

MIT 14.01: Principles of Microeconomics
Sp 2025, Lecture 8: Producer Theory (Part I)

Dave Donaldson

Plan for This Lecture

- Revisit supply side of economy (i.e. firms) but with a richer model of supply than we saw in Lecture #3
- Main focus: how firms use inputs to produce in a profit-maximizing (cost-minimizing) way

Producer Theory: Towards Richer Models of Supply

- In Lecture #3 we saw a model of supply (for any firm f) but it was very simple:
 - Binary decision of whether to sell a unit of good B_f or not, given cost of making that unit c_f
- What is missing from this?
 - What if the goods are better thought of as producible in a fairly continuous manner (e.g. food, rice, cellphone data)?
 - Where do costs come from?
 - Where do *firms* even come from?
- Similarly to the consumer side in Lectures #6-7, keep in mind:
 - Previously each firm f 's decision was binary, and then we got a smooth, upward-sloping aggregate supply function $S(p)$ purely from aggregation across many firms who were different (in their costs)
 - Now we will mostly take an opposite approach: study one firm f , but where f 's decision is continuous. That will lead to a smooth firm-level supply function $S_f(p)$. So we get a smooth supply function even before aggregation across different firms.

Maximizing Profits

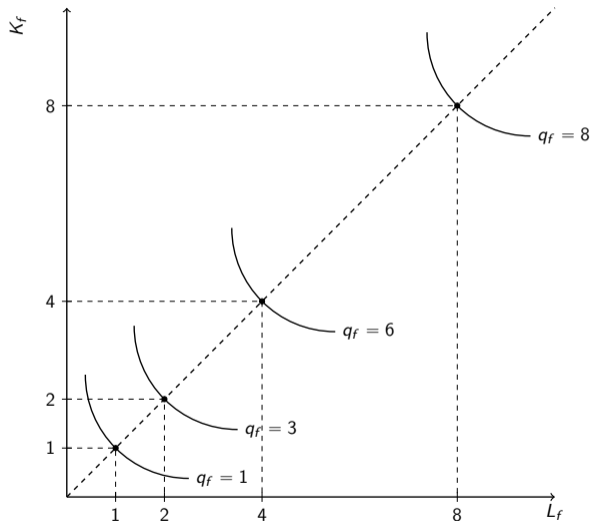
- We will continue to assume (like in Lecture #3) that firms maximize profits
- Suppose firm f can produce a continuous amount of some good (e.g. bednets) denoted $q_f \geq 0$
- And suppose we knew the firm's cost $C_f(q_f)$ of making q_f
 - NB: since our firm only makes one product, for now we won't bother writing things like q_f^A and q_f^B
 - So q_f denotes the number of units of *its* good that firm f makes
- Then this profit-maximizing firm would maximize:

$$\Pi_f \equiv pq_f - C_f(q_f)$$

But Where Do Costs $C_f(q_f)$ Come From?

- Why is it costly to produce? Because firm has to pay for *inputs* in order to produce its output q_f
- What are inputs?
 - We'll focus on two: labor (L_f) and capital (K_f)
 - These are often called *primary factors* of production
 - (One can of course imagine other inputs, but let's keep it simple)
- Inputs relate to the output q_f through a *production function*: $q_f \leq \phi_f(L_f, K_f)$
 - This is a purely “technological” constraint—inputs go in, output comes out
 - We write “ \leq ” just to remind ourselves that the firm could always throw away some q_f (though it usually wouldn't want to)
- Aside on capital:
 - By $q_f \leq \phi_f(L_f, K_f)$ we mean the “flow” of output per unit time
 - Natural to think of the firm “hiring” L_f for that unit of time
 - But K_f is more likely to be (durable) things that the firm *owns* like machines, buildings, or even land
 - So that sometimes brings up discussions of “financial capital” (e.g. how can a firm get funding to buy the physical K_f), but we won't go into that here

The View from Above: Isoquants of the Production Function



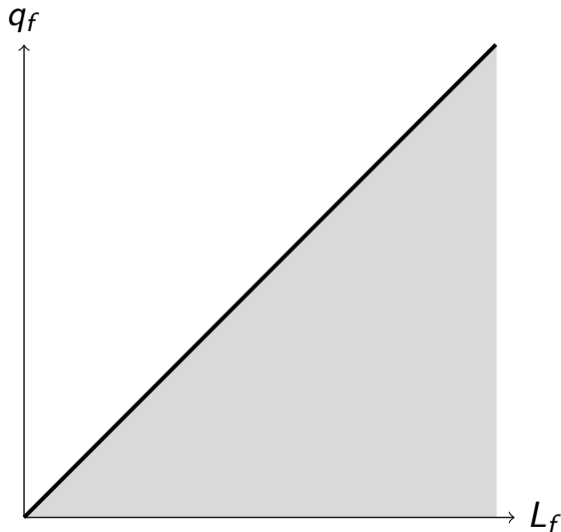
- *Isoquants* are level sets of the production function $q_f = \phi_f(L_f, K_f)$
 - Analogous to *indiff. curves* for the consumer's U function in Lecture #6
- Usually expect that $\phi_f(\cdot)$ is increasing in both arguments (more inputs make more output), as drawn here
- Let the *marginal product of capital* $MPK_f \equiv \frac{\partial \phi_f}{\partial K_f}$ (and similarly for labor)
- Usually expect isoquants convex to origin, as drawn here
 - This implies *diminishing marginal product* for any input, e.g.:

$$\frac{\partial MPK_f}{\partial K_f} = \frac{\partial^2 \phi_f}{\partial K_f^2} < 0$$

Example of a Cobb-Douglas Production Function

- Suppose the production function is $q_f = \phi_f(K_f, L_f) = K_f^\alpha L_f^\beta$
- We expect $\alpha > 0$ and $\beta > 0$ so that $\phi_f(\cdot)$ is increasing in both arguments
- What is the marginal product of capital (and labor)?
 - $MPK_f = \frac{\partial \phi_f}{\partial K_f} = \alpha K_f^{\alpha-1} L_f^\beta$
 - $MPL_f = \frac{\partial \phi_f}{\partial L_f} = \beta K_f^\alpha L_f^{\beta-1}$
- Is the marginal product of capital (or labor) diminishing?
 - $MPK_f = \alpha K_f^{\alpha-1} L_f^\beta$ is decreasing in K_f (i.e. displays diminishing marginal product of capital) as long as $\alpha < 1$
 - $MPL_f = \beta K_f^\alpha L_f^{\beta-1}$ is decreasing in L_f (i.e. displays diminishing marginal product of labor) as long as $\beta < 1$

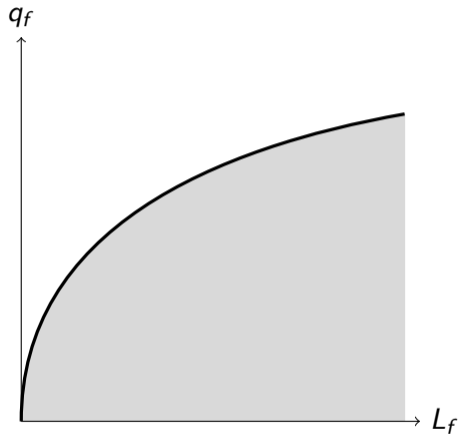
Returns to Scale (RTS)



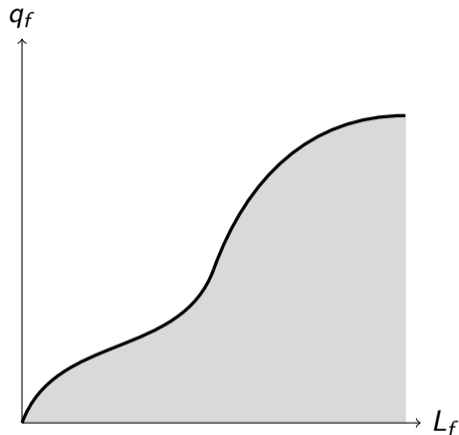
Shaded area satisfies $q_f \leq \phi_f(L_f)$

- Returns to scale (RTS) defined using the following (for any $\lambda > 0$):
 - Let $\phi_f(\lambda K_f, \lambda L_f) = \lambda^r \phi_f(K_f, L_f)$
 - If $r = 1$: constant RTS (CRTS)
 - If $r < 1$: decreasing RTS (DRTS)
 - If $r > 1$: increasing RTS (IRTS)
- E.g., if we double all inputs (i.e. set $\lambda = 2$), will we get more than double the output (i.e. $r > 1$)? If so, IRTS.
- Drawn here: a one-input production function (i.e. $\phi_f(L_f)$) that has CRTS

More Examples of Returns to Scale (RTS)

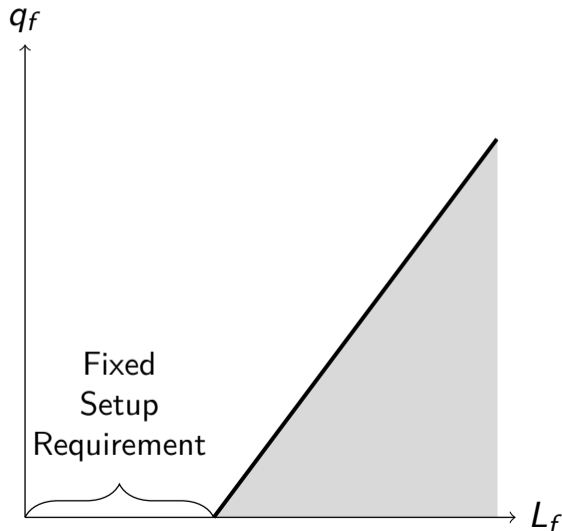


Decreasing returns to scale (DRTS)



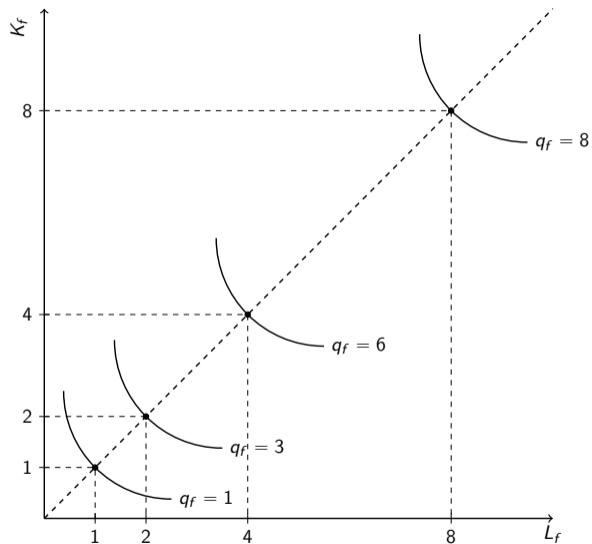
Variable returns to scale (VRTS): i.e. DRTS for low L_f , then IRTS for medium L_f , then DRTS again for high L_f

Fixed Setup Requirement: Important Type of IRTS Production Function



- Here it takes a certain amount of L (the “fixed setup requirement” region) to be able to start producing anything
- After that, increases in output are proportional to increases in inputs, but because of the fixed setup requirement the production function is IRTS everywhere

Visualizing Returns to Scale with Two Inputs



- This has VRTS...
 - $L_f = 1$ to $L_f = 2$: IRTS
 - $L_f = 2$ to $L_f = 4$: CRTS
 - $L_f = 4$ to $L_f = 8$: DRTS

Example of a Cobb-Douglas Production Function

- Suppose the production function is $q_f = \phi_f(K_f, L_f) = K_f^\alpha L_f^\beta$
- What is the return to scale for this case?
 - $\phi_f(\lambda K_f, \lambda L_f) = (\lambda K_f)^\alpha (\lambda L_f)^\beta = \lambda^{\alpha+\beta} K_f^\alpha L_f^\beta = \lambda^{\alpha+\beta} \phi_f(K_f, L_f)$
 - So $\text{RTS} = \alpha + \beta$ for any level of inputs (so can't be VRTS)
 - Hence CRTS if $\alpha + \beta = 1$, DRTS if $\alpha + \beta < 1$, and IRTS if $\alpha + \beta > 1$

Costs

- So far we've seen the firm's *technology* $\phi_f(K_f, L_f)$ – which has nothing to do, per se, with their *costs* of producing
- But suppose that the cost to the firm of using labor is it pays w per unit L_f used
- And suppose the cost of using capital is r per unit K_f used. But:
 - Think of the firm as paying r to *rent* (or “hire”) the K_f for one unit of time
- What if the firm actually owns its capital?
 - Then the true cost to the firm of using these owned inputs should include the foregone opportunities that they could have been put towards outside of the firm
- This correct treatment of owned inputs is an example of the importance of thinking about *opportunity costs* when we study a firm's (or anyone's!) costs:
 - *Economic cost* (what we do here) includes all opportunity costs of using things that the firm owns (e.g if the firm's owner puts in their own work time into production, but the owner could work outside the firm for w , then the owner's work time should be included in the L_f in wL_f)
 - Very different from *accounting costs*, used by accountants. (Why do accountants do that? Partly because most countries don't let firms deduct opportunity costs from their corporate taxes.)

Minimizing Costs

- Since the firm is maximizing profits, we can think of it achieving this in two steps:
 - Step 1: find the cheapest (i.e. cost-minimizing) way of making the amount q_f , given its available technology $q_f = \phi_f(K_f, L_f)$, and taking w and r as given (i.e. the input market is perfectly competitive). Call the achieved minimum-cost value the *cost function* $C_f(w, r, q_f)$.
 - Step 2: find the profit-maximizing value of q_f , given the cost function $C_f(w, r, q_f)$
- For now, we focus on step 1. Mathematically, this is just:

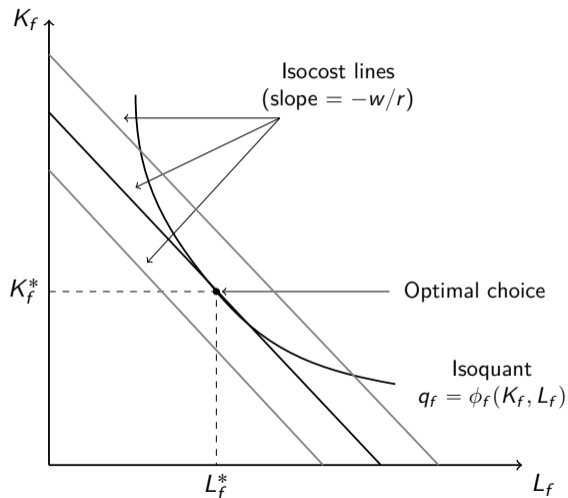
$$\min_{K_f, L_f} wL_f + rK_f \quad \text{s.t.} \quad q_f \leq \phi_f(K_f, L_f) \quad (1)$$

- And then the value achieved by the objective function at the optimum (aka the “value function” of any optimization problem) is the cost function

$$C_f(w, r, q_f) \equiv wL_f^* + rK_f^*$$

where L_f^* and K_f^* solve the problem in (1)

The Cost-Minimization Problem



- Very similar to U-max problem for consumer in Lecture #6
 - Here: moving the *isocost* line inwards until it's tangential to the given isoquant (at given q_f)
 - Consumer: moving the indifference curve outwards until it is tangential to the given budget constraint
- Solution (defining $MRTS =$ *marginal rate of technical substitution*):

$$\frac{MPL_f}{MPK_f} \equiv -MRTS_f = (w/r)$$

- Can also write as $MPL_f/w = MPK_f/r$, which you can think of as equalizing the “bang for the buck” across the two inputs

Return to the Cobb-Douglas Example

- Solve the cost-minimization problem using the Lagrangian:

$$\mathcal{L} = wL_f + rK_f + \lambda(q_f - K_f^\alpha L_f^\beta)$$

- The three FOCs are

$$\frac{\partial \mathcal{L}}{\partial K_f} = r - \lambda^* \alpha (K_f^*)^{\alpha-1} (L_f^*)^\beta = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial L_f} = w - \lambda^* \beta (K_f^*)^\alpha (L_f^*)^{\beta-1} = 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = q_f - (K_f^*)^\alpha (L_f^*)^\beta = 0 \quad (4)$$

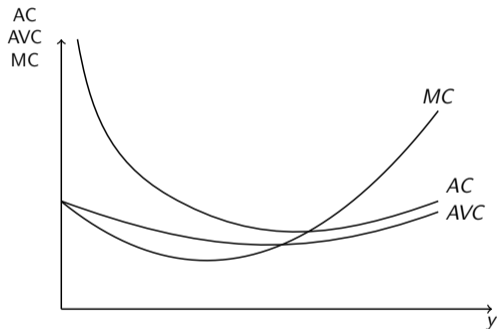
- Find solutions (K_f^*, L_f^*) , plug them back into the objective function $(wL_f + rK_f)$ to get the cost function (i.e. minimized cost = cost of doing the optimal thing):

$$C_f(r, w, q_f) = \kappa r^{\alpha/\gamma} w^{\beta/\gamma} q_f^{1/\gamma}, \quad \text{with } \gamma \equiv \alpha + \beta \quad \text{and} \quad \kappa \equiv \gamma(\alpha^{-\alpha} \beta^{-\beta})^{1/\gamma}$$

More About Cost Functions

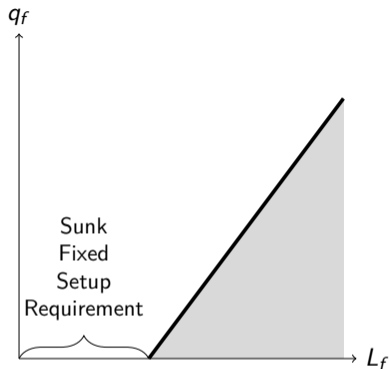
- From now on, we will hold w and r fixed in the background and suppress them, so write the cost function as $C_f(q_f)$
- Can often decompose $C_f(q_f)$ into *fixed costs* (FC_f , which don't depend on output) and *variable costs* ($VC_f(q_f)$, which do depend on output):
$$C_f(q_f) = FC_f + VC_f(q_f)$$
 - Fixed costs arise when there are fixed setup requirements in the production function (like we saw earlier)
- Some useful definitions related to cost functions:
 - *Average cost*: $AC_f(q_f) \equiv C_f(q_f)/q_f$
 - *Average variable cost*: $AVC_f(q_f) \equiv VC_f(q_f)/q_f$
 - *Marginal cost*: $MC_f(q_f) \equiv \frac{\partial C_f(q_f)}{\partial q_f}$
- The way that $C_f(q_f)$ depends on q_f is governed by the returns to scale of the underlying production function. E.g. if $\phi_f(\cdot)$ is CRTS then:
 - $C_f(q_f) = c_f q_f$ for some constant $c_f > 0$
 - So $AC_f(q_f) = AVC_f(q_f) = MC_f(q_f) = c_f$, and all are constant (don't depend on q_f)
 - And of course $FC_f = 0$ since can't have a fixed setup requirement if CRTS

More General Relationship Between Marginal, Average, and Average Variable Costs

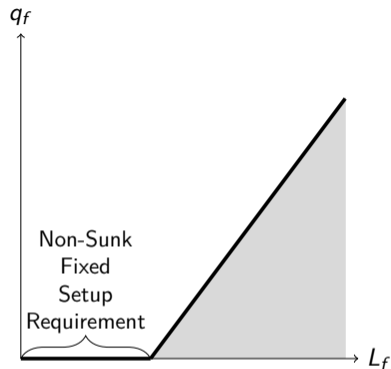


- $MC = AC$ always at the point where AC is minimized. Why?
 - $MC > AC$ implies that AC must be rising
 - $MC < AC$ implies that AC must be falling
- And $MC = AVC$ always at the point where AVC is minimized (by analogous logic)

Sunk vs Non-Sunk Fixed Costs: A Question of Timing



This setup requirement is already *sunk* (i.e. not recoverable) at this point in time, so it is not possible for the firm to choose low amounts of L_f . The firm's $C_f(q_f)$ therefore has a *sunk FC* (i.e. the firm must pay *FC* even if $q_f = 0$).



This setup requirement is *not yet sunk* (i.e. still recoverable) at this point in time, so it is possible for the firm to choose low amounts of L_f . The firm's $C_f(q_f)$ therefore has a *non-sunk FC* (i.e. the firm doesn't have to pay *FC* if it chooses $q_f = 0$).

Return to the Cobb-Douglas Example

- Recall, we saw that the cost function was:

$$C_f(r, w, q_f) = \kappa r^{\alpha/\gamma} w^{\beta/\gamma} q_f^{1/\gamma}$$

- Let's choose units such that $\kappa r^{\alpha/\gamma} w^{\beta/\gamma} = 1$ so that this is: $C_f(q_f) = q_f^{1/\gamma}$
- And let's imagine that there is also a (sunk) fixed cost of F such that the full cost function is: $C_f(q_f) = F + q_f^{1/\gamma}$
 - Note that if F is sunk then even if $q_f = 0$, the firm still has to pay F , i.e. $C_f(0) = F$
- So we can work out:
 - Variable cost:* $VC_f(q_f) \equiv C_f(q_f) - C_f(0) = F + q_f^{1/\gamma} - F = q_f^{1/\gamma}$
 - Average cost:* $AC_f(q_f) \equiv C_f(q_f)/q_f = F/q_f + q_f^{\frac{1-\gamma}{\gamma}}$
 - Average variable cost:* $AVC_f(q_f) \equiv VC_f(q_f)/q_f = q_f^{\frac{1-\gamma}{\gamma}}$
 - Marginal cost:* $MC_f(q_f) \equiv \frac{\partial C_f(q_f)}{\partial q_f} = \frac{1}{\gamma} q_f^{\frac{1-\gamma}{\gamma}}$

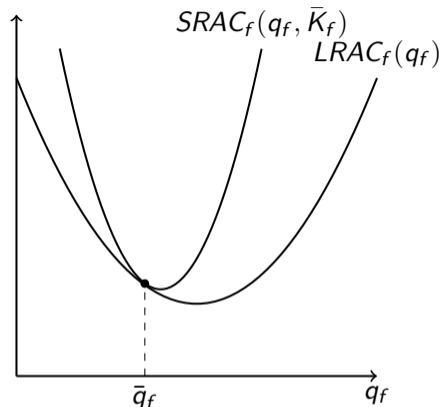
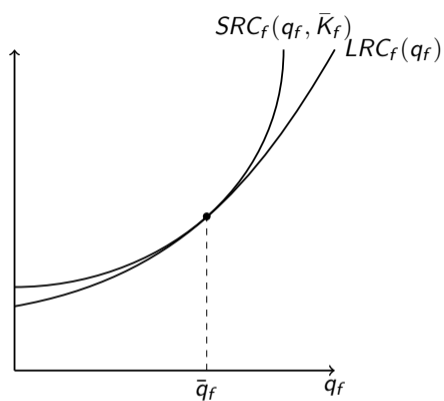
Short-run vs Long-run Cost Functions

- In the “short run” we might imagine that one input (e.g. K_f) cannot be adjusted away from some value \bar{K}_f
 - If K and L are the only inputs, then if K_f is fixed the choice of how much L_f to use to make a given q_f is trivial
 - So here (and only here) let's introduce two types of labor (L^A, L^B), at prices (w^A, w^B), which go into a new prod. fun. $\phi_f(K_f, L_f^A, L_f^B)$
- So the *short-run cost-minimization problem* looks like:

$$\min_{L_f^A, L_f^B} w^A L_f^A + w^B L_f^B + r \bar{K}_f \quad \text{s.t.} \quad q_f \leq \phi_f(\bar{K}_f, L_f^A, L_f^B)$$

- And the solution is the *short-run cost function*: $SRC_f(w^A, w^B, r, q_f, \bar{K}_f)$
- This means that earlier (when K_f was an unrestricted choice) we were solving the *long-run cost-minimization problem*. And hence the cost function we arrived at is really the *long-run cost function*: $LRC_f(w^A, w^B, r, q_f)$
- We can of course have AC and MC versions of long-run and short-run costs too (i.e. SRAC vs LRAC, SRMC vs LRMC)

Short-run vs Long-run Cost Functions



$SRC \geq LRC$ always (since former is strictly constrained version of the latter). But there is always one value of q , here denoted \bar{q}_f , at which the constraint of $K_f = \bar{K}_f$ in the SR problem is exactly what would have been chosen in the LR problem; so $SRC(\bar{q}_f, \bar{K}_f) = LRC(\bar{q}_f)$, i.e. the two curves touch at one point. Same argument applies to the AC case (on right).

Concluding Remarks

- **Key concepts from today's lecture:**
 - Production functions and production concepts (marginal products, isoquants, returns-to-scale)
 - Cost-minimization problem and its solution (the cost function)
 - Different notions of costs (fixed and sunk, fixed and non-sunk, average, average variable, marginal)
 - Short-run vs. long-run cost functions
- **Next lecture:**
 - What will firms' supply decisions be, given their cost functions?