14.581 International Trade
— Lecture 17: Gravity Models (Theory) —

Today's Plan

1. The Simplest Gravity Model: Armington
2. Gravity Models and the Gains from Trade: ACR (2012)
3. Beyond ACR's (2012) Equivalence Result: CR (2013)

1. The Simplest Gravity Model: Armington

The Armington Model

The Armington Model: Equilibrium

- Labor endowments

$$L_i \text{ for } i = 1, \ldots n$$

- CES utility $\Rightarrow$ CES price index

$$P_j^{1-\sigma} = \sum_{i=1}^{n} \left(w_i \tau_{ij}\right)^{1-\sigma}$$

- Bilateral trade flows follow **gravity equation**:

$$X_{ij} = \frac{\left(w_i \tau_{ij}\right)^{1-\sigma}}{\sum_{l=1}^{n} \left(w_l \tau_{lj}\right)^{1-\sigma}} w_j L_j$$

- In what follows $\varepsilon \equiv -\frac{d \ln X_{ij}/X_{jj}}{d \ln \tau_{ij}} = \sigma - 1$ denotes the **trade elasticity**
- Trade balance

$$\sum_i X_{ji} = w_j L_j$$

Why Call It a Gravity Model?!?

- Letting $Y_i = \sum_j X_{ij}$ be country $i's$ total sales and $X_j = \sum_i X_{ij}$ be country $j's$ total expenditures, then

$$Y_i = \sum_j \frac{(w_i \tau_{ij})^{1-\sigma} X_j}{P_j^{1-\sigma}} = w_i^{1-\sigma} \Omega_i^{1-\sigma}$$

where

$$\Omega_i^{1-\sigma} \equiv \sum_j \frac{\tau_{ij}^{1-\sigma} X_j}{P_j^{1-\sigma}}$$

- Solving $w_i^{1-\sigma}$ from $Y_i = w_i^{1-\sigma} \Omega_i^{1-\sigma}$ and plugging into (*) we get

$$X_{ij} = X_j Y_i \tau_{ij}^{1-\sigma} (P_j \Omega_i)^{\sigma-1}$$

- This is the **Gravity Equation**, with bilateral resistance τ_{ij} and multilateral resistance terms p_j (inward) and Ω_i (outward).
  - X_j and Y_i play the role of masses for countries i and j
  - τ_{ij} plays the role of physical distance

The Armington Model: Welfare Analysis

- **Question:**
  Consider a foreign shock: $L_i \to L_i'$ for $i \neq j$ and $\tau_{ij} \to \tau_{ij}'$ for $i \neq j$. How do foreign shocks affect real consumption, $C_j \equiv w_j / P_j$?

- Shephard's Lemma implies

$$d \ln C_j = d \ln w_j - d \ln P_j = -\sum_{i=1}^{n} \lambda_{ij} \left(d \ln c_{ij} - d \ln c_{jj} \right)$$

with $c_{ij} \equiv w_i \tau_{ij}$ and $\lambda_{ij} \equiv X_{ij} / w_j L_j$.

- Gravity implies

$$d \ln \lambda_{ij} - d \ln \lambda_{jj} = -\varepsilon \left(d \ln c_{ij} - d \ln c_{jj} \right).$$

The Armington Model: Welfare Analysis

- Combining these two equations yields

$$d \ln C_j = \frac{\sum_{i=1}^{n} \lambda_{ij} \left(d \ln \lambda_{ij} - d \ln \lambda_{jj} \right)}{\varepsilon}.$$

- Noting that $\sum_i \lambda_{ij} = 1 \Longrightarrow \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0$ then

$$d \ln C_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}.$$

- Integrating the previous expression yields $(\hat{x} = x'/x)$

$$\hat{C}_j = \hat{\lambda}_{jj}^{-1/\varepsilon}.$$

The Armington Model: Welfare Analysis

- In general, predicting $\hat{\lambda}_{jj}$ requires (computer) work
  - We can use exact hat algebra as in DEK (Lecture #3)
  - Gravity equation + data $\{\lambda_{ij}, Y_j\}$, and ε
- But predicting how bad would it be to shut down trade is easy...
  - In autarky, $\lambda_{jj} = 1$. So
$$C_j^A / C_j = \lambda_{jj}^{1/\varepsilon}$$
  - Thus **gains from trade** can be computed as
$$GT_j \equiv 1 - C_j^A / C_j = 1 - \lambda_{jj}^{1/\varepsilon}$$

The Armington Model: Gains from Trade

- Suppose that we have estimated trade elasticity using gravity equation
  - Central estimate in the literature is $\varepsilon = 5$; see Head and Mayer (2013) Handbook chapter
- Using World Input Output Database (2008) to get λ_{jj}, we can then estimate gains from trade:

	λ_{jj}	$\% \ GT_j$
Canada	0.82	3.8
Denmark	0.74	5.8
France	0.86	3.0
Portugal	0.80	4.4
Slovakia	0.66	7.6
U.S.	0.91	1.8

Cheese, really?

2. Gravity Models and the Gains from Trade: ACR (2012)

Motivation

- **New Trade Models**
  - Micro-level data have lead to **new questions** in international trade:
    - How many firms export?
    - How large are exporters?
    - How many products do they export?
  - New models highlight **new margins** of adjustment:
    - From inter-industry to intra-industry to intra-firm reallocations
- **Old question:**
  - How large are the gains from trade (GT)?
- **ACR's question:**
  - How do new trade models affect the magnitude of GT?

ACR's Main Equivalence Result

- ACR focus on gravity models
  - PC: Armington and Eaton & Kortum '02
  - MC: Krugman '80 and many variations of Melitz '03
- Within that class, welfare changes are ($\hat{x} = x'/x$)

$$\hat{C} = \hat{\lambda}^{1/\varepsilon}$$

- **Two sufficient statistics** for welfare analysis are:
  - Share of domestic expenditure, λ;
  - Trade elasticity, ε
- **Two views** on ACR's result:
  - Optimistic: welfare predictions of Armington model are more robust than you thought
  - Pessimistic: within that class of models, micro-level data do not matter

Primitive Assumptions
Preferences and Endowments

- **CES utility**
  - Consumer price index,

$$P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega,$$

- **One factor of production:** labor
  - $L_i \equiv$ labor endowment in country i
  - $w_i \equiv$ wage in country i

Primitive Assumptions
Technology

- **Linear cost function:**

$$C_{ij}\left(\omega, t, q\right) = \underbrace{q w_i \tau_{ij} \alpha_{ij}\left(\omega\right) t^{\frac{1}{1-\sigma}}}_{\text{variable cost}} + \underbrace{w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}\left(\omega\right) m_{ij}\left(t\right)}_{\text{fixed cost}},$$

q : quantity,
τ_{ij} : iceberg transportation cost,
$\alpha_{ij}\left(\omega\right)$: good-specific heterogeneity in variable costs,
ξ_{ij} : fixed cost parameter,
$\phi_{ij}\left(\omega\right)$: good-specific heterogeneity in fixed costs.

Primitive Assumptions
Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = q w_i \tau_{ij} \alpha_{ij}(\omega) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega) m_{ij}(t)$$

$m_{ij}(t)$: cost for endogenous destination specific technology choice, t,

$$t \in [\underline{t}, \overline{t}] \, , \; m_{ij}' > 0, \; m_{ij}'' \geq 0$$

Primitive Assumptions
Technology

- **Linear cost function:**

$$C_{ij}\left(\omega, t, q\right) = q w_i \tau_{ij} \alpha_{ij}\left(\omega\right) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}\left(\omega\right) m_{ij}\left(t\right)$$

- Heterogeneity across goods

$$G_j\left(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n\right) \equiv \left\{\omega \in \Omega \mid \alpha_{ij}\left(\omega\right) \leq \alpha_i,\ \phi_{ij}\left(\omega\right) \leq \phi_i,\ \forall i\right\}$$

Primitive Assumptions

Market Structure

- **Perfect competition**
  - Firms can produce any good.
  - No fixed exporting costs.
- **Monopolistic competition**
  - Either firms in i can pay $w_i F_i$ for monopoly power over a random good.
  - Or exogenous measure of firms, $\overline{N}_i < \overline{N}$, receive monopoly power.
- Let N_i be the measure of goods that can be produced in i
  - Perfect competition: $N_i = \overline{N}$
  - Monopolistic competition: $N_i < \overline{N}$

Macro-Level Restrictions

Trade is Balanced

- Bilateral trade flows are

$$X_{ij} = \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij}(\omega)\, d\omega$$

- **R1** *For any country j,*

$$\sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji}$$

  - Trivial if perfect competition or $\beta = 0$.
  - Non trivial if $\beta > 0$.

- **R2** *For any country* j,

$$\Pi_j / \left(\sum_{i=1}^{n} X_{ji} \right) \text{ is constant}$$

where Π_j : aggregate profits gross of entry costs, $w_j F_j$, (if any)

- Trivial under perfect competition.
- Direct from Dixit-Stiglitz preferences in Krugman (1980).
- Non-trivial in more general environments.

Macro-Level Restriction

CES Import Demand System

- *Import demand system*

$$(\mathbf{w}, \mathbf{N}, \boldsymbol{\tau}) \rightarrow \mathbf{X}$$

- **R3**

$$\varepsilon_j^{ii'} \equiv \partial \ln \left(X_{ij} / X_{jj} \right) / \partial \ln \tau_{i'j} = \begin{cases} \varepsilon < 0 & i = i' \neq j \\ 0 & \textit{otherwise} \end{cases}$$

- Note: symmetry and separability.

- The *trade elasticity* ε is an *upper-level* elasticity: it combines
  - $x_{ij}(\omega)$ (*intensive margin*)
  - Ω_{ij} (*extensive margin*).
- R3 $\implies$ complete specialization.
- R1-R3 are not necessarily independent
  - If $\beta = 0$ then R3 $\implies$ R2.

Macro-Level Restriction

Strong CES Import Demand System (AKA Gravity)

- **R3'** The IDS satisfies

$$X_{ij} = \frac{\chi_{ij} \cdot M_i \cdot (w_i \tau_{ij})^{\varepsilon} \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot M_{i'} \cdot (w_{i'} \tau_{i'j})^{\varepsilon}}$$

  where χ_{ij} is independent of $(\mathbf{w}, \mathbf{M}, \boldsymbol{\tau})$.

- Same restriction on $\varepsilon_j^{ii'}$ as R3 but, but additional structural relationships

Welfare results

- State of the world economy:

$$\mathbf{Z} \equiv (\mathbf{L}, \boldsymbol{\tau}, \boldsymbol{\xi})$$

- *Foreign shocks*: a change from $\mathbf{Z}$ to $\mathbf{Z}'$ with no domestic change.

Equivalence (I)

- **Proposition 1:** *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}.$$

- Implication: 2 sufficient statistics for welfare analysis $\widehat{\lambda}_{jj}$ and ε
- New margins affect structural interpretation of ε
  - ...and composition of gains from trade (GT)...
  - ... but size of GT is the same.

Gains from Trade Revisited

- Proposition 1 is an *ex-post* result... a simple *ex-ante* result:
- **Corollary 1:** *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}.$$

Equivalence (II)

- A stronger ex-ante result for **variable trade costs** under R1-**R3'**:
- **Proposition 2:** *Suppose that R1-R3' hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$$

where

$$\widehat{\lambda}_{jj} = \left[\sum_{i=1}^{n} \lambda_{ij} \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}\right]^{-1},$$

and

$$\widehat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij} \hat{w}_j Y_j \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j}\right)^{\varepsilon}}.$$

- ε and $\{\lambda_{ij}\}$ are sufficient to predict $\widehat{W}_j$ (ex-ante) from $\hat{\tau}_{ij}$, $i \neq j$.

Taking Stock

- ACR consider models featuring:
  - (i) Dixit-Stiglitz preferences;
  - (ii) one factor of production;
  - (iii) linear cost functions; and
  - (iv) perfect or monopolistic competition;

  with three macro-level restrictions:
  - (i) trade is balanced;
  - (ii) aggregate profits are a constant share of aggregate revenues; and
  - (iii) a CES import demand system.
- Equivalence for ex-post welfare changes and GT
  - under R3' equivalence carries to ex-ante welfare changes

3. Beyond ACR's (2012) Equivalence Result:

CR (2013)

Departing from ACR's (2012) Equivalence Result

- **Other Gravity Models:**
  - Multiple Sectors
  - Tradable Intermediate Goods
  - Multiple Factors
  - Variable Markups (ACDR 2012)
  - Economic Geography (Allen and Arkolakis 2014, Redding 2016)
- **Beyond Gravity:**
  - PF's sufficient statistic approach
  - Revealed preference argument (Bernhofen and Brown 2005)
  - More data (Costinot and Donaldson 2011)

Back to Armington

1. Add multiple sectors
2. Add traded intermediates

Multiple sectors, GT

- Nested CES: Upper level EoS ρ and lower level EoS ε_s
- Recall gains for Canada of 3.8%. Now gains can be much higher: $\rho = 1$ implies $GT = 17.4\%$

Tradable intermediates, GT

- Set $\rho = 1$, add tradable intermediates with Input-Output structure
- Labor shares are $1 - \alpha_{j,s}$ and input shares are $\alpha_{j,ks}$ ($\sum_k \alpha_{j,ks} = \alpha_{j,s}$)

Tradable intermediates, GT

	% GT_j	% GT_j^{MS}	% GT_j^{IO}
Canada	3.8	17.4	30.2
Denmark	5.8	30.2	41.4
France	3.0	9.4	17.2
Portugal	4.4	23.8	35.9
U.S.	1.8	4.4	8.3

Combination of micro and macro features

- In Krugman, free entry $\Rightarrow$ scale effects associated with total employment
- In Melitz, additional scale effects associated with sales in each market
- In both models, trade may affect entry and fixed costs
- All these effects do not play a role in the one sector model
- With multiple sectors and traded intermediates, these effects come back

Gains from Trade

....	Canada	China	Germany	Romania	US
Aggregate	**3.8**	**0.8**	**4.5**	**4.5**	**1.8**

Gains from Trade

....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	**17.4**	**4.0**	**12.7**	**17.7**	**4.4**

Gains from Trade

....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	**15.3**	**4.0**	**17.6**	**12.7**	**3.8**

Gains from Trade

....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	15.3	4.0	17.6	12.7	3.8
MS, IO, PC	**29.5**	**11.2**	**22.5**	**29.2**	**8.0**

Gains from Trade

....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	15.3	4.0	17.6	12.7	3.8
MS, IO, PC	29.5	11.2	22.5	29.2	8.0
MS, IO, MC (Krugman)	**33.0**	**28.0**	**41.4**	**20.8**	**8.6**

Gains from Trade

....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	15.3	4.0	17.6	12.7	3.8
MS, IO, PC	29.5	11.2	22.5	29.2	8.0
MS, IO, MC (Krugman)	33.0	28.0	41.4	20.8	8.6
MS, IO, MC (Melitz)	**39.8**	**77.9**	**52.9**	**20.7**	**10.3**

From GT to trade policy evaluation

- Back to $\{\lambda_{ij}, Y_j\}$, ε and $\{\hat{\tau}_{ij}\}$ to get implied $\hat{\lambda}_{jj}$
- This is what CGE exercises do
- Contribution of recent quantitative work:
  - Link to theory—"mid-sized models"
  - Compare models that match same macro data (See Melitz and Redding 13 for a different view)
  - Quantify mechanisms
    - Multiple sectors, tradable intermediates
    - Market structure matters, but in a more subtle way

Still a pretty restrictive class of models...

For Future Research

- **Trade policy in gravity models:**
  - Good approximation to optimal tariff is $1/\varepsilon \approx 20\%$ (related to Gros 87)
  - Large range for which countries gain from tariffs (up to 50%)
  - Small effects of tariffs on other countries
  - Are these numbers we can believe in? If not what are these models missing?
- **Fit of gravity models:**
  - Is model successful in predicting impact of trade liberalization?
  - Are import demand systems in practice very different from those in ACR: cross-price elasticities non-zero? variable diagonal elements?
    - Adao, Costinot, and Donaldson (2016) find that they are
- **What are we missing?**
  - Effects of trade on firm-level productivity
  - Dynamics: trade imbalances, capital accumulation, spillovers
    - EKNR (2016), Caliendo, Dvorkin and Parro (2015)
  - Domestic distortions