Online Supplement to "Why is Trade Not Free? A Revealed Preference Approach"

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In this document, we characterize the competitive equilibrium of our economy (Section OS.1), describe our calibration procedure (Section OS.2), outline an algorithm to solve the equilibrium given a set of trade taxes and exogenous parameters (Section OS.3), and present the expressions used to compute the sensitivity of real income to imports (Section OS.4).

OS.1 Competitive Equilibrium

Prices. For each origin region or country $o \in \mathcal{R}$, destination $d \in \mathcal{R}$, and product $h \in \mathcal{H}_s$ from sector $s \in \mathcal{S}$, p_{odh} denotes the domestic price. Equations (11)-(14) then imply that for all $r \in \mathcal{R}_H$, destination $d \in \mathcal{R}$, and product $h \in \mathcal{H}_s$ from sector $s \in \mathcal{S}$,

$$p_{rdh} = (\theta_{rds})^{-1} p_{rs}, \tag{OS.1}$$

$$p_{rs} = [\alpha_s]^{-\alpha_s} [w_{rs}]^{\alpha_s} \prod_{k \in S} [\alpha_{ks}]^{-\alpha_{ks}} [P_{rk}]^{\alpha_{ks}}, \tag{OS.2}$$

$$P_{rk} = \left[\sum_{c=H,F} \theta^c_{rk} [P^c_{rk}]^{1-\kappa} \right]^{\frac{1}{1-\kappa}}, \tag{OS.3}$$

$$P^c_{rk} = \left[\sum_{v \in \mathcal{H}_k} \theta^c_{rkv} [P^c_{rkv}]^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{OS.4}$$

$$P^c_{rkv} = \left[\sum_{o \in \mathcal{R}_c} \theta^c_{orkv} [p_{orv}]^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{OS.5}$$

Our assumptions that $\theta^H_{rkv} = \bar{\theta}^H_{rk} = 1/|\mathcal{H}_k|$ and $\theta^H_{orkv} = \bar{\theta}^H_{ork}$ then imply that for all $r \in \mathcal{R}_H$ and $k \in \mathcal{S}$,

$$P^H_{rk} = \left[\sum_{o \in \mathcal{R}_H} \bar{\theta}^H_{ork} [(\theta_{ork})^{-1} p_{rk}]^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

Combining the expressions above, we express the equilibrium conditions simply in terms of the region-sector prices p_{rs}. For all $r \in \mathcal{R}_H$ and sector $s \in \mathcal{S}$,

$$p_{rs} = [\alpha_s]^{-\alpha_s}[w_{rs}]^{\alpha_s} \prod_{k \in S} [\alpha_{ks}]^{-\alpha_{ks}} [P_{rk}]^{\alpha_{ks}}, \tag{OS.6}$$

$$P_{rk} = \left[\sum_{c=H,F} \theta^c_{rk} [P^c_{rk}]^{1-\kappa} \right]^{\frac{1}{1-\kappa}}, \tag{OS.7}$$

$$P^H_{rk} = \left[\sum_{o \in \mathcal{R}_H} \tilde{\theta}^H_{ork} [p_{ok}]^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{OS.8}$$

$$P^F_{rk} = \left[\sum_{v \in \mathcal{H}_k} \theta^F_{rkv} [P^F_{rkv}]^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{OS.9}$$

$$P^F_{rkv} = \left[\sum_{o \in \mathcal{R}_F} \theta^F_{orkv} [p_{orv}]^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{OS.10}$$

where $\tilde{\theta}^H_{ork} \equiv \bar{\theta}^H_{ork} / (\theta_{ork})^{1-\sigma}$.

Bilateral Trade Flows. The expressions for prices in equations (OS.6)-(OS.10) and the expressions for technology and preferences in equations (11)-(14) and (15)-(18) imply that the (tariff-inclusive) spending in domestic region $r \in \mathcal{R}_H$ on product $h \in \mathcal{H}_s$ of sector $s \in \mathcal{S}$ from foreign country $i \in \mathcal{R}_F$ is given by

$$X^F_{irsh} = \frac{\theta^F_{irsh} [p_{irh}]^{1-\sigma}}{[P^F_{rsh}]^{1-\sigma}} X^F_{rsh}, \tag{OS.11}$$

$$X^F_{rsh} = \frac{\theta^F_{rsh} [P^F_{rsh}]^{1-\eta}}{[P^F_{rs}]^{1-\eta}} X^F_{rs}, \tag{OS.12}$$

$$X^F_{rs} = \frac{\theta^F_{rs} [P^F_{rs}]^{1-\kappa}}{[P_{rs}]^{1-\kappa}} X_{rs}, \tag{OS.13}$$

where X_{rs} is total expenditure in region r on sector s. Similarly, spending in region r on all products of sector $s \in \mathcal{S}$ from domestic region $o \in \mathcal{R}_H$ is

$$X^H_{ors} = \frac{\tilde{\theta}^H_{ors} [p_{os}]^{1-\sigma}}{[P^H_{rs}]^{1-\sigma}} X^H_{rs}, \tag{OS.14}$$

$$X^H_{rs} = \frac{\theta^H_{rs} [P^H_{rs}]^{1-\kappa}}{[P_{rs}]^{1-\kappa}} X_{rs}. \tag{OS.15}$$

Input Demand. Equation (11) and the above definition of P_{rs} imply that the problem of the representative producer f that produces a product $h \in \mathcal{H}_s$ within a sector $s \in \mathcal{S}$ in region $r \in \mathcal{R}_H$ for use in region $d \in \mathcal{R}$ can be written as

$$\max_{\ell_{rs}(f),\, Q_{rk}(f)} p_{rdh}\theta_{rds}(\ell_{rs}(f))^{\alpha_s} \prod_k (Q_{rk}(f))^{\alpha_{ks}} - w_{rs}\ell_{rs}(f) - \sum_k P_{rk}Q_{rk}(f).$$

This implies

$$w_{rs}\ell_{rs}(f) = \alpha_s Y_{rdh},$$
$$P_{rk}Q_{rk}(f) = \alpha_{ks} Y_{rdh},$$

where $Y_{rdh} = p_{rdh}q(f)$ is the firm's total revenue.

Aggregating across all firms within the same region $r \in \mathcal{R}_H$ and sector $s \in \mathcal{S}$ and applying the labor market clearing condition then implies

$$W_{rs} = \alpha_s Y_{rs}, \tag{OS.16}$$

$$I_{rks} = \alpha_{ks} Y_{rs}, \tag{OS.17}$$

where $W_{rs} \equiv w_{rs}N_{rs}$ and $Y_{rs} \equiv \sum_{d\in\mathcal{R}, h\in\mathcal{H}_s} Y_{rdh}$ are the aggregate value added and revenue of all firms in domestic region r and sector s, and where I_{rks} is the aggregate expenditure of all such firms on intermediate inputs from sector k.

Substituting in for $Q_{rk}(f)$ in each firm's production function and then aggregating for each region-sector, we obtain

$$Y_{rs} = N_{rs}(p_{rs})^{1/\alpha_s} \prod_{k\in\mathcal{S}} [\alpha_{ks}/P_{rk}]^{\alpha_{ks}/\alpha_s}. \tag{OS.18}$$

Final Demand. Equation (15) implies that final expenditure in region r on sector s is

$$F_{rs} = P_{rs}C_{rs} = \gamma_s F_r, \tag{OS.19}$$

where F_r denotes aggregate final spending in r, which must be equal to r's aggregate income,

$$F_r = \sum_{s\in\mathcal{S}} W_{rs} + \sum_{s\in\mathcal{S}} N_{rs}\tau,$$

with

$$\tau = \frac{1}{N} \sum_{i\in\mathcal{R}_F} \sum_{r\in\mathcal{R}_H} \sum_{s\in\mathcal{S}} \sum_{h\in\mathcal{H}_s} \frac{t_{ih}}{p_{irh}} X^F_{irsh}. \tag{OS.20}$$

Finally, the consumption price index is given by

$$P_r^C = \prod_{k\in\mathcal{S}} (P_{rk})^{\gamma_k}. \tag{OS.21}$$

Market Clearing. Total spending of each region r on sector s is

$$X_{rs} = \xi_{rs} + \gamma_s \left(\sum_{k\in\mathcal{S}} \alpha_k Y_{rk} + N_r \tau \right) + \sum_{k\in\mathcal{S}} \alpha_{sk} Y_{rk}, \tag{OS.22}$$

where $N_r \equiv \sum_s N_{rs}$. Domestic demand for goods of sector s produced by region $r \in \mathcal{R}_H$ is

$$D_{rs}^H = \sum_{d\in\mathcal{R}_H} X_{rds}^H. \tag{OS.23}$$

And, using equation (20) and the fact that Home has no export taxes, foreign country i's expenditure $X_{ris}^{M,F} = \sum_{h\in\mathcal{H}_s} p_{rih}^{M,F} q_{rih}^{M,F}$ on goods produced by domestic region r in sector s is

$$X_{ris}^{M,F} = (p_{rs})^{1-1/\psi^{M,F}} (\theta_{ris})^{-(1-1/\psi^{M,F})} \sum_{h\in\mathcal{H}_s} (\theta_{rih}^{M,F})^{1/\psi^{M,F}}.$$

Thus total foreign demand faced by region r in sector s, $D_{rs}^F \equiv \sum_{i\in\mathcal{R}_F} X_{ris}^{M,F}$, is given by

$$D_{rs}^F = \delta_{rs} (p_{rs})^{1-1/\psi^{M,F}}, \tag{OS.24}$$

where

$$\delta_{rs} \equiv \sum_{i\in\mathcal{R}_F} (\theta_{ris})^{-(1-1/\psi^{M,F})} \sum_{h\in\mathcal{H}_s} (\theta_{rih}^{M,F})^{1/\psi^{M,F}}. \tag{OS.25}$$

Domestic good market clearing then requires, for each region r and sector s,

$$Y_{rs} = D_{rs}^F + D_{rs}^H. \tag{OS.26}$$

Finally, by equation (19), market clearing for imports requires that for all $i \in \mathcal{R}_F$, $r \in \mathcal{R}_H$, and $h \in \mathcal{H}$,

$$(p_{irh})^{1+\psi^{X,F}} = t_{ih} (p_{irh})^{\psi^{X,F}} + \theta_{irh}^{X,F} (X_{irsh}^F)^{\psi^{X,F}} \tag{OS.27}$$

with X_{irsh}^M given by equation (OS.11).

OS.2 Calibration

This section describes in detail the procedure we use to calibrate the model of Section 3. Specifically, we show how to calibrate the following parameters:

$$\{\alpha_s, \alpha_{ks}, \gamma_s, \xi_{rs}, \theta_{rds}, \theta^c_{rk}, \theta^c_{rkj}, \theta^c_{orsh}, \theta^{X,F}_{irh}, \theta^{M,F}_{rih}, N_{rs}\}.$$

As a first step, we normalize to one all domestic prices ($p_{odh} = 1$) and wages ($w_{rs} = 1$) in the initial equilibrium. Equation (OS.1)-(OS.5) imply that for all $r \in \mathcal{R}_H$, $h \in \mathcal{H}_s$, $s \in \mathcal{S}$, and $c \in \{H, F\}$, the price indices also satisfy

$$P^c_{rsh} = P^c_{rs} = P_{rs} = 1. \tag{OS.28}$$

Note that this normalization further implies that

$$t_{ih} = t^{\text{av}}_{ih} / (1 + t^{\text{av}}_{ih}),$$

with t^{av}_{ih} denoting ad-valorem import tariffs, as in Section 3.3.

Upper-nest Technology Parameters: $\{\alpha_{ks}, \alpha_s, \theta_{rds}\}$**.** Let Y^{NA}_s be the national gross output of sector s and I^{NA}_{ks} be the national purchases of sector s from sector k. Since (OS.17) holds for all domestic regions, we set α_{ks} equal to sector s's national spending share on inputs from sector k:

$$\alpha_{ks} = \frac{I^{\text{NA}}_{ks}}{Y^{\text{NA}}_s}.$$

Using (OS.16), we then set the value-added share α_s equal to the share of sector s's revenue not spent on inputs:

$$\alpha_s = 1 - \sum_{k \in \mathcal{S}} \alpha_{ks}.$$

Finally, we back out θ_{rds} to be consistent with (OS.1) under our price normalization:

$$\theta_{rds} = [\alpha_s]^{-\alpha_s} \prod_{k \in S} [\alpha_{ks}]^{-\alpha_{ks}}.$$

Upper-nest Preference Parameters: $\{\gamma_s\}$**.** Let F^{NA}_s be the national final spending on sector s. Since (OS.19) holds for all domestic regions, we set γ_s equal to the share of s in national final spending:

$$\gamma_s = \frac{F_s^{\mathbf{NA}}}{\sum_k F_k^{\mathbf{NA}}}.$$

Lower-nest Technology and Preference Parameters: $\{\theta^c_{rs}, \theta^c_{rsh}, \theta^c_{orsh}\}$. Under our price normalization, equations (OS.11)-(OS.13) imply

$$\theta^F_{irsh} = \frac{X^F_{irsh}}{X^F_{rsh}}, \quad \theta^F_{rsh} = \frac{X^F_{rsh}}{X^F_{rs}}, \quad \theta^F_{rs} = \frac{X^F_{rs}}{X_{rs}},$$

with X^F_{irsh} the (tariff-inclusive) value of imports of product $h \in \mathcal{H}_s$ from country i by region r, $X^F_{rsh} = \sum_{i \in \mathcal{R}^F} X^F_{irsh}$, $X^F_{rs} = \sum_{h \in \mathcal{H}_s} X^F_{rsh}$, and X_{rs} the aggregate spending on sector s in region r. In some instances, we observe either $X^F_{rsh} = 0$ or $X^F_{rs} = 0$. If $X^F_{rsh} = 0$, we set $\theta^F_{irsh} = 1/|\mathcal{R}_F|$, and if $X^F_{rs} = 0$, we set $\theta^F_{rsh} = 1/|\mathcal{H}_s|$, without loss of generality.

Likewise, under our price normalization, equations (OS.14)-(OS.15) imply

$$\bar{\theta}^H_{ors} = \frac{X^H_{ors}}{X^H_{rs}} \quad \text{and} \quad \theta^H_{rs} = \frac{X^H_{rs}}{X_{rs}},$$

with X^H_{ors} the domestic trade flows of all products in sector s from region o to region r, and $X^H_{rs} = \sum_{o \in \mathcal{R}_H} X^H_{ors}$. In some instances, we observe $X^H_{rs} = 0$. If so, we set $\bar{\theta}^H_{ors} = 1/|\mathcal{R}_H|$, without loss of generality. Finally, we can calibrate θ^H_{orsh} using our assumption that $\theta^H_{orsh} = \bar{\theta}^H_{ors}$ and θ^H_{rsh} using our assumption that $\theta^H_{rsh} = \bar{\theta}^H_{rs} = 1/|\mathcal{H}_s|$ for all $h \in \mathcal{H}_s$.

Foreign Supply and Demand Shifters: $\{\theta^{X,F}_{irh}, \theta^{M,F}_{rih}\}$. Under our price normalization, (19) and (20) imply

$$\begin{aligned} \theta^{X,F}_{irh} &= (1 + t^{\text{av}}_{ih})^{-1} (X^F_{irsh})^{-\psi^{X,F}}, \\ \theta^{M,F}_{rih} &= (X^{M,F}_{rih})^{\psi^{M,F}}, \end{aligned}$$

with $X^{M,F}_{rih}$ the value of exports of product h from region r to country i.

Residual Spending: $\{\xi_{rs}\}$. We first compute lump-sum transfers using (OS.20):

$$\tau = \frac{1}{N} \sum_{i \in \mathcal{R}_F} \sum_{r \in \mathcal{R}_H} \sum_{s \in \mathcal{S}} \sum_{h \in \mathcal{H}_s} \frac{t^{\text{av}}_{ih}}{1 + t^{\text{av}}_{ih}} X^F_{irsh}.$$

We next compute each region r's total spending on sector s:

$$X_{rs} = X_{rs}^{H} + X_{rs}^{F},$$

where X_{rs}^{H} and X_{rs}^{F} are the total spending of region r on domestic and foreign products from sector s, respectively. Finally, given the levels of spending X_{rs}, gross output Y_{rs}, and transfers $N_r\tau$, we use (OS.22) to solve for ξ_{rs} as:

$$\xi_{rs} = X_{rs} - \gamma_s \left(\sum_{k\in\mathcal{S}} \alpha_k Y_{rk} + N_r \tau \right) - \sum_{k\in\mathcal{S}} \alpha_{sk} Y_{rk}.$$

OS.3 Numerical Algorithm for Equilibrium Computation

This section describes the algorithm that we use to compute equilibria with zero tariffs in Section 4.3, in order to construct instrumental variables, as well as equilibria without redistributive trade protection in Section 5. Given the multiplicative structure of the model, it is convenient to work with ad-valorem tariffs $\{t_{ih}^{\text{av}}\}$. Formally, the equilibrium conditions are those described above, except for the two equations (OS.20) and (OS.27) in which specific tariffs enter explicitly, which we write as

$$\tau = \frac{1}{N} \sum_{i\in\mathcal{R}_F} \sum_{r\in\mathcal{R}_H} \sum_{s\in\mathcal{S}} \sum_{h\in\mathcal{H}_s} \frac{t_{ih}^{\text{av}}}{1+t_{ih}^{\text{av}}} X_{irsh}^{F}, \tag{OS.29}$$

$$(p_{irh})^{1+\psi^{X,F}} = \frac{t_{ih}^{\text{av}}}{1+t_{ih}^{\text{av}}} (p_{irh})^{1+\psi^{X,F}} + \theta_{irh}^{X,F} (X_{irh}^{F})^{\psi^{X,F}}. \tag{OS.30}$$

The algorithm solves for the equilibrium conditional on arbitrary ad-valorem tariffs $\{\tilde{t}_{ih}^{\text{av}}\}$ as well as the calibrated values of $\{\alpha_s, \alpha_{ks}, \gamma_s, \xi_{rs}, \theta_{rds}, \theta_{rk}^{c}, \theta_{rkj}^{c}, \theta_{orsh}^{c}, \theta_{irh}^{X,F}, \theta_{rih}^{M,F}, N_{rs}\}$ and $\{\kappa, \eta, \sigma, \psi^{X,F}, \psi^{M,F}\}$. For the counterfactual with zero tariffs, we set $\tilde{t}_{ih}^{\text{av}} = 0$. For the counterfactual without redistributive trade protection, we set $\tilde{t}_{ih}^{\text{av}} = t'_{ih}/(1-t_{ih})$, where t'_{ih} is described in equation (25) and t_{ih} is the specific tariff in the initial equilibrium.[1]

[1]Since we have normalized import prices to one in the initial equilibrium and set $\psi^{X,F} = 0$, we note that import prices in the counterfactual equilibrium without redistribution must satisfy $p'_{rih} = 1 - t_{ih} + t'_{ih}$. Hence they do not vary across regions r, as already imposed in equations (OS.29) and (OS.30). Since the ad-valorem tariff equivalent to the specific tariff t'_{ih} must also satisfy $t'_{irh}/p'_{irh} = \tilde{t}_{irh}^{\text{av}}/(1+\tilde{t}_{irh}^{\text{av}})$, we further get $\tilde{t}_{ih}^{\text{av}} = t'_{ih}/(1-t_{ih})$, as stated above.

OS.3.1 Solving for Domestic Demand

A key step in characterizing equilibrium outcomes is to solve for the domestic spending $\{X_{rs}\}$ as a function of $\{p_{rs}\}$ and $\{P^F_{rs}\}$. This is a fixed point problem because spending (non-linearly) affects tariff revenue, which in turn affects spending.

We begin by characterizing tariff revenue as a function of p_{rs} and P^F_{rs} and X_{rs}. From the expressions in Section OS.1, we get that

$$X^F_{irsh} = \frac{\theta^F_{irh}[p_{irh}]^{1-\sigma}}{[P^F_{rsh}]^{1-\sigma}} X^F_{rsh},$$

$$p_{irh} = \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1}{1+\psi^{X,F}}} (X^F_{irsh})^{\frac{\psi^{X,F}}{1+\psi^{X,F}}}. \tag{OS.31}$$

Thus,

$$p_{irh} = \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1}{1+\psi^{X,F}\sigma}} (\theta^F_{irsh})^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}} \left(\frac{X^F_{rsh}}{[P^F_{rsh}]^{1-\sigma}}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}},$$

$$X^F_{irsh} = \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1-\sigma}{1+\psi^{X,F}\sigma}} (\theta^F_{irsh})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}} \left(\frac{X^F_{rsh}}{[P^F_{rsh}]^{1-\sigma}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}.$$

Let us write

$$X^F_{irsh} = \varphi_{irsh} \left(\frac{X^F_{rsh}}{[P^F_{rsh}]^{1-\sigma}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}, \tag{OS.32}$$

$$\text{where} \quad \varphi_{irsh} \equiv \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1-\sigma}{1+\psi^{X,F}\sigma}} (\theta^F_{irsh})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}. \tag{OS.33}$$

Since $(P^F_{rsh})^{1-\sigma} = \sum_{i\in\mathcal{R}_F} \theta^F_{irsh}[p_{irh}]^{1-\sigma}$,

$$P^F_{rsh} = (X^F_{rsh})^{\frac{\psi^{X,F}}{1+\psi^{X,F}}} \left[\sum_{i\in\mathcal{R}_F} \varphi_{irsh}\right]^{\frac{1+\psi^{X,F}\sigma}{(1+\psi^{X,F})(1-\sigma)}}. \tag{OS.34}$$

Thus,

$$X^F_{irsh} = \frac{\varphi_{irsh}}{\sum_{o\in\mathcal{R}_F}\varphi_{orsh}} X^F_{rsh}. \tag{OS.35}$$

Recall that $X^F_{rsh} = \frac{\theta^F_{rsh}[P^F_{rsh}]^{1-\eta}}{[P^F_{rs}]^{1-\eta}} X^F_{rs}$. So, by substituting P^F_{rsh} into the expression above, we get that

$$X^F_{rsh} = (\theta^F_{rsh})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\eta}} \left[\sum_{i\in\mathcal{R}_F} \varphi_{irsh}\right]^{\frac{(1+\psi^{X,F}\sigma)(1-\eta)}{(1+\psi^{X,F}\eta)(1-\sigma)}} \left[\frac{X^F_{rs}}{[P^F_{rs}]^{1-\eta}}\right]^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\eta}}. \tag{OS.36}$$

Since $[P^F_{rs}]^{1-\eta} = \sum_{h\in\mathcal{H}_s} \theta^F_{rsh}[P^F_{rsh}]^{1-\eta}$,

$$P^F_{rs} = \left[X^F_{rs}\right]^{\frac{\psi^{X,F}}{1+\psi^{X,F}}} [\mu_{rs}]^{\frac{1+\psi^{X,F}\eta}{(1+\psi^{X,F})(1-\eta)}}, \tag{OS.37}$$

$$\text{where} \quad \mu_{rs} \equiv \sum_{h\in\mathcal{H}_s} (\theta^F_{rsh})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\eta}} \left[\sum_{i\in\mathcal{R}_F} \varphi_{irsh}\right]^{\frac{(1+\psi^{X,F}\sigma)(1-\eta)}{(1+\psi^{X,F}\eta)(1-\sigma)}}. \tag{OS.38}$$

Finally, $X^F_{rs} = \frac{\theta^F_{rs}[P^F_{rs}]^{1-\kappa}}{[P_{rs}]^{1-\kappa}} X_{rs}$ implies that

$$X^F_{rs} = (\theta^F_{rs})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}} [\mu_{rs}]^{\frac{(1+\psi^{X,F}\eta)(1-\kappa)}{(1+\psi^{X,F}\kappa)(1-\eta)}} \left(\frac{X_{rs}}{[P_{rs}]^{1-\kappa}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}}. \tag{OS.39}$$

Thus, substituting (OS.39) into (OS.37), we obtain

$$P^F_{rs} = \zeta_{rs} \left(\frac{X_{rs}}{[P_{rs}]^{1-\kappa}}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\kappa}}, \tag{OS.40}$$

$$\text{where} \quad \zeta_{rs} \equiv (\theta^F_{rs})^{\frac{\psi^{X,F}}{1+\psi^{X,F}\kappa}} [\mu_{rs}]^{\frac{1+\psi^{X,F}\eta}{(1+\psi^{X,F}\kappa)(1-\eta)}}. \tag{OS.41}$$

Combining (OS.35), (OS.34), (OS.39), and (OS.40), we can also write

$$X^F_{irsh} = \varphi_{irsh}(\theta^F_{rsh})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\eta}} \left[\sum_{o\in\mathcal{R}_F} \varphi_{orsh}\right]^{\frac{(1+\psi^{X,F})(\sigma-\eta)}{(1+\psi^{X,F}\eta)(1-\sigma)}} \times (\theta^F_{rs})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}} [\mu_{rs}]^{\frac{(1+\psi^{X,F})(\eta-\kappa)}{(1+\psi^{X,F}\kappa)(1-\eta)}} \left(\frac{X_{rs}}{[P_{rs}]^{1-\kappa}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}}, \tag{OS.42}$$

and, using $T_{rs} \equiv \sum_{h\in\mathcal{H}_s} \sum_{i\in\mathcal{R}_F} \frac{\tilde{t}^{\text{av}}_{ih}}{1+\tilde{t}^{\text{av}}_{ih}} X^F_{irsh}$,

$$T_{rs} = \left(\frac{X_{rs}}{[P_{rs}]^{1-\kappa}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}} \varphi^R_{rs}, \tag{OS.43}$$

with

$$\varphi_{rs}^{R} \equiv (\theta_{rs}^{F})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}} [\mu_{rs}]^{\frac{(1+\psi^{X,F})(\eta-\kappa)}{(1+\psi^{X,F}\kappa)(1-\eta)}} \times \sum_{h\in\mathcal{H}_s} (\theta_{rsh}^{F})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\eta}} \left(\sum_{i\in\mathcal{R}_F} \varphi_{irsh}\right)^{\frac{(1+\psi^{X,F})(\sigma-\eta)}{(1+\psi^{X,F}\eta)(1-\sigma)}} \left(\sum_{i\in\mathcal{R}_F} \frac{\tilde{t}_{ih}^{\text{av}}}{1+\tilde{t}_{ih}^{\text{av}}} \varphi_{irsh}\right). \tag{OS.44}$$

Combining (OS.22) and (OS.43), we can solve for domestic expenditure $\{X_{rs}\}$ as the solution to:

$$X_{rs} - \sum_{d\in\mathcal{R}_H} \sum_{k\in\mathcal{S}} \hat{e}_{rs,dk} (X_{dk})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}} = \hat{X}_{rs}, \tag{OS.45}$$

with

$$\hat{X}_{rs} = \xi_{rs} + \sum_{k\in\mathcal{S}} (\gamma_s \alpha_k + \alpha_{sk}) Y_{rk},$$
$$\hat{e}_{rs,dk} = \gamma_s \frac{N_r}{N} \varphi_{dk}^{R} \left([P_{dk}]^{\kappa-1}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}}, \tag{OS.46}$$

where both $\{Y_{rk}\}$ and $\{P_{dk}\}$ are only functions of $\{p_{rs}\}$ and $\{P_{rs}^{F}\}$ via (OS.18) and (OS.7)–(OS.8).

OS.3.2 Algorithm

We use the following algorithm to solve for the competitive equilibrium of the model.

i. Compute parameters that are invariant to prices: φ_{rs}^{R} from (OS.44), ζ_{rs} from (OS.41), μ_{rs} from (OS.38), φ_{irsh} from (OS.33), and δ_{rs} from (OS.25).

ii. We have an outer loop (indexed by a). Guess $P_{rs}^{F,a=0}$ using (OS.40):

$$P_{rs}^{F,a=0} = \zeta_{rs} \left(\tilde{E}_{rs}^{0}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\kappa}} \tag{OS.47}$$

where we use a pre-determined choice of the sector-level demand shifter $\tilde{E}_{rs}^{0} \equiv X_{rs}^{0}(P_{rs}^{0})^{\kappa-1}$ (which we take to be the value in some observed initial equilibrium).

iii. Given $P_{rs}^{F,a}$, we have a middle loop (indexed by b) that solves for p_{rs}^{a}.

   (a) We guess $p_{rs}^{a,b=0}$.

(b) Given $\{P^{F,a}_{rs}, p^{a,b}_{rs}\}$, compute the following vectors of region-sector variables (with length $|\mathcal{R}_H| \cdot |\mathcal{S}|$ and same ordering of sectors and regions for all variables).

   i. Domestic region-sector price index $P^{H,a,b}_{rs}$ by substituting $p^{a,b}_{rs}$ into (OS.8).

   ii. Region-sector price index $P^{a,b}_{rs}$ by substituting $P^{F,a}_{rs}$ and $P^{H,a,b}_{rs}$ into (OS.7).

   iii. Region-sector supply $Y^{a,b}_{rs}$ by substituting $p^{a,b}_{rs}$ and $P^{a,b}_{rk}$ into (OS.18).

   iv. Region-sector foreign demand $D^{F,a,b}_{rs}$ by substituting $p^{a,b}_{rs}$ into (OS.24).

   v. Region-sector spending $X^{a,b}_{rs}$ using (OS.45). Here, we have an inner fixed-problem algorithm (indexed by c):

      A. Compute $\hat{X}^{a,b} = [\hat{X}_{rs}]_{|\mathcal{R}_F|\cdot|\mathcal{S}|\times 1}$ and $\hat{e}^{a,b} \equiv [\hat{e}^{a,b}_{rs,dk}]_{|\mathcal{R}_F|\cdot|\mathcal{S}|\times|\mathcal{R}_F|\cdot|\mathcal{S}|}$ by plugging $Y^{a,b}_{rk}$ and $P^{a,b}_{dk}$ into (OS.46).

      B. Guess that $X^{a,b,c=0} = \sum_{d=0}^{\bar{d}} (\hat{e}^{a,b})^d \hat{X}^{a,b}$.[2] Given $X^{a,b,c}$, compute

$$\widetilde{X}^{a,b,c}_{rs} \equiv \sum_{d\in\mathcal{R}_H} \sum_{k\in\mathcal{S}} \hat{e}^{a,b}_{rs,dk} \left(X^{a,b,c}_{d,k} \right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F_\kappa}}} + \hat{X}^{a,b}_{r,s},$$

$$Xerr_{rs} \equiv X^{a,b,c}_{rs} - \widetilde{X}^{a,b,c}_{rs}.$$

      If $max_{r,s}|Xerr_{rs}| < tol$, then we set $X^{a,b} = X^{a,b,c}$. Otherwise, we repeat the step with

$$X^{a,b,c+1}_{rs} = X^{a,b,c}_{rs} - \chi^X \left(X^{a,b,c}_{rs} - \widetilde{X}^{a,b,c}_{rs} \right).$$

      for $\chi^X > 0$ small enough. Note that, given our initial guess of $X^{a,b,c=0}$, this converges in a single step if $\psi^{X,F} = 0$.

   vi. Region-sector domestic spending $X^{H,a,b}_{rs}$ by substituting $P^{H,a,b}_{rs}$, $P^{a,b}_{rs}$ and $X^{a,b}_{rs}$ into (OS.14).

   vii. Region-sector domestic demand $D^{H,a,b}_{rs}$ by substituting $p^{a,b}_{rs}$, $X^{H,a,b}_{rs}$ and $P^{H,a,b}_{rs}$ into (OS.14) and (OS.23).

   viii. Region-sector excess supply:

$$Yerr^{a,b}_{rs} \equiv \frac{Y^{a,b}_{rs} - (D^{F,a,b}_{r,s} + D^{H,a,b}_{rs})}{Y^{a,b}_{rs}}.$$

(c) If $\max_{r,s}\{|Yerr^{a,b}_{rs}|\} < tol$, then proceed to step (iv) by setting $p^{a}_{rs} = p^{a,b}_{rs}$. If not,

[2] $\bar{d}$ is a numerical parameter that can in principle be set to infinity. In practice, we set $\bar{d} = 5$.

repeat (iii.b) with the updated guesses for prices:

$$\ln p_{rs}^{a,b+1} = \ln p_{rs}^{a,b} - \chi^{H} Yerr_{rs}^{a,b}$$

for $\chi^H > 0$ small enough. Intuitively, supply is larger than demand when $Yerr_{rs}^{a,b} > 0$, so we reduce domestic prices in region r sector s until convergence.

iv. Given the demand shifter $E_{rs}^{a} = X_{rs}^{a}(P_{rs}^{a})^{\kappa-1}$, we compute the error in the import price index of each region-sector,

$$Perr_{rs} \equiv |P_{rs}^{F,a} - \zeta_{rs} \left(E_{rs}^{a}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\kappa}}|.$$

If $\max_{r,s}\{|Perr_{rs}|\} < tol$, then stop. If not, repeat step (iii) with the updated guess for prices:

$$P_{rs}^{F,a+1} = \zeta_{rs} \left(\tilde{E}_{rs}^{a+1}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\kappa}},$$

with

$$\tilde{E}_{rs}^{a+1} = \tilde{E}_{rs}^{a} - \chi^{F}(\tilde{E}_{rs}^{a} - E_{rs}^{a})$$

for $\chi^F > 0$ small enough.

v. Upon convergence, we compute X_{irsh}^{F} using (OS.32), (OS.36), and (OS.39), import prices p_{irh} using (OS.31), import quantity $m_{irh} = X_{irsh}^{F}/p_{irh}$, region-sector value-added W_{rs} and wages $w_{rs} = W_{rs}/N_{rs}$ using (OS.16), region-level consumption price index P_{r}^{C} using (OS.21), and per-capita lump-sum transfers $\tau = \frac{1}{N}\sum_{r\in\mathcal{R}_H, s\in\mathcal{S}} T_{rs}$ with T_{rs} given by (OS.43).

OS.4 Analytical Jacobian Matrices

OS.4.1 Jacobian Matrices with Respect to Imports

In this section, we derive analytical expressions for the derivatives of imports, wages, consumption price indices, and country-specific terms-of-trade with respect to imports of each country-product variety. We rely throughout on the assumption, already imposed in our calibration, that foreign export supply curves are perfectly elastic: $\psi^{X,F} = 0$. Combined with our price normalization, this assumption implies that for every foreign country $i \in \mathcal{R}_F$, every region $r \in \mathcal{R}_H$, and every product $h \in \mathcal{H}$, the change in import prices associated with a small change in specific tariffs dt_{ih} is the same as the change associated with an ad-valorem tariff $d\log(1+t_{ih}^{\text{av}})$, i.e.,

$$d\log p_{irh} = d\log(1+t_{ih}^{\text{av}}) = dt_{ih}. \tag{OS.48}$$

For analytical convenience, we can thus start by deriving Jacobian matrices with respect to dt_{ih} by computing derivatives with respect to $d\log(1+t_{ih}^{\text{av}})$ instead, as we do below.

Domestic Prices and Expenditures. We first compute the derivatives of domestic prices p_{rs} and expenditures X_{rs} with respect to $d\log(1+t_{ih}^{\text{av}})$ that are common to all domestic regions. Substituting (OS.18), (OS.24), and (OS.23) into the domestic market clearing condition (OS.26) and differentiating implies

$$\frac{1}{\alpha_s} d\log p_{rs} - \sum_{k\in\mathcal{S}} \frac{\alpha_{ks}}{\alpha_s} d\log P_{rk} = \frac{D_{rs}^F}{Y_{rs}} \left(1 - 1/\psi^{M,F}\right) d\log p_{rs} + \sum_{d\in\mathcal{R}_H} \frac{X_{rds}^H}{Y_{rs}} d\log X_{rds}^H. \tag{OS.49}$$

Next, we substitute in for $d\log P_{rk}$ using the differentiated version of (OS.7) derived from (OS.8)–(OS.10) and (OS.48),

$$d\log P_{rk} = \sum_{o\in\mathcal{R}_H} \frac{X_{ork}^H}{X_{rk}} d\log p_{ok} + \sum_{h\in\mathcal{H}_k} \sum_{i\in\mathcal{R}_F} \frac{X_{irkh}^F}{X_{rk}} d\log(1+t_{ih}^{\text{av}}), \tag{OS.50}$$

and we substitute for $d\log X_{rds}^H$ using the differentiated version of (OS.14) derived from (OS.7) and (OS.50),

$$\begin{aligned} d\log X_{rds}^H = {} & (1-\sigma) d\log p_{rs} + \sum_{j\in\mathcal{R}_H} \left[(\sigma-1) - (\kappa-1)\frac{X_{ds}^F}{X_{ds}}\right] \frac{X_{jds}^H}{X_{ds}^H} d\log p_{js} \\ & + (\kappa-1) \sum_{h\in\mathcal{H}_s} \sum_{i\in\mathcal{R}_F} \frac{X_{idsh}^F}{X_{is}} d\log(1+t_{ih}^{\text{av}}) + d\log X_{ds}. \end{aligned} \tag{OS.51}$$

In vector notation, letting $d\log p \equiv \{d\log p_{rs}\}$, $d\log X \equiv \{d\log X_{rs}\}$, and $d\log(1+t) \equiv \{d\log(1+t^{\text{av}}_{ih})\}$, this implies

$$\begin{aligned}
&\mathcal{E}^p_p d\log p = \mathcal{E}^p_X d\log X + \mathcal{E}^p_{1+t} d\log(1+t) \qquad \text{(OS.52)}\\
\text{where}\quad &[\mathcal{E}^p_p]_{rs,ok} \equiv \mathbb{1}_{r=o,s=k}\left[\frac{1}{\alpha_s} - \left(1-1/\psi^{M,F}\right)\frac{D^F_{rs}}{Y_{rs}} - (1-\sigma)\frac{X^H_{rs}}{Y_{rs}}\right]\\
&\quad + \mathbb{1}_{s=k}\sum_{j\in\mathcal{R}_H}\frac{X^H_{rjs}}{Y_{rs}}\left[(1-\sigma)-(1-\kappa)\frac{X^F_{js}}{X_{js}}\right]\frac{X^H_{ojs}}{X^H_{js}} - \frac{\alpha_{ks}}{\alpha_s}\frac{X^H_{ork}}{X_{rk}},\\
&[\mathcal{E}^p_X]_{rs,dk} \equiv \mathbb{1}_{s=k}\frac{X^H_{rds}}{Y_{rs}},\\
&[\mathcal{E}^p_{1+t}]_{rs,ih} \equiv \frac{\alpha_{s(h)s}}{\alpha_s}\frac{X^F_{irs(h)h}}{X_{rs(h)}} + \mathbb{1}_{h\in\mathcal{H}_s}\sum_{d\in\mathcal{R}_H}\frac{X^H_{rds}}{Y_{rs}}(\kappa-1)\frac{X^F_{idsh}}{X_{ds}},
\end{aligned}$$

with $s(h)\in\mathcal{S}$ the sector to which product h belongs.

We now turn to the regional demand equation (OS.22). Substituting (OS.18), (OS.20), and (OS.48) into (OS.22) and differentiating implies

$$\begin{aligned}
d\log X_{rs} = &\sum_{k\in\mathcal{S}}\frac{(\gamma_s\alpha_k+\alpha_{sk})Y_{rk}}{X_{rs}}\left(\frac{1}{\alpha_k}d\log p_{rk} - \sum_{\ell\in\mathcal{S}}\frac{\alpha_{\ell k}}{\alpha_k}d\log P_{r\ell}\right) \qquad \text{(OS.53)}\\
&+\frac{\gamma_s N_r/N}{X_{rs}}\sum_{i\in\mathcal{R}_F}\sum_{d\in\mathcal{R}_H}\sum_{k\in\mathcal{S}}\sum_{h\in\mathcal{H}_k}\left(\frac{X^F_{idkh}}{1+t^{\text{av}}_{ih}}d\log(1+t^{\text{av}}_{ih}) + \frac{t^{\text{av}}_{ih}}{1+t^{\text{av}}_{ih}}X^F_{idkh}d\log X^F_{idkh}\right).
\end{aligned}$$

Next, we substitute in for $d\log P_{r\ell}$ using (OS.50), and we substitute for $d\log X^F_{idkh}$ using a differentiated version of (OS.42) derived from (OS.11), (OS.9)–(OS.10), (OS.27), and (OS.50),

$$\begin{aligned}
d\log X^F_{idkh} = &(1-\sigma)d\log(1+t^{\text{av}}_{ih}) + (\sigma-\eta)\sum_{o\in\mathcal{R}_F}\frac{X^F_{odkh}}{X^F_{dkh}}d\log(1+t^{\text{av}}_{oh}) \qquad \text{(OS.54)}\\
&+\sum_{v\in\mathcal{H}_k}\sum_{o\in\mathcal{R}_F}\left((\eta-1)-(\kappa-1)\frac{X^H_{dk}}{X_{dk}}\right)\frac{X^F_{odkv}}{X^F_{dk}}d\log(1+t^{\text{av}}_{ov})\\
&+(\kappa-1)\sum_{r\in\mathcal{R}_H}\frac{X^H_{rdk}}{X_{dk}}d\log p_{rk} + d\log X_{dk}
\end{aligned}$$

In vector notation, this implies

$$
\begin{aligned}
\mathcal{E}_X^X d\log X =& \mathcal{E}_p^X d\log p + \mathcal{E}_{1+t}^X d\log(1+t), \qquad \text{(OS.55)}\\
\text{where}\quad [\mathcal{E}_X^X]_{rs,dk} \equiv& -\mathbb{1}_{r=d,s=k}\frac{\gamma_s N_r/N}{X_{rs}}T_{dk},\\
[\mathcal{E}_p^X]_{rs,ok} \equiv& \mathbb{1}_{r=o}\left(\gamma_s+\alpha_{sk}/\alpha_k\right)\frac{Y_{rk}}{X_{rs}}\\
&-\frac{X^H_{ork}}{X_{rk}}\sum_{\ell\in\mathcal{S}}\alpha_{k\ell}\left(\gamma_s+\alpha_{s\ell}/\alpha_\ell\right)\frac{Y_{r\ell}}{X_{rs}}+\frac{\gamma_s N_r/N}{X_{rs}}(\kappa-1)\sum_{d\in\mathcal{R}_H}T_{dk}\frac{X^H_{odk}}{X_{dk}},\\
[\mathcal{E}_{1+t}^X]_{rs,ih} \equiv& -\frac{X^F_{irs(h)h}}{X_{rs(h)}}\sum_{k\in\mathcal{S}}\alpha_{s(h)k}\left(\gamma_s+\alpha_{sk}/\alpha_k\right)\frac{Y_{rk}}{X_{rs}}\\
&+\frac{\gamma_s N_r/N}{X_{rs}}\frac{\left(1+(1-\sigma)t^{\text{av}}_{ih}\right)\sum_{d\in\mathcal{R}_H}X^F_{ids(h)h}}{1+t^{\text{av}}_{ih}}\\
&+\frac{\gamma_s N_r/N}{X_{rs}}(\sigma-\eta)\sum_{d\in\mathcal{R}_H}T_{ds(h)h}\frac{X^F_{ids(h)h}}{X^F_{ds(h)h}}\\
&+\frac{\gamma_s N_r/N}{X_{rs}}\sum_{d\in\mathcal{R}_H}T_{ds(h)}\left((\eta-1)-(\kappa-1)\frac{X^H_{ds(h)}}{X_{ds(h)}}\right)\frac{X^F_{ids(h)h}}{X^F_{ds(h)}},
\end{aligned}
$$

with $T_{dkh} \equiv \sum_{i\in\mathcal{R}_F}\frac{t^{\text{av}}_{ih}}{1+t^{\text{av}}_{ih}}X^F_{idkh}$ and $T_{dk} \equiv \sum_{h\in\mathcal{H}_k}T_{dkh}$. Finally, we combine (OS.52) and (OS.55) to compute the Jacobian matrices of domestic prices and expenditures with respect to tariffs

$$
\frac{d\log p}{d\log(1+t)} = \left(\mathcal{E}_p^p-\mathcal{E}_X^p(\mathcal{E}_X^X)^{-1}\mathcal{E}_p^X\right)^{-1}\left(\mathcal{E}_X^p(\mathcal{E}_X^X)^{-1}\mathcal{E}_{1+t}^X+\mathcal{E}_{1+t}^p\right), \qquad \text{(OS.56)}
$$

$$
\frac{d\log X}{d\log(1+t)} = (\mathcal{E}_X^X)^{-1}\left(\mathcal{E}_p^X\frac{d\log p}{d\log(1+t)}+\mathcal{E}_{1+t}^X\right). \qquad \text{(OS.57)}
$$

Imports, Wages, Consumer Prices, and Terms-of-Trade. We now use the Jacobian matrices described in (OS.56) and (OS.57) in order to compute the Jacobian matrices of imports, wages, consumer prices, and terms of trade with respect to tariffs.

We begin with imports. The quantity of imports of each product $h\in\mathcal{H}$ from each origin $i\in\mathcal{R}_F$ is equal to

$$
m_{ih} = \sum_{r\in\mathcal{R}_H}X^F_{irs(h)h}/p_{irh}.
$$

Under our assumption that $\psi^{X,F}=0$, the Jacobian of imports is closely related to that of import expenditures derived in (OS.54). In vector notation, we have

$$
\begin{aligned}
\frac{d\log m}{d\log(1+t)} &= \mathcal{E}^{m}_{p}\frac{d\log p}{d\log(1+t)} + \mathcal{E}^{m}_{X}\frac{d\log X}{d\log(1+t)} + \mathcal{E}^{m}_{1+t} \\
\text{where}\quad [\mathcal{E}^{m}_{p}]_{ih,rk} &\equiv \mathbb{1}_{k=s(h)}(\kappa-1)\sum_{d\in\mathcal{R}_H}\frac{m_{idh}}{m_{ih}}\frac{X^{H}_{rds(h)}}{X_{ds(h)}}, \\
[\mathcal{E}^{m}_{X}]_{ih,rk} &\equiv \mathbb{1}_{k=s(h)}\frac{m_{irh}}{m_{ih}}, \\
[\mathcal{E}^{m}_{1+t}]_{ih,jv} &\equiv -\mathbb{1}_{i=j,h=v}\sigma + \mathbb{1}_{h=v}(\sigma-\eta)\sum_{r\in\mathcal{R}_H}\frac{m_{irh}}{m_{ih}}\frac{X^{F}_{jrs(h)h}}{X^{F}_{rs(h)h}} \\
&\quad + \mathbb{1}_{s(h)=s(v)}\sum_{r\in\mathcal{R}_H}\frac{m_{irh}}{m_{ih}}\left((\eta-1)-(\kappa-1)\frac{X^{H}_{rs(h)}}{X_{rs(h)}}\right)\frac{X^{F}_{jrs(h)v}}{X^{F}_{rs(h)}}.
\end{aligned}
$$

Next, we consider wages. Since value added has a constant share in gross output and factors are immobile, $d\log w_{rs} = d\log Y_{rs}$. Using this observation, differentiating (OS.18), substituting for $d\log P_{rk}$ using (OS.50), we obtain in vector notation,

$$
\begin{aligned}
\frac{d\log w}{d\log(1+t)} &= \mathcal{E}^{w}_{p}\frac{d\log p}{d\log(1+t)} + \mathcal{E}^{w}_{1+t}, \\
\text{where}\quad [\mathcal{E}^{w}_{p}]_{rs,ok} &\equiv \mathbb{1}_{r=i,s=k}\frac{1}{\alpha_s} - \frac{\alpha_{ks}}{\alpha_s}\frac{X^{H}_{ork}}{X_{rk}} \\
[\mathcal{E}^{w}_{1+t}]_{rs,ih} &\equiv -\frac{\alpha_{s(h)s}}{\alpha_s}\frac{X^{F}_{irs(h)h}}{X_{rs(h)}}.
\end{aligned}
$$

Next, we consider consumer price indices. Differentiating (OS.21) and substituting for $d\log P_{rk}$ using (OS.50), we obtain in vector notation,

$$
\begin{aligned}
\frac{d\log P^{C}}{d\log(1+t)} &= \mathcal{E}^{P^C}_{p}\frac{d\log p}{d\log(1+t)} + \mathcal{E}^{P^C}_{1+t}, \\
\text{where}\quad [\mathcal{E}^{P^C}_{p}]_{r,ok} &= \gamma_k\frac{X^{H}_{ork}}{X_{rk}}, \\
[\mathcal{E}^{P^C}_{1+t}]_{r,ih} &= \gamma_{s(h)}\frac{X^{F}_{irs(h)h}}{X_{rs(h)}}.
\end{aligned}
$$

Finally, we consider terms of trade. Under the assumption that $\psi^{X,F} = 0$, the change in any country i's terms of trade is equal to

$$d\text{ToT}_i \equiv -\sum_{s\in\mathcal{S}}\sum_{h\in\mathcal{H}_s}\sum_{r\in\mathcal{R}_H} p_{rih}^{M,F} x_{rih} d\log p_{rih}^{M,F}.$$

As there are no export taxes, $d\log p_{rih}^{M,F} = d\log p_{rih}$, and by (OS.1), $d\log p_{rih} = d\log p_{rs(h)}$. In vector notation, this implies

$$\frac{d\text{ToT}}{d\log(1+t)} = \mathcal{E}_p^{ToT}\frac{d\log p}{d\log(1+t)},$$
$$\text{where}\quad [\mathcal{E}_p^{ToT}]_{i,rk} = -\sum_{h\in\mathcal{H}_k} p_{rih}^{M,F} x_{rih}.$$

Finally, let $d\overline{\text{ToT}} \equiv \sum_{i\in\mathcal{R}_F} d\text{ToT}_i$ denote aggregate ToT effects. The expression above implies

$$\frac{d\overline{\text{ToT}}}{d\log(1+t)} = \mathcal{E}_p^{\overline{ToT}}\frac{d\log p}{d\log(1+t)},$$
$$\text{where}\quad [\mathcal{E}_p^{\overline{ToT}}]_{1,rk} = -\sum_{i\in\mathcal{R}_F}\sum_{h\in\mathcal{H}_k} p_{rih}^{M,F} x_{rih}.$$

From Tariff to Import Changes. The last step of our derivation is to convert the Jacobian matrices above—which are derivatives with respect to tariff changes—into the Jacobian matrices that enter our estimating equation—which are derivatives with respect to import changes. We do so by multiplying each original Jacobian matrix by the inverse of the Jacobian matrix of imports with respect to tariffs:

$$\frac{d\log w}{d\log m} = \frac{d\log w}{d\log(1+t)}\left[\frac{d\log m}{d\log(1+t)}\right]^{-1},$$
$$\frac{d\log P^C}{d\log m} = \frac{d\log P^C}{d\log(1+t)}\left[\frac{d\log m}{d\log(1+t)}\right]^{-1},$$
$$\frac{d\text{ToT}}{d\log m} = \frac{d\text{ToT}}{d\log(1+t)}\left[\frac{d\log m}{d\log(1+t)}\right]^{-1},$$
$$\frac{d\overline{\text{ToT}}}{d\log m} = \frac{d\overline{\text{ToT}}}{d\log(1+t)}\left[\frac{d\log m}{d\log(1+t)}\right]^{-1}.$$

OS.4.2 Jacobian Matrices with Respect to Foreign Tariffs

We now turn to the analytical expression for the derivative of wages with respect to foreign tariffs, which we use for the model-testing exercise in Section 3.5. As already discussed in footnote 25, changes in foreign ad valorem tariffs, $d \log(1 + t_{ih}^{F,\text{av}})$, are equivalent to changes in foreign import demand shifters, $d \log \theta_{rih}^{M,F} = -d \log(1 + t_{ih}^{F,\text{av}})$ for all $r \in \mathcal{R}_H$. We can therefore characterize Jacobian matrices with respect to foreign tariffs by characterizing Jacobian matrices with respect to changes in foreign import demand shifters that are uniform across domestic regions, as we do below. In line with our previous analysis, we maintain the assumption that foreign export supply curves are perfectly elastic: $\psi^{X,F} = 0$.

Domestic Prices. We compute the derivative of domestic prices p_{rs} with respect to foreign import demand shifters by expanding the domestic market clearing condition (OS.26) and the regional demand equation (OS.22).

First, note that unlike domestic tariffs, foreign import demand shifters only affect the regional demand equation (OS.22) indirectly through domestic prices p_{rs} and expenditures X_{rs}. For such shocks, (OS.55) therefore simplifies into

$$\mathcal{E}_X^X d \log X = \mathcal{E}_p^X d \log p. \tag{OS.58}$$

Second, we turn to the domestic market clearing condition, which foreign demand shifters directly affect through Home exports. This leads us to a modified version of the differentiated domestic market clearing condition in (OS.49),

$$\begin{aligned} \frac{1}{\alpha_s} d \log p_{rs} - \sum_{k \in \mathcal{S}} \frac{\alpha_{ks}}{\alpha_s} d \log P_{rk} = & \sum_{i \in \mathcal{R}_F} \sum_{h \in \mathcal{H}_s} \frac{p_{rih}^{M,F} q_{rih}^{M,F}}{Y_{rs}} \left(1 - \frac{1}{\psi^{M,F}}\right) d \log p_{rs} \\ & + \sum_{i \in \mathcal{R}_F} \sum_{h \in \mathcal{H}_s} \frac{p_{rih}^{M,F} q_{rih}^{M,F}}{Y_{rs}} \frac{1}{\psi^{M,F}} d \log \theta_{rih}^{M,F} + \sum_{d \in \mathcal{R}_H} \frac{X_{rds}^H}{Y_{rs}} d \log X_{rds}^H. \end{aligned} \tag{OS.59}$$

Next, we substitute for $d \log P_{rk}$ using (OS.50) and for $d \log X_{ris}^H$ using (OS.51) (while setting $d \log(1 + t) = 0$, since domestic tariffs are held fixed). In vector notation, this implies

$$\mathcal{E}_p^p d \log p = \mathcal{E}_X^p d \log X + \mathcal{E}_{\theta^{M,F}}^p d \log \theta^{M,F},$$
$$\text{where} \quad [\mathcal{E}_{\theta^{M,F}}^p]_{rs,ih} \equiv \mathbb{1}_{s(h)=s} 1/\psi^{M,F} \frac{p_{rih}^{M,F} q_{rih}^{M,F}}{Y_{rs}},$$

with $d \log \theta^{M,F} = \left\{ d \log \theta_{ih}^{M,F} \right\}$. Note that without risk of confusion, we omit the region index r from the vector of foreign import demand shifters as a short-hand for $d \log \theta_{rih}^{M,F} = d \log \theta_{ih}^{M,F}$ for all $r \in \mathcal{R}_H$.

Finally, we combine (OS.58) and (OS.59) to compute the Jacobian matrix $d \log p / d \log \theta^{M,F}$

:

$$\frac{d\log p}{d\log\theta^{M,F}} = \left(\mathcal{E}^{p}_{p} - \mathcal{E}^{p}_{X}(\mathcal{E}^{X}_{X})^{-1}\mathcal{E}^{X}_{p}\right)^{-1}\mathcal{E}^{p}_{\theta^{M,F}}. \tag{OS.60}$$

Wages. We now use the Jacobian matrices of domestic prices with respect to foreign import demand shifters described in (OS.60) in order to compute the Jacobian of wages. Conveniently, foreign import demand shifters only affect wages through domestic prices, leading to

$$\frac{d\log w}{d\log\theta^{M,F}} = \mathcal{E}^{w}_{p}\frac{d\log p}{d\log\theta^{M,F}}.$$