MIT 14.582: PhD International Economics II
Sp 2026, Lecture 17: Economic Geography and Urban Economics (Theory II)

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Plan for Today's Lecture

- A first look at "quantitative spatial models" in economic geography

What do we mean by "quantitative spatial model"?

- "Spatial": draws on international trade, economic geography, urban economics
- "Quantitative"—basic characteristics:
  - Features that make it more "natural" to connect to real world data points on spatial units:
    - Many regions
    - Lots of exogenous heterogeneity: local characteristics (climate, natural resources, institutions, geographic position in space, etc.)
    - Spatial mobility frictions (trade/migration costs), as seems important
  - Estimate/calibrate parameters (elasticities and location-specific exogenous characteristics)
  - Goal is often to perform model-consistent, quantitative counterfactual exercises
- Recent overviews: Allen and Arkolakis (2025) and Redding (2025) chapters in *Handbook of Regional and Urban Econ*
- Today we will cover a great example from the "regional" side (Allen and Arkolakis, 2014 QJE). Next lecture will cover a great example from the "urban" side (Ahlfeldt et al, 2015 ECMA).

Allen and Arkolakis (2014 QJE) Setup

- Basically a special case (but now for $N \geqslant 2$) of the model we saw last lecture (but we will now use AA's notation). That is:

- No agriculture ($\beta = 1$, $\sum_i L_i^A = 0$, in our old notation)

- Only sector is the "manufacturing" sector from KLR in previous lecture
  - Armington-differentiated (EoS $= \varepsilon + 1$)
  - Uses labor, paid w_i
  - CRTS production but with EES, productivity: $A_i = \bar{A}_i L_i^{\psi}$ (NB: AA write $\psi = \alpha$)
  - Trade costs: $\tau_{ij} \geqslant 1$, $\tau_{ii} = 1$

- Free labor mobility:
  - $W_i = \frac{w_i}{P_i} u_i$, with amenity $u_i = \bar{u}_i L_i^{-\delta}$ (NB: AA write $\delta = -\beta$)
  - $W_i = W$, for some endogenous W, in all locations with $L_i > 0$
  - Aggregate labor supply fixed: $\sum_i L_i = \bar{L}$

Equilibrium

- As before, will consider an equilibrium to occur when...
- Markets clear:
$$w_i L_i = \sum_j X_{ij}, \tag{1}$$
- Trade is balanced:
$$w_i L_i = \sum_j X_{ji} \tag{2}$$
- Welfare is equalized:
$$W_i = W \tag{3}$$
- And the aggregate labor market clears
$$\sum_i L_i = \bar{L}$$
- Will focus on "regular" equilibria (i.e. $L_i > 0$ for all i)

Equilibrium equations

- Recall that X_{ij} is given by gravity trade equation:

$$X_{ij} = \tau_{ij}^{-\varepsilon} \left(\frac{w_i}{A_i} \right)^{-\varepsilon} w_j L_j P_j^{\varepsilon}, \text{ with } P_j^{-\varepsilon} \equiv \sum_i \tau_{ij}^{-\varepsilon} \left(\frac{w_i}{A_i} \right)^{-\varepsilon}$$

- Then substitute this into (1), (2), and (3) and use $W_i = \frac{w_i}{P_i} u_i$, $A_i = \bar{A}_i L_i^{\psi}$, and $u_i = \bar{u}_i L_i^{-\delta}$. Get everything in terms of (w_i, L_i, W).

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- Then this reduces to two systems of equations (but NB: AA show that if τ_{ij} is symmetric then this can actually be written as one system):

$$L_i^{1-\psi\varepsilon} w_i^{\varepsilon+1} = W^{-\varepsilon} \sum_j \tau_{ij}^{-\varepsilon} \bar{A}_i^{\varepsilon} \bar{u}_j^{\varepsilon} L_j^{1-\delta\varepsilon} w_j^{\varepsilon+1} \qquad (4)$$

$$w_i^{-\varepsilon} L_i^{\delta\varepsilon} = W^{-\varepsilon} \sum_j \tau_{ji}^{-\varepsilon} \bar{A}_j^{\varepsilon} u_i^{\varepsilon} w_j^{\varepsilon} L_j^{\psi\varepsilon} \qquad (5)$$

- Non-linear system of $2N$ eq. with $2N+1$ unknowns ($\{w_i, L_i\}$, W).

A special case: no spillovers ($\psi = \delta = 0$)

- Then equilibrium system becomes:

$$\left(L_i w_i^{\varepsilon+1}\right) = W^{-\varepsilon} \sum_{j \in \mathcal{N}} \tau_{ij}^{-\varepsilon} \bar{A}_i^{\varepsilon} \bar{u}_j^{\varepsilon} \left(L_j w_j^{\varepsilon+1}\right)$$

$$\left(w_i^{-\varepsilon}\right) = W^{-\varepsilon} \sum_{j \in \mathcal{N}} \tau_{ji}^{-\varepsilon} \bar{A}_j^{\varepsilon} u_i^{\varepsilon} \left(w_j^{-\varepsilon}\right)$$

- Define vectors $x_i \equiv L_i w_i^{\varepsilon+1}$, $y_i \equiv w_i^{-\varepsilon}$ and matrix $\mathbf{T} \equiv \left[\tau_{ij}^{-\varepsilon} \bar{A}_i^{\varepsilon} \bar{u}_j^{\varepsilon}\right]_{ij}$. Then in matrix notation we have:

$$W^{\varepsilon}\mathbf{x} = \mathbf{T}\mathbf{x}$$

$$W^{\varepsilon}\mathbf{y} = \mathbf{T}'\mathbf{y}$$

- *Implication*: x_i and y_i are the eigenvectors of matrix $\mathbf{T}$ and its transpose, respectively, with W^{ε} its (common) eigenvalue. Perron-Frobenius theorem: if $\mathbf{T}$ is a strictly positive matrix then positive (i.e. economically interesting) eigenvectors are unique and a simple iterative procedure will find them.

The general case: with spillovers ($\psi \neq 0$, $\delta \neq 0$)

- Recall the equilibrium system:

$$W^{\varepsilon} L_i^{1-\psi\varepsilon} w_i^{\varepsilon+1} = \sum_{j\in\mathcal{N}} T_{ij} L_j^{1-\delta\varepsilon} w_j^{\varepsilon+1}$$

$$W^{\varepsilon} L_i^{\delta\varepsilon} w_i^{-\varepsilon} = \sum_{j\in\mathcal{N}} T_{ji} L_j^{\psi\varepsilon} w_j^{-\varepsilon}$$

- Now use change of variables: $x_i \equiv L_i^{1-\psi\varepsilon} w_i^{\varepsilon+1}$, $y_i \equiv L_i^{\delta\varepsilon} w_i^{-\varepsilon}$.
- Then system becomes:

$$W^{\varepsilon} x_i = \sum_{j\in\mathcal{N}} T_{ij} x_j^{a_{11}} y_j^{a_{12}} \tag{6}$$

$$W^{\varepsilon} y_i = \sum_{j\in\mathcal{N}} T_{ji} x_j^{a_{21}} y_j^{a_{22}} \tag{7}$$

with the definition:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \equiv \begin{pmatrix} 1+\delta\varepsilon & \varepsilon+1 \\ \psi\varepsilon & -\varepsilon \end{pmatrix} \begin{pmatrix} 1-\psi\varepsilon & \varepsilon+1 \\ \delta\varepsilon & -\varepsilon \end{pmatrix}^{-1}$$

Aside: Allen, Arkolakis and Li (AER:I 2024)

- Consider a system of H *systems* of equations (each of size N) in the NH unknowns x_{ih}, where system h is given by:

$$x_{ih} = \sum_j f_{ijh}(x_{j1}, ...x_{jH}) \tag{8}$$

- **Theorem:** (AAL, 2024)
  - Let $\epsilon_{ijh,jh'}(x_j) \equiv \frac{\partial \ln f_{ijh}(x_j)}{\partial \ln x_{jh'}}$, let the $H \times H$ matrix $\boldsymbol{A}$ with the (h, h') element given by $\boldsymbol{A}_{h,h'} \equiv \max_{i,j,x_j} \left|\epsilon_{ijh,jh'}(x_j)\right|$, and let $\rho(\cdot)$ denote the spectral radius (largest eigenvalue) matrix operator.
  - Then, if $\rho(\boldsymbol{A}) < 1$, there exists a unique regular equilibrium solution to (8).
  - Further, in this case the unique solution can be found by a simple iterative procedure from any starting guess.

- So in many ways, as long as $\rho(\boldsymbol{A}) < 1$ many of the simplicities of the no-spillovers (linear) case carry over to these more complicated models with (weak) positive externalities

Applying AAL to AA

- System in (6) and (7) fits into the AAL theorem with

$$\boldsymbol{A} \equiv \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{9}$$

- This will satisfy $\rho(\boldsymbol{A}) < 1$ for low values of ψ and $-\delta$
  - Indeed, for "reasonable" values of ε, the condition $\rho(\boldsymbol{A}) < 1$ is pretty close to simply $\psi < \delta$

- What about *necessary* conditions for uniqueness?
  - As we saw last lecture, KLR (2024) showed that in the AA (2014) model $\psi < \delta$ is not necessary for uniqueness away from autarky.
  - But AAL show that if $\rho(\boldsymbol{A}) \geqslant 1$ then there always exists *some* value of fundamentals (e.g. things like τ_{ij}, $\bar{A}_i$, $\bar{u}_i$ in AA) such that multiplicity of regular equilibria occurs.
  - So $\rho(\boldsymbol{A}) \geqslant 1$ is necessary if we are completely agnostic about fundamentals. This may be useful in some estimation problems (e.g. for applying some of the results in Tishara Garg's 2025 JMP).

Applying these results

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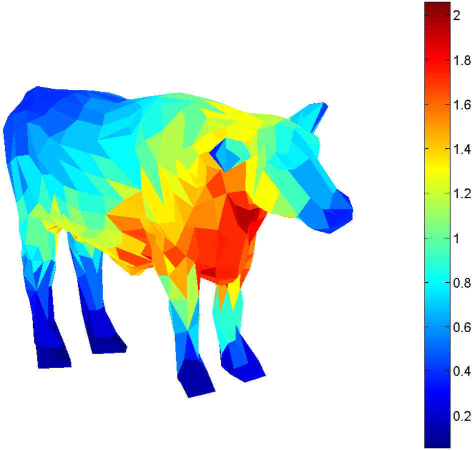
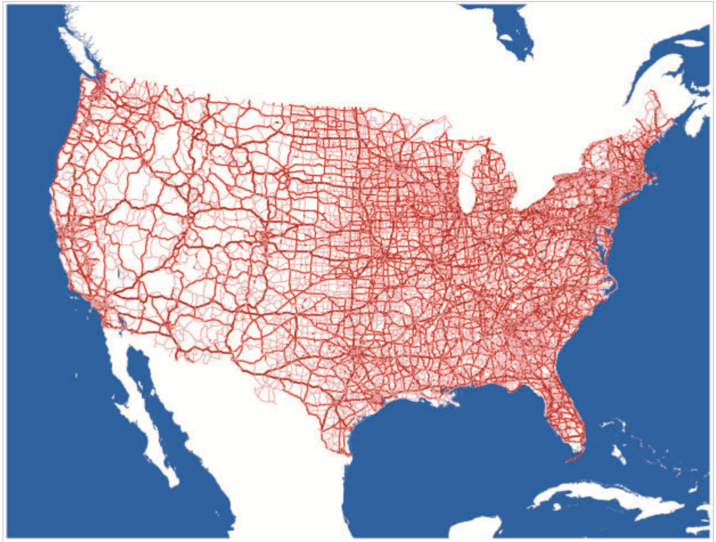


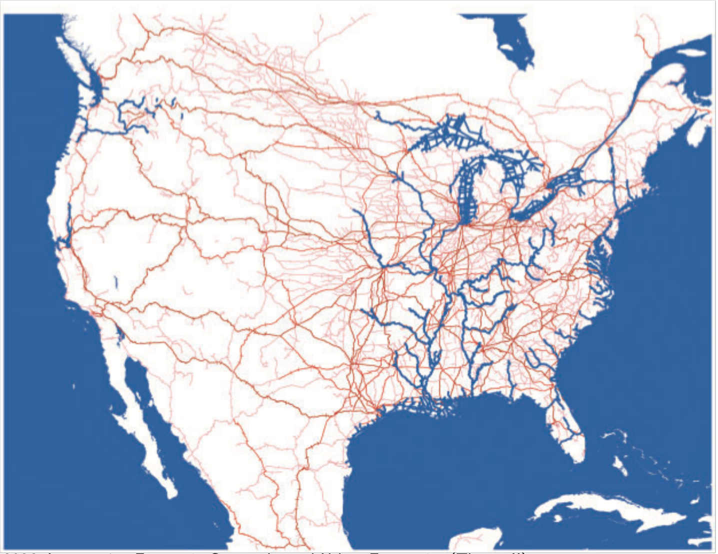
Applying these results

- AA go on to show how, with knowledge of $(\psi, \delta, \varepsilon)$, can either:
  1. With data on X_{ij}, apply usual "exact hat" approach to studying counterfactuals. (Math for proportional changes is the same as that for equilibrium, above.)
  2. Or, with knowledge of τ_{ij}, and data on (w_i, L_i), "invert the model" to back out set of $(\bar{A}_i, \bar{u}_i)$. Then solve for new equilibrium as result of any change in exogenous variables.
- AA pursue option #2 here—interested in high-resolution context, for which trade data X_{ij} not available.
  - Take elasticities from prior estimation in the literature:
    - $\psi = 0.1$: guided by ballpark we saw in Econ Geography lectures 2-3
    - $\delta = 0.3$: model isomorphic to one with fixed housing stock in each location, and with C-D prefs across good and housing; under this view, $\delta = 0.3$ matches data on housing expenditure
    - $\varepsilon = 8$: following EK 2002; higher than for international trade, which may be plausible—modulo the comments from last lecture about ag. vs. manuf. elasticities)
  - Estimate τ_{ij} via "FMM" procedure based on CFS (2007) US trade data
  - US County-level data on (w_i, L_i)

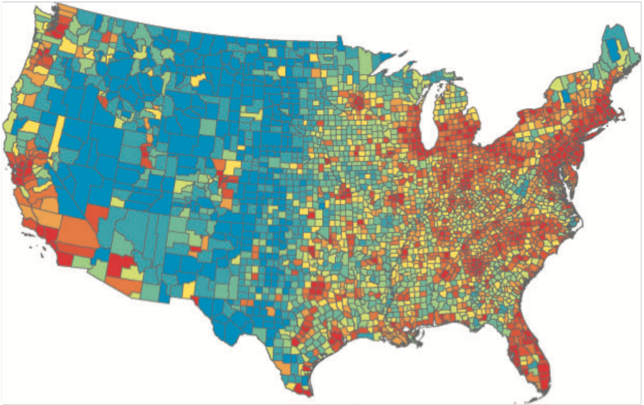
The transportation network (used for estimating τ_{ij})—Highways (Interstate system and minor)



The transportation network (used for estimating τ_{ij})—Rail and Water

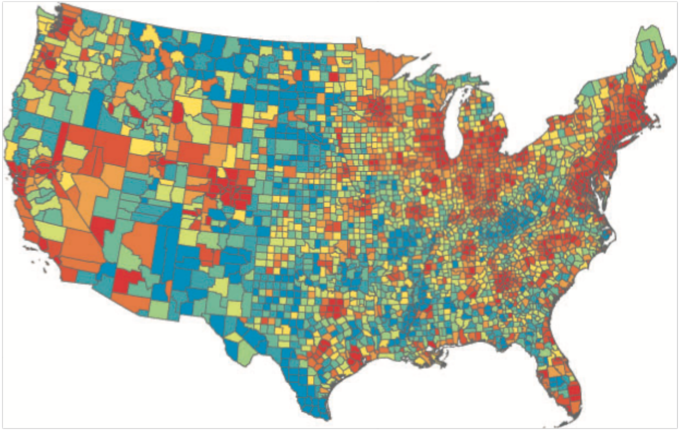


Data on L_i



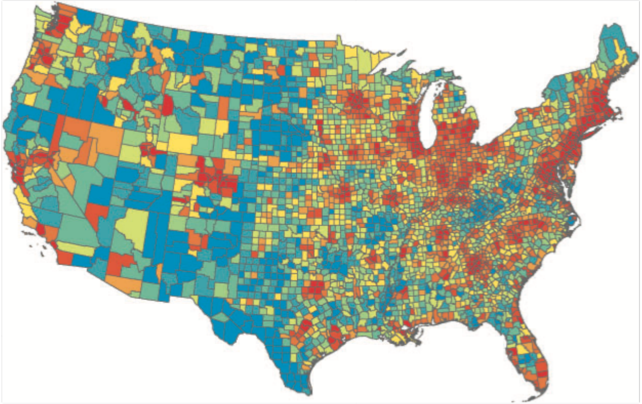
Population density

Data on w_i



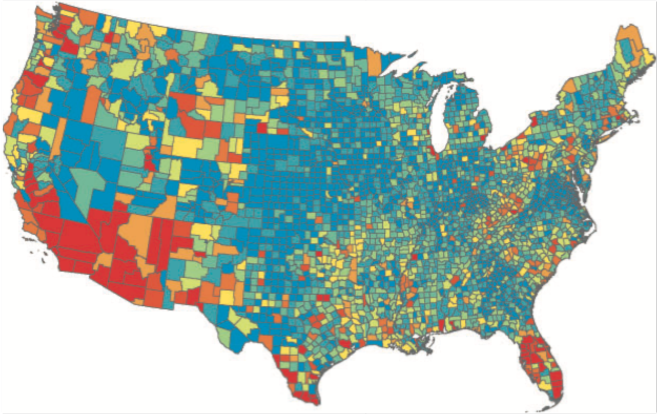
Wages

Result of model inversion $(\psi, \delta, \varepsilon) + \{w_i, L_i\} \Rightarrow \{\bar{A}_i\}$



Exogenous productivity

Result of model inversion $(\psi, \delta, \varepsilon) + \{w_i, L_i\} \Rightarrow \{\bar{u}_i\}$



Exogenous amenity

Counterfactual Simulation: remove interstate highway system

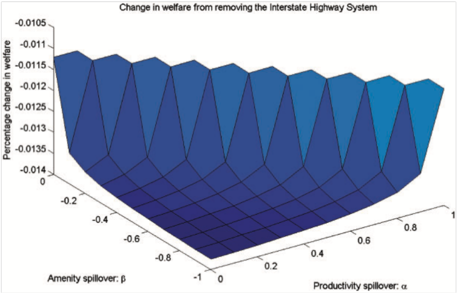


FIGURE XIX

Estimated Decline in Welfare from Removing the Interstate Highway System

This figure shows the estimated decline in welfare (in percentage terms) from the removal of the IHS for each combination of productivity spillover strength $\alpha \in [0, 1]$ and $\beta \in [-1, 0]$ such that $\alpha + \beta \leq 0$.

Implies that IHS passes a simple cost-benefit test (c. 2007). Simulation says annual benefits are $\approx$ \$200 B; perpetuity cost of building the system (at 5% cost of capital) is annual $\approx$ \$30 B, and maintenance is annual $\approx$ \$70 B.