MIT 14.582 PhD International Economics II — Lecture 14: Economic Geography and Urban Economics (Theory I) —

Dave Donaldson

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Plan for Today's Lecture

- Introduction to simple economic geography models
- A more detailed look at a core economic geography model (Kucheryyavy, Lyn and Rodriguez-Clare, 2021)

Economic Geography Theory

- Previous 2 lectures have established the plausibility of agglomeration externalities

- A number of questions present themselves:
  - Q1: What determines spatial allocation of economic activities? Fundamentals? Spillovers?
  - Q2: When is spatial concentration of economic activities sustainable?
  - Q3: How do changes in market integration (trade costs, migration costs) affect the answers to the previous question?
  - Q4: Is the market allocation of activity to space efficient? Equitable?

- We will now begin towards an understanding of these questions, with the help of some simple theory

A Toy Model

- We'll start with a simple example today to build intuition.
  - Next two lectures will show the frontier of class of models that build on this setup.
- Consider Armington model with:
  - Two locations $i = 1, 2$
  - Perfect labor mobility between the two locations
  - External economies of scale (EES) in production
- Utility (among people who reside in i):

$$U_i = \Big[\sum_{j=1,2} (c_{ij})^{\frac{\sigma-1}{\sigma}} \Big]^{\frac{\sigma}{\sigma-1}}, \text{ with } \sigma > 1$$

- Technology (for any firm using amount of labor input l in location i):

$$q_i = A(L_i) l_i, \text{ with } A(L_i) = L_i^{\psi} \text{ and } \psi > 0$$

where L_i = total employment in location i

- Total labor in fixed supply: $L_1 + L_2 = L$

Free Trade Equilibrium

- We'll focus first on free trade equilibrium:
  - Firms maximize profits taking prices and total employment levels as given; consumers maximize utility; markets clear.
- Equilibrium conditions can be reduced to intersection of (relative) labor supply and (relative) labor demand
  - Same idea as trade in factor services in 14.581, but with labor supply now a (perfectly) elastic function (of relative *real* wages)
- **Relative labor supply** (since $p_{k1} = p_{k2}$ in any region k, by free trade):

$$\frac{w_2}{w_1} = 1 \text{ if } \frac{L_2}{L_1} \in (0, \infty)$$

- **Relative labor demand:**

$$\frac{Q_2}{Q_1} = \left(\frac{p_2}{p_1}\right)^{-\sigma} \iff \frac{A(L_2)L_2}{A(L_1)L_1} = \left(\frac{w_2/A(L_2)}{w_1/A(L_1)}\right)^{-\sigma}$$

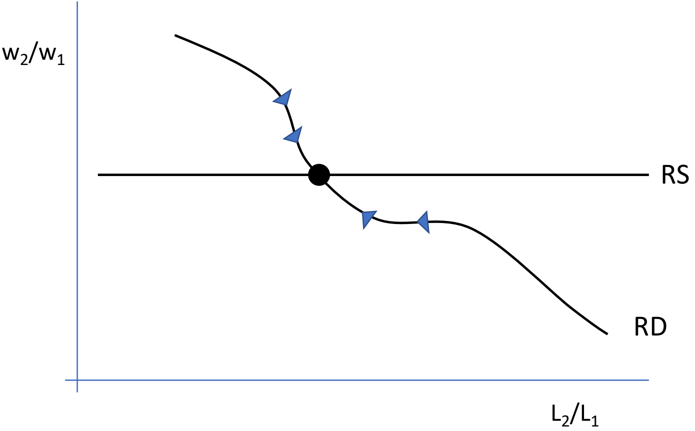
$$\iff \frac{w_2}{w_1} = \left(\frac{L_2}{L_1}\right)^{-\frac{1+\psi(1-\sigma)}{\sigma}}$$

A Quick Detour: Stable and Unstable Equilibria

- Most economic geography models (like trade models) are static
- How do we think about stability in this context?
- Fujita, Krugman and Venables (1999) propose to assume the following ad-hoc dynamics:
  - At any date t, $\frac{w_{2,t}}{w_{1,t}} > 1 \Rightarrow \frac{d(L_{2,t}/L_{1,t})}{dt} > 0$ and $\frac{w_{2,t}}{w_{1,t}} < 1 \Rightarrow \frac{d(L_{2,t}/L_{1,t})}{dt} < 0$
  - This is like quantity tatonnement:
    - In the short-run, i.e. for fixed labor allocation, the economy is in equilibrium
    - In the long-run if welfare not equalized across locations, people tend to move where welfare is higher
- **Definition:** *A competitive equilibrium is stable if, starting from an arbitrary distribution of labor around that equilibrium, movements along the relative labor demand curve satisfying the previous ad-hoc dynamic would lead back to that same equilibrium*

Free Trade in a Neoclassical Economy: $\psi = 0$

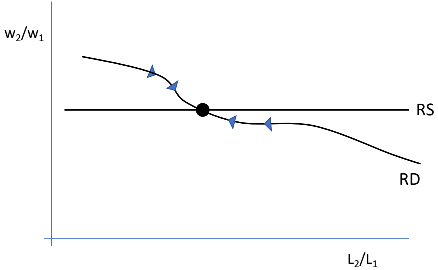
- Recall, inverse RD curve is $\frac{w_2}{w_1} = \left(\frac{L_2}{L_1}\right)^{-\frac{1+\psi(1-\sigma)}{\sigma}}$



- Unique stable equilibrium is interior: no economic concentration

Free Trade with Weak IRS: $\psi > 0$ but $\psi(\sigma - 1) < 1$

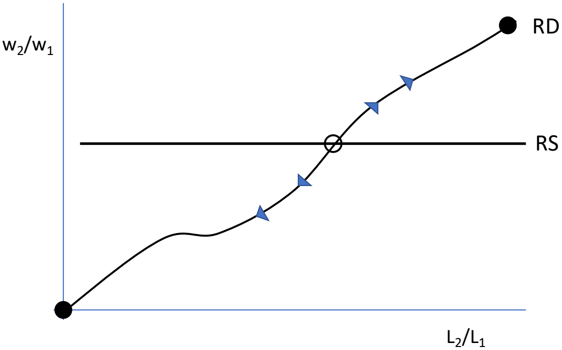
- Recall, inverse RD curve is $\frac{w_2}{w_1} = \left(\frac{L_2}{L_1}\right)^{-\frac{1+\psi(1-\sigma)}{\sigma}}$



- Relative labor demand curve is now flatter
  - Since larger region is becoming relatively more productive, a smaller decrease in relative wages is required to incentivize firms to hire more workers there
- But relative labor demand curve remains downward slopping
  - Unique stable equilibrium is still interior: no economic concentration

Free Trade with Strong IRS: $\psi(\sigma - 1) > 1$

- Recall, inverse RD curve is $\frac{w_2}{w_1} = \left(\frac{L_2}{L_1}\right)^{-\frac{1+\psi(1-\sigma)}{\sigma}}$



- Relative labor demand curve is upward slopping:
  - Two stable equilibria are corners: economic concentration
  - One unstable interior equilibrium

Spatial Concentration and IRS (Q2)

- **Summary so far:** *If EES large enough (i.e. $\psi(\sigma - 1) > 1$), then only stable equilibria are corner equilibria (extreme spatial concentration). Otherwise, unique equilibrium = interior equilibrium .*

- Aside on $\sigma > 1$ assumption: note that $\psi(\sigma - 1) > 1 \iff$ "EES large enough" only if $\sigma > 1$:
  - If $\sigma < 1$, then an increase in ψ makes relative labor demand curve *more* downward sloping...
    - Recall impact of productivity shocks in two-sector closed economy
  - Since we do not have monopolistic competition, a priori no theoretical reason to restrict attention to $\sigma > 1$
  - On empirical grounds, one may argue that $\sigma > 1$ is relevant...
    - True if we think about aggregate trade elasticity between countries (standard estimates > 1).
    - ...but within a country, can't we imagine that many rural location export agricultural goods in exchange for manufacturing goods? If so, perhaps the elasticity of substitution between agriculture and manufacturing (typically estimated < 1) should apply?

Autarky Equilibrium

- What about the role of trade costs (Q3)?
- To start, consider extreme case where the two regions cannot trade
- **Relative inverse (real) labor supply:**

$$\frac{w_2/p_2}{w_1/p_1} = 1 \text{ if } \frac{L_2}{L_1} \in (0, \infty)$$

- **Relative inverse (real) labor demand is:**

$$\frac{p_2}{p_1} = \frac{w_2/A(L_2)}{w_1/A(L_1)} \iff \frac{w_2/p_2}{w_1/p_1} = \left(\frac{L_2}{L_1}\right)^{\psi}$$

- Under autarky, real wage = aggregate productivity at each location
- Suppose that $\psi(\sigma - 1) < 1$. As we go from free trade to autarky:
  - RD goes from being downward-sloping to upward-sloping (if $\psi > 0$)
  - So we go from unique (stable) interior equilibrium to multiple (stable) corner equilibria
  - That is: trade integration goes *against* spatial concentration

Equilibrium with Arbitrary Trade Costs

- More generally, suppose that there are iceberg trade costs, $\tau \geq 1$
- **Relative (real) labor supply:**

$$\frac{w_2 / P_2}{w_1 / P_1} = 1 \text{ if } \frac{L_2}{L_1} \in (0, \infty)$$

where P_i is CES price index at location i such that

$$P_i^{1-\sigma} = \left[\frac{w_i}{A(L_i)}\right]^{1-\sigma} + \left[\frac{\tau w_j}{A(L_j)}\right]^{1-\sigma}$$

- **RS Curve=** Solution $\frac{w_2}{w_1}$ of

$$\frac{\left(\tau \frac{w_1}{w_2}\right)^{1-\sigma} + \left(\frac{L_2}{L_1}\right)^{-\psi(1-\sigma)}}{1 + \left(\tau \frac{w_2}{w_1}\right)^{1-\sigma}\left(\frac{L_2}{L_1}\right)^{-\psi(1-\sigma)}} = 1 \tag{1}$$

Equilibrium with Arbitrary Trade Costs (Continued)

- **Relative (real) labor demand:**

$$\frac{p_2 Q_2}{p_1 Q_1} = \frac{\left(\frac{\tau p_2}{P_1}\right)^{1-\sigma} w_1 L_1 + \left(\frac{p_2}{P_2}\right)^{1-\sigma} w_2 L_2}{\left(\frac{p_1}{P_1}\right)^{1-\sigma} w_1 L_1 + \left(\frac{\tau p_1}{P_2}\right)^{1-\sigma} w_2 L_2}$$

$$\iff \frac{w_2 L_2}{w_1 L_1} = \frac{\left(\frac{\tau w_2}{P_1 A(L_2)}\right)^{1-\sigma} w_1 L_1 + \left(\frac{w_2}{P_2 A(L_2)}\right)^{1-\sigma} w_2 L_2}{\left(\frac{w_1}{P_1 A(L_1)}\right)^{1-\sigma} w_1 L_1 + \left(\frac{\tau w_1}{P_2 A(L_1)}\right)^{1-\sigma} w_2 L_2}$$

- **RD Curve=** Solution $\frac{w_2}{w_1}$ of

$$\frac{\left(\frac{(\frac{\tau w_2}{w_1})^{1-\sigma}(\frac{L_2}{L_1})^{-\psi(1-\sigma)}}{1+(\frac{\tau w_2}{w_1})^{1-\sigma}(\frac{L_2}{L_1})^{-\psi(1-\sigma)}}\right)^{1-\sigma} \frac{w_1 L_1}{w_2 L_2} + \left(\frac{(\frac{w_2}{\tau w_1})^{1-\sigma}(\frac{L_2}{L_1})^{-\psi(1-\sigma)}}{1+(\frac{w_2}{\tau w_1})^{1-\sigma}(\frac{L_2}{L_1})^{-\psi(1-\sigma)}}\right)^{1-\sigma}}{\left(\frac{1}{1+(\frac{\tau w_2}{w_1})^{1-\sigma}(\frac{L_2}{L_1})^{-\psi(1-\sigma)}}\right)^{1-\sigma} + \left(\frac{1}{1+(\frac{w_2}{\tau w_1})^{1-\sigma}(\frac{L_2}{L_1})^{-\psi(1-\sigma)}}\right)^{1-\sigma} \frac{w_2 L_2}{w_1 L_1}} = 1 \tag{2}$$

Equilibrium with Arbitrary Trade Costs (Continued)

- In line with analysis of free trade and autarky equilibria in the $\psi(\sigma - 1) < 1$ case, one can show (see Kucheryavyy, Lin, and Rodriguez-Clare, 2021) the existence of a cut-off for trade costs,

$$\tau^* = \left[\frac{2 + \psi}{\psi(1 + 2\sigma)} \right]^{1/\sigma},$$

  such that there exists a unique (stable) interior equilibrium iff $\tau < \tau^*$

- **Heuristic Proof:**
  - Differentiate (1) and (2) to compute $\frac{d(w_2/w_1)}{d(L_2/L_1)}|_{RS}$ and $\frac{d(w_2/w_1)}{d(L_2/L_1)}|_{RD}$
  - Compare $\frac{d(w_2/w_1)}{d(L_2/L_1)}|_{RS}$ and $\frac{d(w_2/w_1)}{d(L_2/L_1)}|_{RD}$ at symmetric eq $(\frac{w_2}{w_1} = 1, \frac{L_2}{L_1} = 1)$
  - τ^* is s.t. we switch from $\left|\frac{d(w_2/w_1)}{d(L_2/L_1)}|_{RS}\right| <$ to $> \left|\frac{d(w_2/w_1)}{d(L_2/L_1)}|_{RD}\right|$

Equilibrium with Arbitrary Trade Costs (Continued)

- In line with analysis of free trade and autarky equilibria in the case $\psi(\sigma - 1) < 1$, one can show (see Kucheryavyy, Lin, and Rodriguez-Clare 2021) the existence of a cut-off for trade costs,

$$\tau^* = \left[\frac{2 + \psi}{\psi(1 + 2\sigma)} \right]^{1/\sigma},$$

  such that there exists a unique (stable) interior equilibrium iff $\tau < \tau^*$
- **Intuition:**
  - In an Armington world, trade is a source of diminishing marginal returns (recall Acemoglu and Ventura, 2000)
  - Under free trade, we assume that this force is the dominant one ($\psi(\sigma - 1) < 1$), which leads to a unique interior equilibrium
  - As trade costs rise, source of diminishing marginal returns become less and less important and multiplicity of equilibria arises
- More on KLR (2021) below!

Lower Trade Costs $\Rightarrow$ Less Spatial Concentration?

- In toy model, trade integration = force against spatial concentration
- **Questions:**
  - Is this a robust feature of economic geography models, that arises from IRS and factor mobility?
  - Is this specific to the way trade has been modeled?
- Answer depends on whether trade is a source of diminishing marginal returns. In Armington, it is, but it does not have to be:
  - In Ventura (1997), trade leads to "factor price insensitivity" (to factor endowment shocks) and removes diminishing marginal returns to capital accumulation
  - In Matsuyama (1992), a small open economy behaves like a closed economy with linear utility
  - In such environments, we expect the opposite comparative static results
- To study this in more detail, need an extended model

An extended model (KLR 2021)

- Like above: 2 locations
- But now 2 sectors:
  - "Manufacturing":
    - Armington-differentiated ($\varepsilon = \sigma - 1$ from above)
    - Uses freely mobile labor, paid w_i
    - Productivity $A_i = \bar{A}_i L_i^{\psi}$
    - Trade costs $\tau_{ij} \geq 1$, $\tau_{ii} = 1$
  - "Agriculture":
    - Competitive and homogeneous good
    - Uses agricultural labor, in fixed supply $\bar{L}_i^A$
    - Same CRTS technology (productivity = 1) in both locations.
    - Always produced in both locations. So $p_i^A = w_i^A$.
    - Freely traded, so $w_i^A = w^A$ for all i
- *Preferences:* Nested CES (Cobb-Douglas upper-tier, with weight $\beta > 0$ on the manuf. good). And residential amenities with congestion externality. So:

$$U_i = \frac{w_i}{P_i^{\beta} (w^A)^{1-\beta}} u_i, \qquad \text{with} \qquad u_i \equiv \bar{u}_i L_i^{-\delta}$$

Equilibrium conditions

- Manufacturing goods market clearing:

$$w_i L_i = \sum_n \lambda_{ni} \beta (w_n L_n + w^A \bar{L}_n)$$
$$\lambda_{ni} \equiv \bar{A}_i^{\varepsilon} L_i^{\varepsilon\psi} (w_i \tau_{ni})^{-\varepsilon} P_n^{\varepsilon}$$

- Agricultural goods market clearing:

$$\sum_i w^A \bar{L}_i^A = (1-\beta) \sum_i (w_i L_i + w^A \bar{L}_i^A)$$

- Migration (for some $\bar{U}$):

$$L_i \geq 0, \qquad \bar{U} - U_i \geq 0, \qquad L_i(\bar{U} - U_i) = 0$$

- Manufacturing labor market clearing:

$$\sum_i L_i = \bar{L}$$

Special Cases

- This model nests a number of highly influential models as special cases...
- Define:

$$\alpha \equiv \frac{(\psi - \delta)\varepsilon}{1 + \delta}$$

- Krugman (JPE 1991) "Core-periphery model":
  - Just set $\alpha = 1$. But in practice, K (1991) uses Dixit-Stiglitz microfoundations, so: $\delta = 0$ and $\psi = 1/\varepsilon$.
  - And symmetric: $\bar{A}_i = 1$, for all i; $\bar{u}_i = 1$, for all i
- Allen and Arkolakis (QJE 2014):
  - Actually have arbitrary N locations (even infinite number)
  - But $\beta = 1$, $\sum_i \bar{L}_i^A = 0$ (what we'll call "no agriculture")
  - We will cover the $N > 2$ version of this in next lecture...

Regular/irregular equilibria

- AA (2014) define:
  - *Regular equilibrium*: $L_i > 0$ for all i
  - *Irregular equilibrium*: $L_i = 0$ for at least one i
- KLR's **Proposition 1:**
  - If $\alpha < 1$: all equilibria are regular
  - If $\alpha > 1$: both irregular outcomes ($L_1 = 0$ and $L_2 = \bar{L}$; and vice versa) are equilibria
  - (If $\alpha = 1$: it's complicated)
- This nicely generalizes what we had already seen for the simple model above

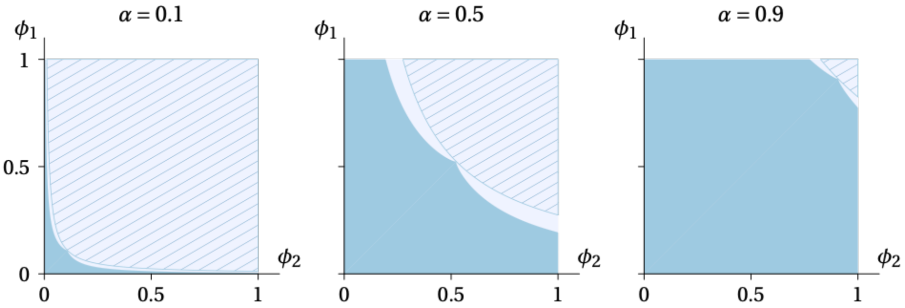
Costly trade

- Define "free-ness" of trade: $\phi_1 \equiv \tau_{12}^{-\varepsilon}$ etc. Costly trade case is then where $\phi_1\phi_2 < 1$
- KLR's **Proposition 5 & Corollary 1:** With costly trade:
  - for $\alpha \leq 0$, there is a unique regular equilibrium
  - for $0 < \alpha < 1$, there is a unique regular equilibrium whenever trade costs are low enough ($\phi_1\phi_2 \to 1$), or α is low enough, or β is low enough
  - for $\alpha \neq 1$, there exists no more than 5 regular equilibria
  - for $\alpha = 1$, there exists no more than 3 regular equilibria
- So the broad idea that at lower trade costs we can sustain a interior/regular equilibrium even at positive EES continues to hold

Two special (but useful, and famous) cases

- Next steps will explore special cases where we can learn more
- **Part 1**: AA (2014) economy (i.e. $\beta = 1$)
  - (a) Symmetric fundamentals (i.e. $G = 1$) but asymmetric TCs (i.e. $\phi_1 \neq \phi_2$)
  - (b) Symmetric TCs (i.e. $\phi_1 = \phi_2$) but asymmetric fundamentals parameterized by $G \equiv \left(\frac{\bar{u}_1}{\bar{u}_2}\right)^{\alpha+\varepsilon} \left(\frac{\bar{A}_1}{\bar{A}_2}\right)^{\varepsilon}$
- **Part 2**: Krugman (1991) economy (i.e. $\beta < 1$)
  - Focus on $\alpha = 1$ case
  - Also symmetric economies:
    - $G = 1$,
    - $\gamma \equiv \frac{L_1^A}{L_1^A + L_2^A} = 0.5$
    - $\phi_1 = \phi_2 = \phi$ (i.e. $\tau_{12} = \tau_{21} = \tau$)

Part 1(a): AA (2014) economy with symmetric fundamentals but asymmetric TCs



(a) Varying ϕ_1 and ϕ_2 with $G = 1$.

See full legend on next slide (but essentially, dark blue $\iff$ multiplicity). So, loosely speaking, it's the product $\phi_1\phi_2$ that governs the uniqueness boundary...but need lower TCs to drive uniqueness when α is high (as in our toy model which had $\phi_1 = \phi_2$).

Part 1(b): AA (2014) economy with symmetric TCs but asymmetric fundamentals

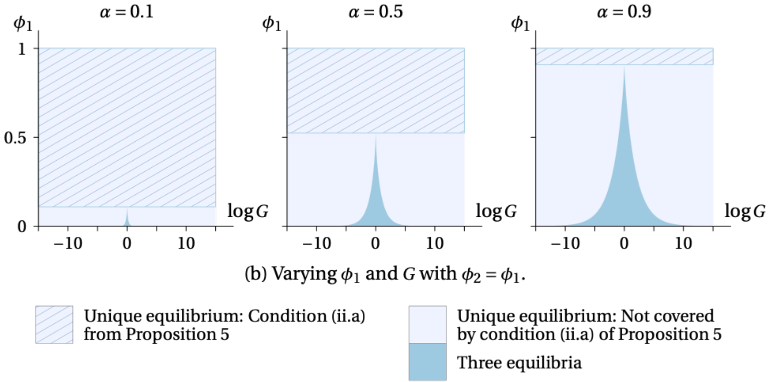


Figure 3: Uniqueness/multiplicity areas in case (ii) of Proposition 7 for AA economy: $0 < \alpha < 1$ and $\varepsilon = 5$.

So at any $\alpha < 1$, asymmetric fundamentals ($G \neq 1$), and/or low trade costs (high ϕ_1) supports uniqueness. What is the intuition?

Part 2: Krugman (1991) economy

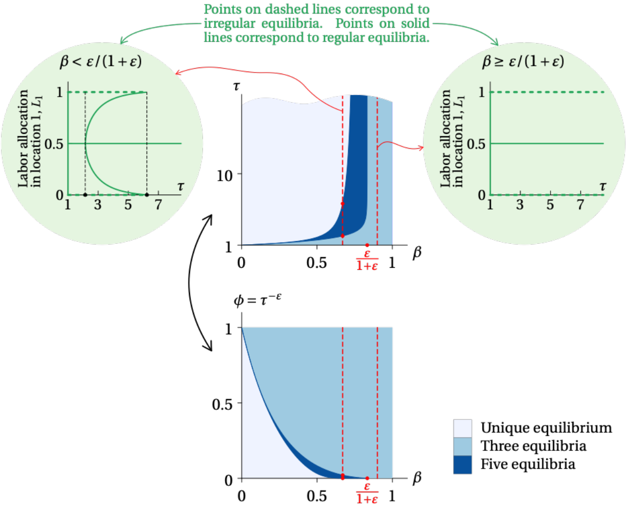


Figure 7: Symmetric Krugman case, $\varepsilon = 5$, $\alpha = 1$, $\overline{L} = 1$, and $\tau_{12} = \tau_{21} \equiv \tau$. Both regular and irregular equilibria are counted in the figures.

(Case with $\beta < \frac{\varepsilon}{1+\varepsilon}$ often referred to as the "no black hole condition".)

Part 2: Krugman (1991) economy

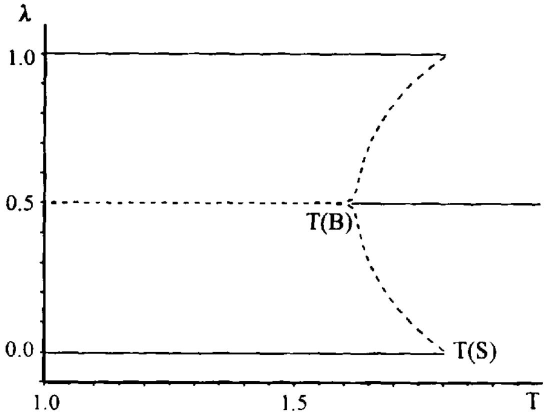


Figure 5.4
Core-periphery bifurcation

- From Fujita, Krugman, Venables (1999) book
- $\lambda \equiv L_1/\bar{L}_2$; $T \equiv \tau^{\varepsilon}$
- Often called the "tomahawk diagram"

Part 2: Krugman (1991) economy

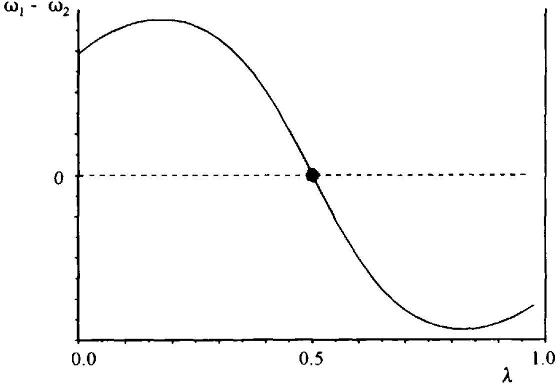


Figure 5.1
Real wage differentials, T = 2.1

($\omega_i \equiv$ real wage in i.) High trade costs $\Rightarrow$ unique equilibrium

Part 2: Krugman (1991) economy

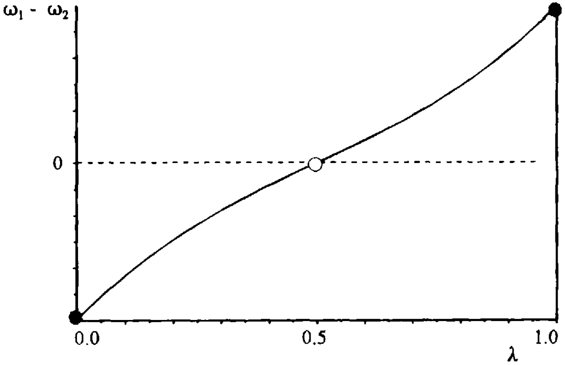


Figure 5.2
Real wage differentials, $T = 1.5$

Low trade costs $\Rightarrow$ three equilibria (one regular)

Part 2: Krugman (1991) economy

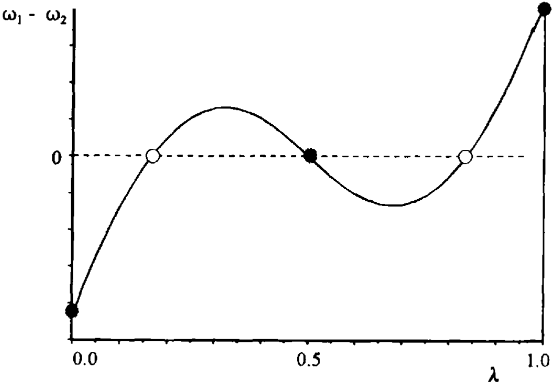


Figure 5.3
Real wage differentials, $T = 1.7$

Medium trade costs $\Rightarrow$ five equilibria (three regular)

The role of agriculture ($\beta < 1$)

- Notice what has happened here:
  - Without agriculture (i.e. AA economy): get uniqueness (despite $0 < \alpha < 1$) when τ is low
  - However, with agriculture: get uniqueness (despite $0 < \alpha < 1$) when τ is *high*

- What is going on?

The Case for Lower Trade Costs ⇒ More Concentration

- The usual intuition offered: agricultural labor can't move (and world consumers always need the ag. goods) so the demand propped up by ag. income in each region acts as a "dispersion force"
- But the "terms of trade as DRTS" view offers a different intuition:
  - First, imperfect substitutability between the two goods is a source of diminishing marginal returns in the manufacturing sector, and this source does not disappear as trade costs go to infinity
  - Second, as trade costs rise the aggregate trade elasticity (a weighted average of ε and "1") falls towards 1 (as weight on ag. trade rises). So DRTS due to ToT gets *stronger*. (We are reducing trade costs but more so in the high elasticity good.)
- When β high, ag. income almost irrelevant, so ToT motive (ε) is force for uniqueness...gets stronger with low τ
- But when β low, ag. income is important, high τ means that manufacturing output must stay near the immobile ag. labor

Further Discussion

- This point helps to clarify some disparate effects seen in the literature...
- Helpman (1998) presents a model that replaces Krugman's agriculture sector (recall: freely tradable) with a housing sector (i.e. non-tradable), and flips the Krugman (1991) result, as we would expect
- FKV's Chapter 7 is devoted to exploration of trade costs (and relaxing the homogeneous good assumption) in the ag. sector. Argue that effects in Krugman (1991) continue to hold for low ag. trade costs

Krugman (1991) model: symmetric and $\alpha < 1$

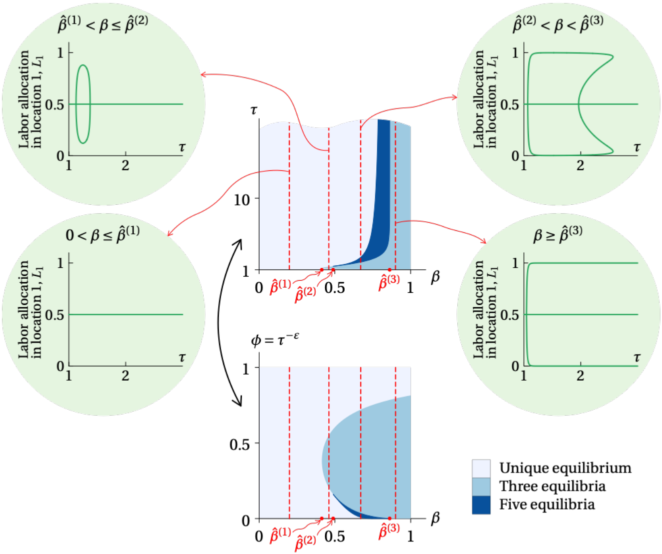
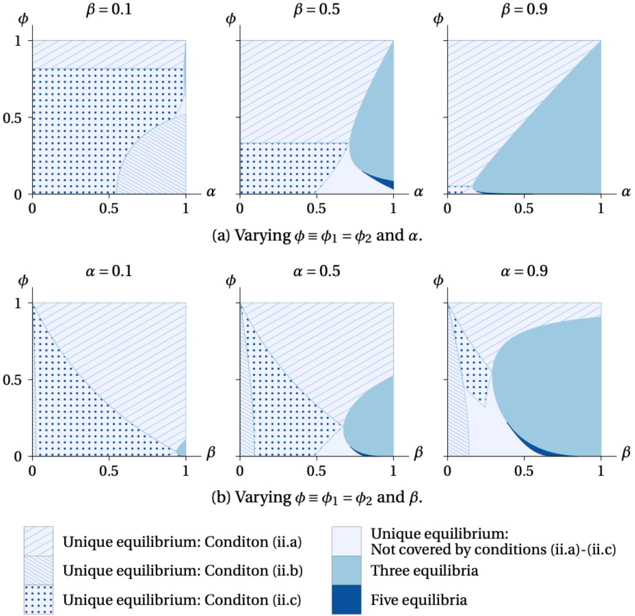
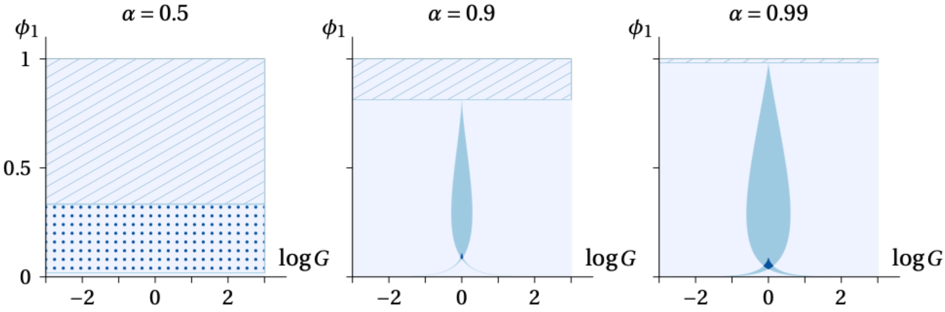


Figure 8: Symmetric case with $\varepsilon = 5$, $\alpha = 0.8$, and $\overline{L} = 1$. All equilibria are regular.

Krugman (1991) model: symmetric and $\alpha < 1$

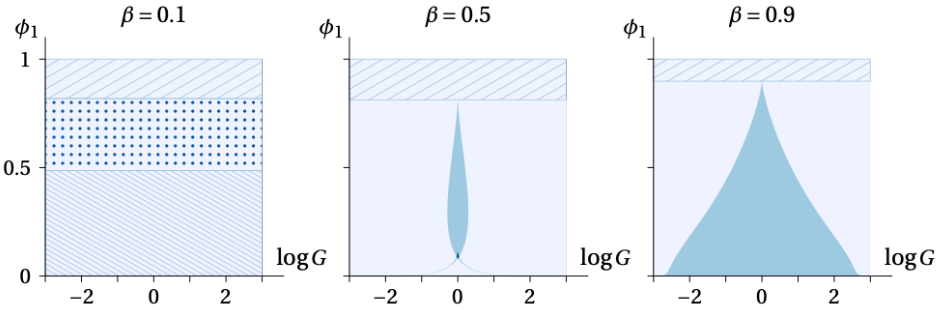


Krugman (1991) model: asymmetric and $\alpha < 1$



(c) Varying ϕ_1 and G with $\phi_2 = \phi_1$ and $\gamma = 0.5$.

Krugman (1991) model: asymmetric and $\alpha < 1$



(c) Varying ϕ_1 and G with $\phi_2 = \phi_1$ and $\gamma = 0.5$.

Alternative Models in the Literature

- Krugman and Venables (QJE 1995):
  - No factor mobility, but retain HME-like setup (2 sectors, differing amounts of IRTS, trade costs)
  - But add input-output linkages
  - Generates the same sort of "circular causation" that can lead to multiplicity
  - NB: 2023 version of KLR discusses simple IO linkages too
- Baldwin, Forslid, Martin, Ottaviano, Robert-Nicoud (2003) book:
  - Considers models with "footloose entrepreneurs", "footloose capitalists", accumulation of capital, accumulation of ideas (i.e. endogenous growth)
  - Same basic idea of models we have seen: multiplicity when you combine local IRTS and something that acts as an endogenous amplifying factor (mobile factors, new inputs made from immobile factors, new factors, new ideas, etc) that reinforces the effects of IRTS
  - Basic question is always whether agglomeration is IRTS on net (race between production externalities, congestion, and ToT effects)