, but not in general
  - For foreign factor services, one can create a Hicks-composite good (whose price get arbitrarily large under autarky)
  - For domestic factor services, no way around the fact that relative autarky prices need to be computed

- **Measurement:**
  - Global input-output linkages makes it harder to measure spending on foreign factor services (Recall Johnson and Noguera 2012)
  - Global input-output linkages also create distinction between foreign and traded factor services (all traded factors disappear under autarky)

- **Welfare:**
  - Whose expenditure function? What if there are winners and losers from trade? How should we trade-off gains and losses?