MIT 14.01: Principles of Microeconomics
Sp 2025, Lecture 9: Producer Theory (Part II)

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Plan for This Lecture

- Continue with our deeper dive into producer theory
- Now we'll study firm-level supply: how much quantity to sell (if any at all)

Firm-level Supply Decision

- We've introduced the cost function $C_f(q_f)$ – the cost to the firm of producing any given value of q_f in the optimal (cost-minimizing) way
- But which value of q_f will the firm choose? (How much q_f will firm f *supply*?)
- Solve firm's profit-maximization problem (for a competitive firm, i.e. taking p as given). Maximize $\Pi_f \equiv pq_f - C_f(q_f)$, i.e.:

$$\max_{q_f \geqslant 0} pq_f - C_f(q_f) \quad \Rightarrow \quad \text{FOC:} \quad p = MC_f(q_f^*)$$

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- Can invert this to get *firm-level supply function* $q_f = S_f(p)$
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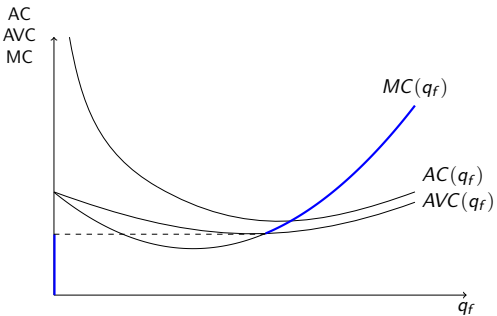
  - Just like in Lecture #3 but now for a continuous q_f
- But we do have to be careful about:
  1. The second-order condition (i.e. is $\frac{\partial^2 \Pi_f(q_f^*)}{\partial q_f^2} < 0$?)
  2. The interior optimum condition (i.e. since $q_f \geqslant 0$, FOC doesn't have to hold if optimum is $q_f = 0$)
  3. Solution indeterminate: if CRTS then MC_f is constant (at all q_f) so whenever $p = MC_f$ the firm would be indifferent about supplying any value of $q_f \geqslant 0$

Firm-level Supply Function $S_f(p)$



- The second-order condition check:
  - If $p = MC_f(q_f)$ at two values of q_f, choose the one where MC is rising
- The interior optimum condition check:
  - Don't produce at all (i.e. set $q_f = 0$) in region where $p < AVC_f(q_f)$
  - Intuition: if $p < AC_f(q_f)$ then the firm is losing money ($\Pi_f < 0$); can happen in presence of sunk fixed costs. But if $p < AVC_f(q_f)$ at all q_f then actually better to shut down ($q_f = 0$) than lose more money on *variable* production.
- Bottom line: $q_f = S_f(p)$ is shown in blue
  - For $q_f > 0$, this is the set of q_f values for which p equals MC, and MC is not falling, and p not below AVC. Otherwise, $q_f = 0$.

Return to the Cobb-Douglas Example from Lecture #8

- We saw that the total cost function (given that fixed cost was sunk) was $C_f(q_f) = F + q_f^{1/\gamma}$
- Recall that we worked out:
  - *Average cost*: $AC_f(q_f) \equiv C_f(q_f)/q_f = F/q_f + q_f^{\frac{1-\gamma}{\gamma}}$
  - *Average variable cost*: $AVC_f(q_f) \equiv VC_f(q_f)/q_f = q_f^{\frac{1-\gamma}{\gamma}}$
  - *Marginal cost*: $MC_f(q_f) \equiv \frac{\partial C_f(q_f)}{\partial q_f} = \frac{1}{\gamma} q_f^{\frac{1-\gamma}{\gamma}}$
- So the supply function $q_f = S_f(p)$ can be found by applying the 3 conditions above (and let's assume $\gamma < 1$, i.e. DRTS):
  1. Need $p = MC_f(q_f)$: $q_f = \delta p^{\frac{\gamma}{1-\gamma}}$, if define $\delta \equiv (\gamma)^{\frac{\gamma}{1-\gamma}}$
  2. But with $q_f = 0$ whenever $MC(q_f)$ is falling: here, this actually never happens (for any $q_f \geqslant 0$), due to $\gamma < 1$.
  3. And with $q_f = 0$ whenever $p < AVC(q_f)$: here, with $\gamma < 1$, we have $MC(q_f) \geqslant AVC(q_f)$ for all q_f. So, since $p = MC$, we know that $p \geqslant AVC(q_f)$ holds for all q_f.
  4. Hence $S_f(p) = \delta p^{\frac{\gamma}{1-\gamma}}$ for all $q_f \geqslant 0$
- Non-DRTS ($\gamma \geqslant 1$) cases are nasty – don't worry about them

Firm-Level Elasticity of Supply

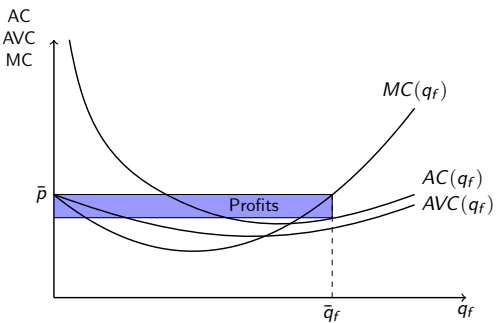
- Just as with the consumer case (in Lecture #7) we can define the price elasticity of supply for firm f as:

$$\varepsilon_{S_f,p} \equiv \frac{p}{S_f(p)} \frac{\partial S_f(p)}{\partial p} = \frac{\partial \ln S_f(p)}{\partial \ln p} \approx \frac{\%\text{ change in } S_f}{\%\text{ change in } p}$$

Return to the Cobb-Douglas Example from Lecture #8

- We saw above that for this case (assuming $\gamma < 1$) the firm-level supply function is: $S_f(p) = \delta p^{\frac{\gamma}{1-\gamma}}$ for all $q_f \geqslant 0$
- So the firm's elasticity of supply is: $\frac{\gamma}{1-\gamma}$
- Note how the elasticity is increasing (supply curve gets flatter) as γ approaches 1 from below. This is the usual case: $S_f(p)$ is upward-sloping due to DRTS, but it gets flatter as we approach CRTS.

Firm-Level Profits

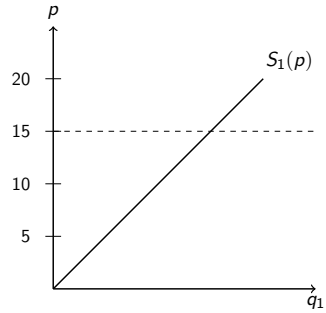


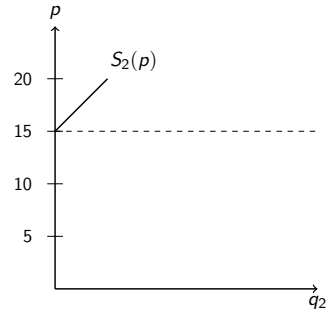
- Profits are given by:

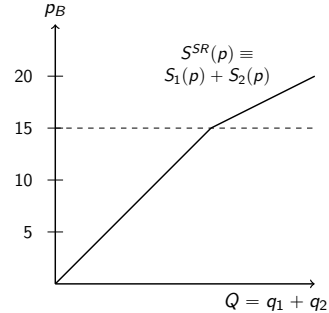
$$\begin{aligned}\Pi_f(q_f) &= pq_f - C_f(q_f) \\ &= [p - AC_f(q_f)]q_f\end{aligned}$$

- Drawn here for case of $\Pi_f(\bar{q}_f) > 0$
- But can have $\Pi_f(\bar{q}_f) = 0$ if happen to have $p = AC_f(\bar{q}_f)$
  - One case where that happens is CRTS production. Then $MC_f = AC_f$, so $p = MC_f = AC_f$, and so $\Pi_f(q_f) = 0$ for all q_f
  - But could happen for other cases too
- And recall that $\Pi_f(\bar{q}_f) < 0$ is possible:
  - With $\bar{q}_f > 0$ if $AVC_f(\bar{q}_f) < p < AC_f(\bar{q}_f)$
  - And with $\bar{q}_f = 0$ if $p < \min_{q_f} AVC_f(q_f)$

Fixed Entry Aggregate Supply Function

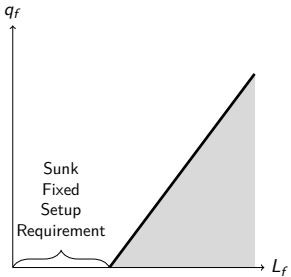


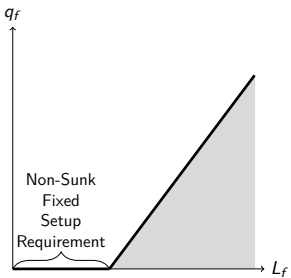




- *Fixed entry* (or *"short-run"*) *aggregate* (or *"industry"*) *supply function*:
  - Defined as $Q = S^{SR}(p) \equiv \sum_{f=1}^{N_F} S_f(p)$ (just like in Lecture #3), so holds constant the set of firms (e.g. fact that there are N_F of them). $N_F = 2$ in figure here.
  - Also holds (w, r) constant, just as we did for each $S_f(p)$ above
- But now, assuming N_F is large, $Q = S^{SR}(p)$ is smooth because:
  - Each firm's $S_f(p)$ is continuous (q_f no longer discrete like it was in Lecture #3)
  - And: the firms might have heterogenous $S_f(p)$, like they did in Lecture #3

Sunk vs Non-Sunk Fixed Costs





- Recall the distinction we introduced in Lecture #8: whether the fixed cost has been paid (i.e. is not recoverable) at the time that we are studying this firm
- If FC_f is the cost of the fixed setup requirement then:
  - When FC is sunk: $C(q_f) = FC_f + VC(q_f)$
  - When FC is non-sunk: $C(q_f) = \min\{0, FC_f + VC(q_f)\}$
- In either case, FC_f is crucial for measuring Π_f. But in both cases, if $q_f > 0$ then FC_f has no effect on the supply function (i.e. the choice of how much q_f)
- In the sunk case, FC_f also has no effect on *whether* to choose $q_f = 0$ or $q_f > 0$
  - Thinking that it does is called the "sunk cost fallacy"
  - E.g. "I don't like these new shoes I just bought (and can't sell). But I'll wear them anyway because I spent so much money on them."

Free Entry Equilibrium (i.e. Making N_F Endogenous)

- Now we suppose that:
  - There is one common $\phi(K_f, L_f)$ that any potential entrant could use if they wanted to (e.g the "best" one)
  - There is a large number of these potential entrants
  - There is *free entry* (aka no barriers to entry) for these potential entrants
  - All inputs (K_f, L_f) can be adjusted freely (so also no barrier to exit)
  - ... and, for simplicity, the firms' (common) average cost function $AC(q_f)$ is U-shaped

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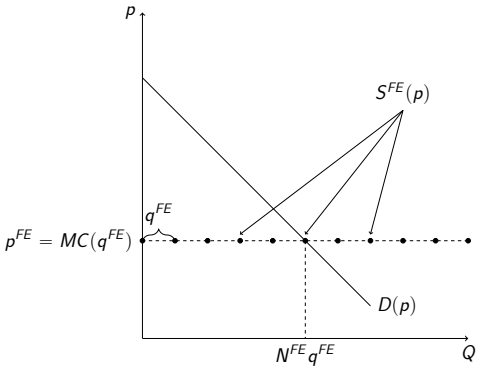
- The *free entry* (aka *"long-run"*) *equilibrium* is one where, given a demand curve $D(\cdot)$, there are N_F^{FE} firms such that:
  1. Firms behave perfectly competitively: each produces $q^{FE} > 0$ such that $p^{FE} = MC(q^{FE})$
  2. Firms make zero profits: $p^{FE} = AC(q^{FE})$ [But recall: "profit" is "economic profit"—costs must include opportunity costs!]
  3. Markets clear $N_F^{FE} q^{FE} = D(p^{FE})$

- #1 and #2 imply that $MC(q^{FE}) = AC(q^{FE})$, so q^{FE} is at the minimum AC—the *minimum efficient scale*. So per-firm output q^{FE} doesn't depend on p!

Free Entry Equilibrium (and Aggregate Long-run Supply Function)



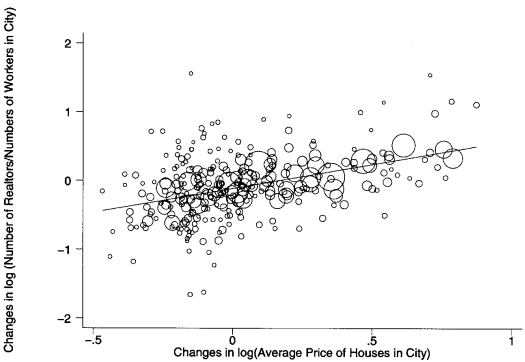
- For illustrative purposes only, this figure has:
  1. An unrealistically low N_F^{FE}
  2. Purely coincidently, the $D(\cdot)$ curve happens to pass through $N_F^{FE}q^{FE}$ for an integer number N_F^{FE} (but if N_F^{FE} were "large" this would be approximately true)

- The *aggregate long-run supply function* $S^{FE}(\cdot)$ is (if we imagine smoothing over the dots) perfectly elastic
  - As if the firms in the industry had CRTS technologies (even though they don't – each has U-shaped AC!)
  - Get CRTS-like behavior on aggregate due to entrants' ability to freely replicate $\phi(K_f, L_f)$ – no source of scarcity (DRTS) to cause $S^{FE}(\cdot)$ to be upward-sloping

Economic Rents

- Suppose we observe a long-run supply curve $S^{FE}(\cdot)$ that *is* upward-sloping
- Then clearly there would be economic profits to be made (for some firms), even in the long-run. We call such "long-run economic profits" *economic rents*.
- How could that arise? There must be a barrier to entry in some sense, e.g.:
  - Government policy somehow prevents free entry (e.g. taxi medallions, liquor licenses)
  - A limited number of special firms truly do have lower-cost technologies – like in our unit-supply model of Lecture #3 – that can't be copied by others
- However, sometimes it may seem like $S^{FE}(\cdot)$ is upward-sloping, but only because there is some scarce input whose price rises (and that we are erroneously failing to hold constant inside $S^{FE}(\cdot)$) as the industry produces more
  - E.g. supply of apartments in Manhattan is upward-sloping but only really because land is a key input, land in Manhattan is scarce, and if the apartment industry tried to expand the number of apartments then the price of land (an input) would rise
  - That is, $S^{FE}(w, r, p)$ is upward-sloping but $S^{FE}(w, r, r_{\text{Land}}, p)$ is not
  - (In fact, the term "rent" comes from David Ricardo's c. 1818 ideas about land!)

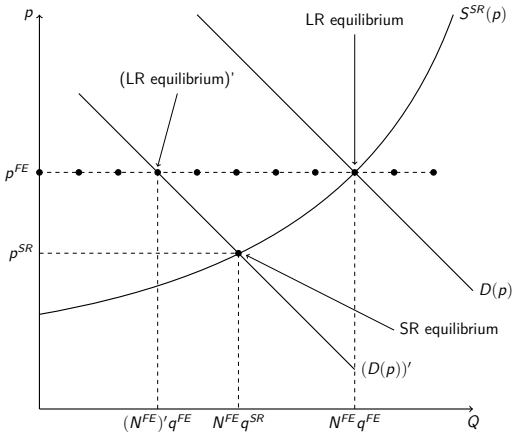
Does Entry Respond to Profit Opportunities?



Hsieh and Moretti (*J. Pol. Econ.*, 2003)

- Real estate agent industry: very little barriers to entry
- When US city house prices went up in 90s (and hence agents' profits per sale went up), the number of real estate agents grew
- In fact, the slope of this log-log line is ≈ 1 (i.e. proportional growth)
- Of course, this is merely the *correlation* between Δ log(house price) and Δ log(per-capita real estate agents) – what confounders make you worried that we cannot say this is evidence of causation?

Comparative Statics in the Short- and Long-Run



- Suppose there is a negative demand shock (from $D(p)$ to $(D(p))'$). What will happen?
- In the "short run", there is adjustment along the fixed-entry supply curve $S^{SR}(p)$:
  - N_F is fixed at N^{FE}: no exit/entry
  - Quantity per firm falls from q^{FE} to q^{SR}
  - Q falls from $N^{FE}q^{FE}$ to $N^{FE}q^{SR}$
  - Price falls from p^{FE} to p^{SR}
- Some firms must have $\Pi_f < 0$ profits at (q^{SR},p^{SR}) – previously they all had $\Pi_f = 0$
- "Long run": some firms exit (we now have $(N^{FE})'$ firms), price rises back to p^{FE}, q rises back to q^{FE}, and Q falls further (from $N^{FE}q^{SR}$ to $(N^{FE})'q^{FE}$).

Concluding Remarks

- **Key Concepts from today's lecture:**
  - Firm-level supply function in a competitive market: quantity is strictly positive where price equals marginal cost, marginal costs not decreasing, and price exceeds average variable costs; otherwise, quantity is zero.
  - Firm-level elasticity of supply
  - Firm-level profits (and how they can be negative in the short-run)
  - Sunk vs. non-sunk fixed costs
  - Fixed-entry (short-run) aggregate supply function: where number of firms is fixed
  - Free entry equilibrium (and long-run aggregate supply function): when number of (identical) firms responds via exit/entry such that firms make zero profits in long-run
  - Economic rents: when firms make profits in the long-run
  - Short-run vs. long-run comparative statics

- **Next lecture:**
  - We have so far taken input/factor prices (w, r) as given
  - How are they determined? Factor market equilibrium...