Today’s Plan

1. The Simplest Gravity Model: Armington
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The Armington Model
The Armington Model: Equilibrium

- Labor endowments

\[ L_i \text{ for } i = 1, \ldots, n \]

- CES utility \( \Rightarrow \) CES price index

\[ P_j^{1-\sigma} = \sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma} \]

- Bilateral trade flows follow gravity equation:

\[ X_{ij} = \frac{(w_i \tau_{ij})^{1-\sigma}}{\sum_{l=1}^{n} (w_l \tau_{lj})^{1-\sigma}} w_j L_j \]

- In what follows \( \varepsilon \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \sigma - 1 \) denotes the trade elasticity

- Trade balance

\[ \sum_i X_{ji} = w_j L_j \]
Why Call It a Gravity Model?!?

- Letting $Y_i = \sum_j X_{ij}$ be country $i$’s total sales and $X_j = \sum_i X_{ij}$ be country $j$’s total expenditures, then

$$Y_i = \sum_j \frac{(w_i \tau_{ij})^{1-\sigma} X_j}{P_j^{1-\sigma}} = w_i^{1-\sigma} \Omega_i^{1-\sigma}$$

where

$$\Omega_i^{1-\sigma} \equiv \sum_j \frac{\tau_{ij}^{1-\sigma} X_j}{P_j^{1-\sigma}}$$

- Solving $w_i^{1-\sigma}$ from $Y_i = w_i^{1-\sigma} \Omega_i^{1-\sigma}$ and plugging into (*) we get

$$X_{ij} = X_j Y_i \tau_{ij}^{1-\sigma} (P_j \Omega_i)^{\sigma-1}$$

- This is the **Gravity Equation**, with bilateral resistance $\tau_{ij}$ and multilateral resistance terms $p_j$ (inward) and $\Omega_i$ (outward).
  - $X_j$ and $Y_i$ play the role of masses for countries $i$ and $j$
  - $\tau_{ij}$ plays the role of physical distance
Question:
Consider a foreign shock: \( L_i \rightarrow L_i' \) for \( i \neq j \) and \( \tau_{ij} \rightarrow \tau_{ij}' \) for \( i \neq j \).
How do foreign shocks affect real consumption, \( C_j \equiv w_j / P_j \)?

Shephard’s Lemma implies
\[
d \ln C_j = d \ln w_j - d \ln P_j = - \sum_{i=1}^{n} \lambda_{ij} \left( d \ln c_{ij} - d \ln c_{jj} \right)
\]
with \( c_{ij} \equiv w_i \tau_{ij} \) and \( \lambda_{ij} \equiv X_{ij} / w_j L_j \).

Gravity implies
\[
d \ln \lambda_{ij} - d \ln \lambda_{jj} = -\varepsilon \left( d \ln c_{ij} - d \ln c_{jj} \right).
\]
The Armington Model: Welfare Analysis

- Combining these two equations yields

\[ d \ln C_j = \frac{\sum_{i=1}^{n} \lambda_{ij} (d \ln \lambda_{ij} - d \ln \lambda_{jj})}{\varepsilon}. \]

- Noting that \( \sum_i \lambda_{ij} = 1 \implies \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0 \) then

\[ d \ln C_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}. \]

- Integrating the previous expression yields \( (\hat{x} = x'/x) \)

\[ \hat{C}_j = \hat{\lambda}_{jj}^{-1/\varepsilon}. \]
In general, predicting $\hat{\lambda}_{jj}$ requires (computer) work

- We can use exact hat algebra as in DEK (Lecture #3)
- Gravity equation + data $\{\lambda_{ij}, Y_j\}$, and $\varepsilon$

But predicting how bad would it be to shut down trade is easy...

- In autarky, $\lambda_{jj} = 1$. So

$$C_j^A / C_j = \lambda_{jj}^{1/\varepsilon}$$

- Thus gains from trade can be computed as

$$GT_j \equiv 1 - C_j^A / C_j = 1 - \lambda_{jj}^{1/\varepsilon}$$
Suppose that we have estimated trade elasticity using gravity equation

- Central estimate in the literature is $\varepsilon = 5$; see Head and Mayer (2013) Handbook chapter

Using World Input Output Database (2008) to get $\lambda_{jj}$, we can then estimate gains from trade:

<table>
<thead>
<tr>
<th>Country</th>
<th>$\lambda_{jj}$</th>
<th>% $GT_j$</th>
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</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.82</td>
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<tr>
<td>U.S.</td>
<td>0.91</td>
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</tr>
</tbody>
</table>
Cheese, really?
Motivation

- **New Trade Models**
  - Micro-level data have lead to **new questions** in international trade:
    - How many firms export?
    - How large are exporters?
    - How many products do they export?
  - New models highlight **new margins** of adjustment:
    - From inter-industry to intra-industry to intra-firm reallocations

- **Old question:**
  - How large are the gains from trade (GT)?

- **ACR’s question:**
  - How do new trade models affect the magnitude of GT?
ACR’s Main Equivalence Result

- ACR focus on gravity models
  - PC: Armington and Eaton & Kortum ’02
  - MC: Krugman ’80 and many variations of Melitz ’03
- Within that class, welfare changes are \( \hat{x} = x'/x \)
  \[
  \hat{C} = \hat{\lambda}^{1/\varepsilon}
  \]

- Two sufficient statistics for welfare analysis are:
  - Share of domestic expenditure, \( \lambda \);
  - Trade elasticity, \( \varepsilon \)

- Two views on ACR’s result:
  - Optimistic: welfare predictions of Armington model are more robust than you thought
  - Pessimistic: within that class of models, micro-level data do not matter
Primitive Assumptions
Preferences and Endowments

- **CES utility**
  - Consumer price index,

  \[ P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega, \]

- **One factor of production:** labor
  - \( L_i \equiv \text{labor endowment in country } i \)
  - \( w_i \equiv \text{wage in country } i \)
**Linear cost function:**

\[ C_{ij}(w, t, q) = q w_i \tau_{ij} \alpha_{ij}(w) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^\beta \zeta_{ij} \phi_{ij}(w) m_{ij}(t), \]

- \( q \): quantity,
- \( \tau_{ij} \): iceberg transportation cost,
- \( \alpha_{ij}(w) \): good-specific heterogeneity in variable costs,
- \( \zeta_{ij} \): fixed cost parameter,
- \( \phi_{ij}(w) \): good-specific heterogeneity in fixed costs.
Primitive Assumptions
Technology

- **Linear cost function:**

\[
C_{ij}(\omega, t, q) = qw_i \tau_{ij} \alpha_{ij}(\omega) t^{\frac{1}{1-\sigma}} + \omega_i^{1-\beta} \omega_j^\beta \xi_{ij} \phi_{ij}(\omega) m_{ij}(t)
\]

\[m_{ij}(t) : \text{cost for endogenous destination specific technology choice, } t,\]

\[t \in [t, \bar{t}] , \ m_{ij}' > 0 , \ m_{ij}'' \geq 0\]
Primitive Assumptions
Technology

- **Linear cost function:**
  \[
  C_{ij} (\omega, t, q) = q w_i \tau_{ij} \alpha_{ij} (\omega) t^{1-\sigma} + w_i^{1-\beta} w_j^\beta \xi_{ij} \phi_{ij} (\omega) m_{ij} (t)
  \]

- **Heterogeneity across goods**
  \[
  G_j (\alpha_1, \ldots, \alpha_n, \phi_1, \ldots, \phi_n) \equiv \{ \omega \in \Omega \mid \alpha_{ij} (\omega) \leq \alpha_i, \phi_{ij} (\omega) \leq \phi_i, \forall i \}\]
**Primitive Assumptions**

**Market Structure**

- **Perfect competition**
  - Firms can produce any good.
  - No fixed exporting costs.

- **Monopolistic competition**
  - Either firms in $i$ can pay $w_i F_i$ for monopoly power over a random good.
  - Or exogenous measure of firms, $N_i < N$, receive monopoly power.

Let $N_i$ be the measure of goods that can be produced in $i$

- Perfect competition: $N_i = N$
- Monopolistic competition: $N_i < N$
Bilateral trade flows are

\[ X_{ij} = \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij}(\omega) \, d\omega \]

**R1** For any country \( j \),

\[ \sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji} \]

- Trivial if perfect competition or \( \beta = 0 \).
- Non trivial if \( \beta > 0 \).


- **R2** *For any country* $j$,

\[
\frac{\Pi_j}{\left(\sum_{i=1}^{n} X_{ji}\right)} \text{ is constant}
\]

where $\Pi_j$ : aggregate profits gross of entry costs, $w_jF_j$, (if any)

- Trivial under perfect competition.
- Direct from Dixit-Stiglitz preferences in Krugman (1980).
- Non-trivial in more general environments.
Macro-Level Restriction

CES Import Demand System

- **Import demand system**
  \[(w, N, \tau) \rightarrow X\]

- **R3**
  \[
  \varepsilon_{ij}^i' \equiv \frac{\partial \ln (X_{ij} / X_{jj})}{\partial \ln \tau_{i'j}} = \begin{cases} 
  \varepsilon < 0 & i = i' \neq j \\
  0 & \text{otherwise}
  \end{cases}
  \]

- Note: symmetry and separability.
The *trade elasticity* $\varepsilon$ is an *upper-level* elasticity: it combines

- $x_{ij}(\omega)$ (*intensive margin*)
- $\Omega_{ij}$ (*extensive margin*).

R3 $\implies$ complete specialization.

R1-R3 are not necessarily independent

- If $\beta = 0$ then R3 $\implies$ R2.
Macro-Level Restriction

Strong CES Import Demand System (AKA Gravity)

- **R3’** The IDS satisfies

\[ X_{ij} = \frac{\chi_{ij} \cdot M_i \cdot (w_i \tau_{ij})^\varepsilon \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot M_{i'} \cdot (w_{i'} \tau_{i'j})^\varepsilon} \]

where \( \chi_{ij} \) is independent of \((w, M, \tau)\).

- Same restriction on \( \varepsilon_{ij}' \) as R3 but, but additional structural relationships
Welfare results

- State of the world economy:
  \[ Z \equiv (L, \tau, \xi) \]

- *Foreign shocks*: a change from \( Z \) to \( Z' \) with no domestic change.
Proposition 1: Suppose that R1-R3 hold. Then

\[ \hat{W}_j = \hat{\lambda}_{jj}^{1/\varepsilon}. \]

Implication: 2 sufficient statistics for welfare analysis \(\hat{\lambda}_{jj}\) and \(\varepsilon\)

New margins affect structural interpretation of \(\varepsilon\)

...and composition of gains from trade (GT)...

... but size of GT is the same.
Proposition 1 is an *ex-post* result... a simple *ex-ante* result:

**Corollary 1:** Suppose that R1-R3 hold. Then

\[ \hat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}. \]
Equivalence (II)

- A stronger ex-ante result for **variable trade costs** under R1-R3':

  - **Proposition 2:** *Suppose that R1-R3’ hold. Then*

    \[ \hat{W}_j = \hat{\lambda}_{jj}^{1/\varepsilon} \]

    where

    \[ \hat{\lambda}_{jj} = \left[ \sum_{i=1}^{n} \lambda_{ij} (\hat{w}_i \hat{\tau}_{ij})^\varepsilon \right]^{-1}, \]

    and

    \[ \hat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij} \hat{w}_j Y_j (\hat{w}_i \hat{\tau}_{ij})^\varepsilon}{Y_i \sum_{i' = 1}^{n} \lambda_{i'i'} (\hat{w}_{i'} \hat{\tau}_{i'j})^\varepsilon}. \]

- \( \varepsilon \) and \( \{\lambda_{ij}\} \) are sufficient to predict \( \hat{W}_j \) (ex-ante) from \( \hat{\tau}_{ij}, i \neq j \).
ACR consider models featuring:

(i) Dixit-Stiglitz preferences;
(ii) one factor of production;
(iii) linear cost functions; and
(iv) perfect or monopolistic competition;

with three macro-level restrictions:

(i) trade is balanced;
(ii) aggregate profits are a constant share of aggregate revenues; and
(iii) a CES import demand system.

Equivalence for ex-post welfare changes and GT

under R3’ equivalence carries to ex-ante welfare changes
Departing from ACR’s (2012) Equivalence Result

- **Other Gravity Models:**
  - Multiple Sectors
  - Tradable Intermediate Goods
  - Multiple Factors
  - Variable Markups (ACDR 2012)
  - Economic Geography (Allen and Arkolakis 2014, Redding 2016)

- **Beyond Gravity:**
  - PF’s sufficient statistic approach
  - Revealed preference argument (Bernhofen and Brown 2005)
  - More data (Costinot and Donaldson 2011)
1. Add multiple sectors

2. Add traded intermediates
Nested CES: Upper level EoS $\rho$ and lower level EoS $\varepsilon_s$

Recall gains for Canada of 3.8%. Now gains can be much higher: $\rho = 1$ implies $GT = 17.4\%$
Set $\rho = 1$, add tradable intermediates with Input-Output structure

Labor shares are $1 - \alpha_{j,s}$ and input shares are $\alpha_{j,ks}$ ($\sum_k \alpha_{j,ks} = \alpha_{j,s}$)
### Tradable intermediates, GT

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Combination of micro and macro features

- In Krugman, free entry \(\Rightarrow\) scale effects associated with total employment
- In Melitz, additional scale effects associated with sales in each market
- In both models, trade may affect entry and fixed costs
- All these effects do not play a role in the one sector model
- With multiple sectors and traded intermediates, these effects come back
## Gains from Trade

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(Week 10)
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</table>
Back to \{\lambda_{ij}, Y_j\}, \varepsilon \text{ and } \{\hat{t}_{ij}\} \text{ to get implied } \hat{\lambda}_{jj}

This is what CGE exercises do

Contribution of recent quantitative work:

- Link to theory—“mid-sized models”
- Compare models that match same macro data (See Melitz and Redding 13 for a different view)
- Quantify mechanisms
  - Multiple sectors, tradable intermediates
  - Market structure matters, but in a more subtle way
Still a pretty restrictive class of models...
For Future Research

- **Trade policy in gravity models:**
  - Good approximation to optimal tariff is \( \frac{1}{\varepsilon} \approx 20\% \) (related to Gros 87)
  - Large range for which countries gain from tariffs (up to 50%)
  - Small effects of tariffs on other countries
  - Are these numbers we can believe in? If not what are these models missing?

- **Fit of gravity models:**
  - Is model successful in predicting impact of trade liberalization?
  - Are import demand systems in practice very different from those in ACR: cross-price elasticities non-zero? variable diagonal elements?
    - Adao, Costinot, and Donaldson (2016) find that they are

- **What are we missing?**
  - Effects of trade on firm-level productivity
  - Dynamics: trade imbalances, capital accumulation, spillovers
  - Domestic distortions