MIT 14.01: Principles of Microeconomics
Sp 2025, Lecture 12: General Equilibrium (part II)

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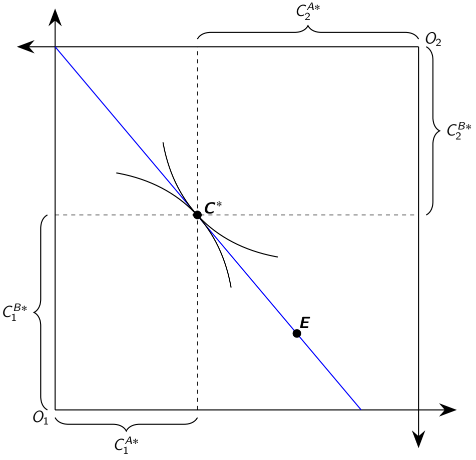
Plan for Today

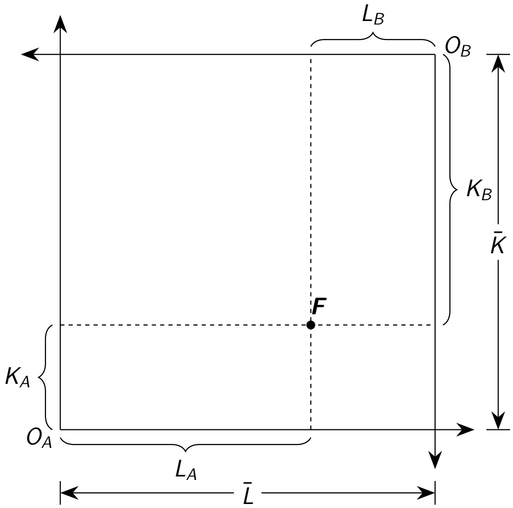
- Continuing our investigation of general equilibrium
- But now for the case of economies in which both goods are both produced (rather than being endowments) and consumed

Where Did the Edgeworth Box Come From?



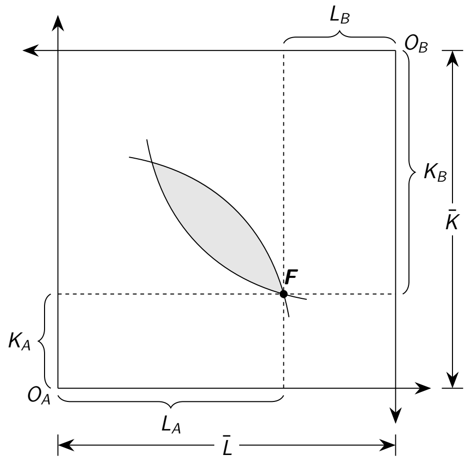
- Recall from Lecture #11, our model of a general equilibrium *exchange economy* had:
  - Two agents (1 and 2) have endowments (***E***) of each of two goods (A and B) and trade them in a competitive market to end up at ($\boldsymbol{C}^*$)
- But where did these endowments ***E*** of the two goods come from?
- Today we will consider a general equilibrium *production economy*:
  - Firms use K and L to produce A and B
  - Consumers use their income to buy goods A and B from the firms
  - Consumers get their income from selling their endowments of K and L to the firms
  - (Recall the circular flow diagram of Lecture #10)

Factor Allocations for Production



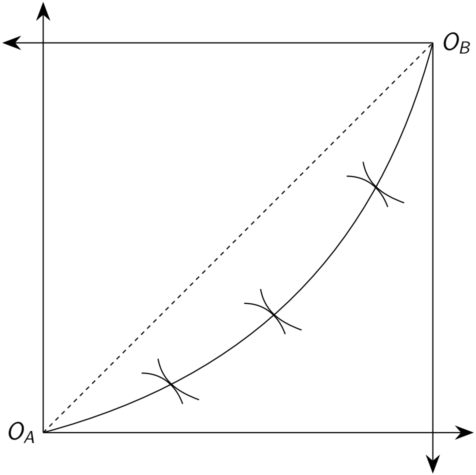
- Suppose that there are two firms with production functions:
  - For making good A: $Q_A = \phi_A(K_A, L_A)$
  - For making good B: $Q_B = \phi_B(K_B, L_B)$
  - Both technologies ϕ_f are constant returns to scale
- And suppose that the total factor endowments in the economy are $\bar{K}$ and $\bar{L}$
- This figure illustrates (with the rotated axis trick we saw from the Edgeworth Box in Lecture #11) all the feasible allocations of factors to firms A and B, letting:
  - $\boldsymbol{F}_A \equiv (K_A, L_A)$ and $\boldsymbol{F}_B \equiv (K_B, L_B)$
  - $\boldsymbol{F} \equiv (\boldsymbol{F}_A, \boldsymbol{F}_B)$
  - NB: $\boldsymbol{F}$ is not the endowment point; it's an allocation of the factors to production

Production Efficiency



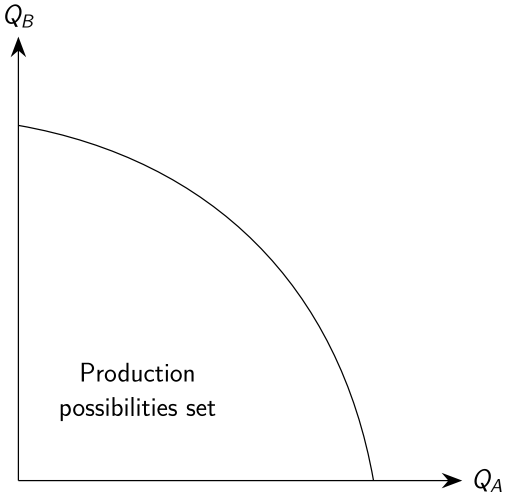
- Associated with the factor allocation $\boldsymbol{F}$ is an output bundle (Q_A, Q_B)
- Here we draw the two firms' isoquants associated with all of the points that yield the same amount of outputs as $\boldsymbol{F}$
- Is $\boldsymbol{F}$ a "good" place for the economy to allocate its factors? *Production Efficiency*:
  - When factors are allocated to firms in such a way that it is impossible to produce more of one good without producing the same or less of some other good
- So $\boldsymbol{F}$ is not productively efficient: all allocations in shaded region would have more Q_A and/or Q_B

Production Efficiency



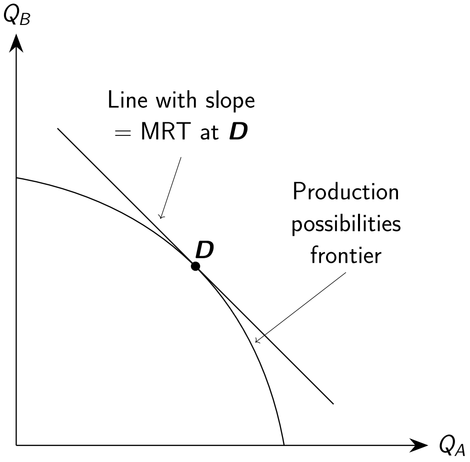
- This figure shows the set of productively efficient factor allocations
- These are points at which $MRTS_A = MRTS_B$
  - Recall from Lecture #8 that MRTS is the "marginal rate of technical substitution", with $MRTS_f \equiv -\frac{MPL_f}{MPK_f} \equiv -\frac{\partial Q_f/\partial L_f}{\partial Q_f/\partial K_f}$ = slope of isoquant
- If $\phi_A(\cdot)$ is different from $\phi_B(\cdot)$ the set of efficient allocations has to be either above or below the diagonal (as shown). Why?
  - If $\phi_A(\cdot) = \phi_B(\cdot)$ then efficient to be on diagonal
  - Otherwise, can deviate from the diagonal and do better by exploiting the fact that $\phi_A(\cdot)$ is different from $\phi_B(\cdot)$
  - NB: this implies that the MRTS is different at every efficient allocation

Production Possibilities Set



- *Production possibilities set:* all feasible bundles of output (Q_A, Q_B) – each corresponds to a feasible factor allocation
- *Production possibility frontier* (PPF): the outer ("north-east") boundary of this set.
  - Corresponds to all productively efficient uses of factors.
  - Think of moving the factors out of B and into A, as you slide down the PPF
- PPF is always (for non-IRTS production functions) weakly concave to the origin like here. Why? Same logic as previous slide:
  - PPF would be linear if no heterogeneity across goods in ways to use factors
  - But with heterogeneity, can do better than linear as move factors from B to A

Marginal Rate of Transformation



- *Marginal rate of transformation* (MRT): The slope of the PPF at a given allocation. That is:

$$MRT \equiv \frac{dQ_B}{dQ_A}$$

- And one can show that:

$$MRT = -\frac{MPL_B}{MPL_A} = \frac{MPK_B}{MPK_A}$$

- Intuition: Sliding down the PPF (i.e. producing more A and less B)
  - Do that by moving L from B into A, so relative *MPL*s matter
  - Same argument applies to K
  - Production efficiency requires that the tradeoff for L is the same as that for K

Factor Market Equilibrium

- Recall from Lecture #8 that a profit-maximizing firm that buys inputs on competitive input markets would solve

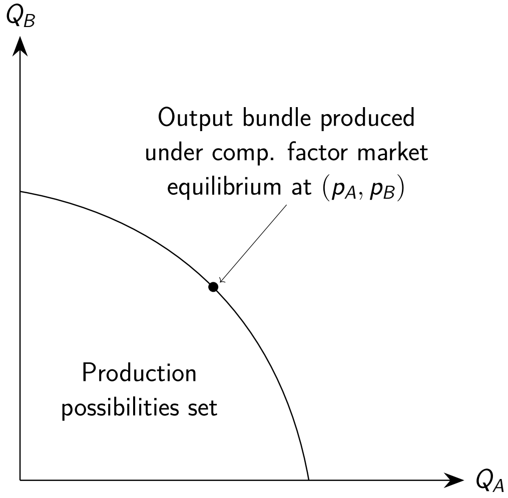
$$\min_{L_f, K_f} wL_f + rK_f \quad \text{s.t.} \quad Q_f \leqslant \phi_f(K_f, L_f)$$

- And the solution involves $MRTS_f = -w/r$. If firm also faces output price p_f then choice of factor usage (e.g. labor) follows unconditional factor demand function (e.g. $L_f^{UD}(w, r, p_f)$).
- *Competitive factor market equilibrium (given goods prices)*: If firms take factor prices (w, r) as given and factor markets clear then equilibrium (for any given goods prices (p_A, p_B)) will be the solution $(w^{\text{eq}}, r^{\text{eq}})$ to

$$\begin{aligned} L_A^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_A) + L_B^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_B) &= \bar{L} \\ K_A^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_A) + K_B^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_B) &= \bar{K} \end{aligned}$$

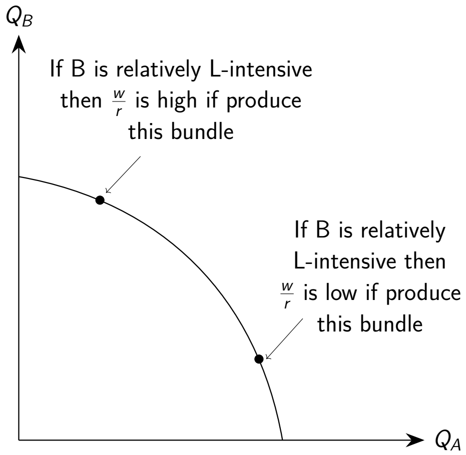
- That is, firms' unconditional factor demands (from Lecture #10) equal factor supply (i.e. the factor endowments)

Getting to Production Efficiency



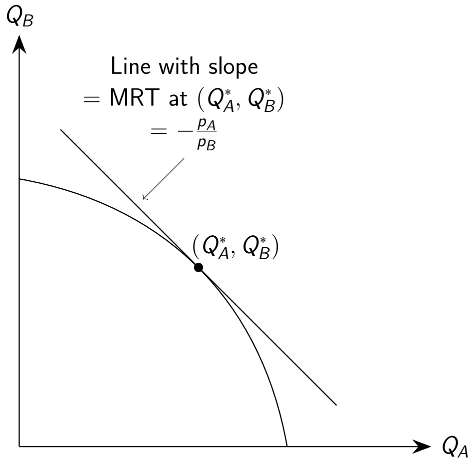
- Since all firms set $MRTS_f = -w/r$, production efficiency (i.e. $MRTS_A = MRTS_B$) occurs whenever all firms face the same value of w/r
- *First Welfare Theorem for a Production Economy (part I)*:
  - Absent factor market failures (taxes, missing markets, etc.) any competitive factor market equilibrium (given goods prices) will be productively efficient.
- So competitive factor markets cause production to take place on the PPF

Aside: the PPF and Earnings Inequality



- But note that (as we saw above) each point on the PPF has a different MRTS
  - So each point will have a different set of relative factor prices w/r
  - So earnings inequality is determined by where on the PPF production takes place
- Which way will it go?
  - Depends on how $\phi_A(\cdot) \neq \phi_B(\cdot)$
  - Since they are both CRTS can confine attention to how intensively they each use K and L
- Suppose that B is relatively L-intensive:
  - Then at high Q_B the total demand for L is relatively high, so w/r will be high.
  - And at low Q_B the total demand for L is relatively low, so w/r will be low.

Full General Equilibrium with Production: Supply Side



- If the output market that firms sell into is also competitive then the output bundle produced (Q_A^*, Q_B^*) will be tangential to the line with slope $-p_A/p_B$

- Why? Price-taking firms choose (Q_f, K_f, L_f) to maximize profits:

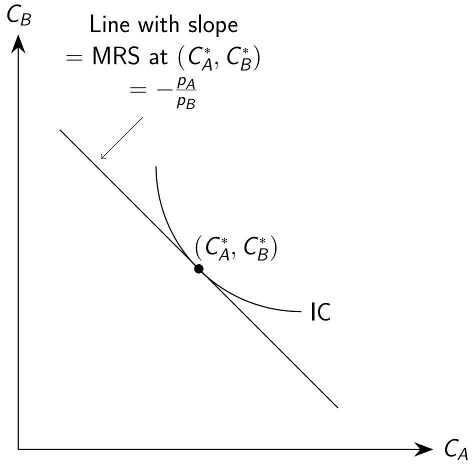
$$\max_{L_f, K_f} p_f \phi(K_f, L_f) - wL_f - rK_f$$

$$\Rightarrow \quad w = p_f MPL_f \quad r = p_f MPK_f$$

$$\Rightarrow \quad \frac{p_A}{p_B} = \frac{MPL_B}{MPL_A} = \frac{MPK_B}{MPK_A} \equiv -MRT$$

- And note that due to CRTS, firms make zero profit and hence $w\bar{L} + r\bar{K} = p_A Q_A^* + p_B Q_B^*$

Full General Equilibrium with Production: Demand Side



- Suppose there is one (type of) consumer and they own all of the factors in the economy. So their income is $y = w\bar{L} + r\bar{K}$.
- If they take goods prices (p_A, p_B) as given and choose (C_A, C_B) to maximize some $U(C_A, C_B)$ subject to the budget constraint $y = p_A C_A + p_B C_B$ we know they'll choose (C_A^*, C_B^*) such that:

$$MRS = -\frac{p_A}{p_B} \quad \text{and } w\bar{L} + r\bar{K} = p_A C_A^* + p_B C_B^*$$

- That is, as usual, their indifference curve is tangential to their budget constraint (but now their income is equal to $w\bar{L} + r\bar{K}$)

Full General Equilibrium with Production: Putting it Together

- A *competitive general equilibrium with production*: A set of goods prices $(p_A^{\text{eq}}, p_B^{\text{eq}})$ and factor prices $(w^{\text{eq}}, r^{\text{eq}})$ such that
  - Factor markets clear:

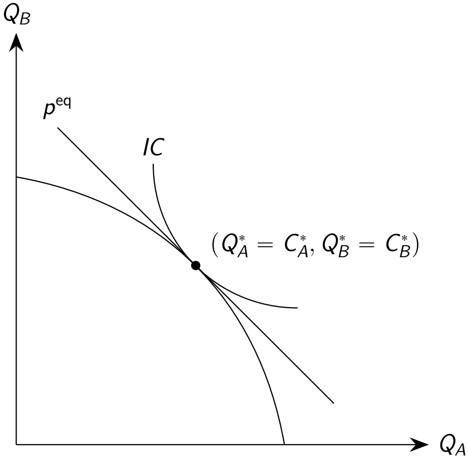
$$\begin{aligned} L_A^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_A^{\text{eq}}) + L_B^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_B^{\text{eq}}) &= \bar{L} \\ K_A^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_A^{\text{eq}}) + K_B^{UD}(w^{\text{eq}}, r^{\text{eq}}, p_B^{\text{eq}}) &= \bar{K} \end{aligned}$$

  - And goods markets clear:

$$\begin{aligned} S_A(w^{\text{eq}}, r^{\text{eq}}, p_A^{\text{eq}}) &= C_A^*(y, p_A^{\text{eq}}, p_B^{\text{eq}}) \\ S_B(w^{\text{eq}}, r^{\text{eq}}, p_B^{\text{eq}}) &= C_B^*(y, p_A^{\text{eq}}, p_B^{\text{eq}}) \\ \text{with } y &= w\bar{L} + r\bar{K} \end{aligned}$$

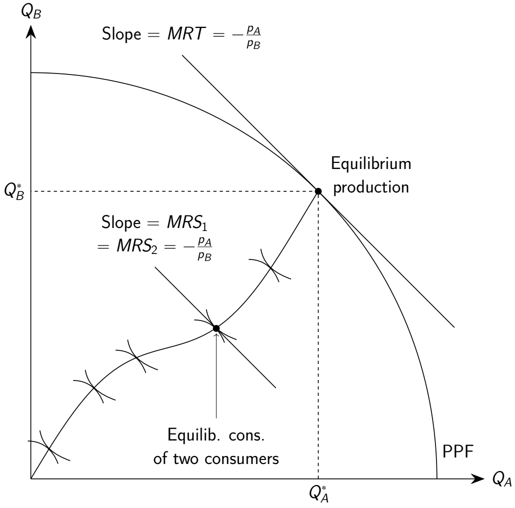
- So it's like the factor market equilibrium (given goods prices) that we saw earlier, but now goods market clearing pins down the good prices (but with feedback, since factor prices determine incomes, which determine goods demand)
- As in Lecture #11, only relative prices pinned down (so set one of the 4 prices to =1) and one of the 4 equations is redundant (Walras' Law)

Full General Equilibrium with Production: Putting it Together



- As illustrated here, the price (p_A^{eq}, p_B^{eq}) of the competitive general equilibrium with production is a line of slope $-\frac{p_A^{eq}}{p_B^{eq}}$, where the consumer's IC is tangential to the PPF
- This happens at the quantities that are optimal for producers (e.g. Q_A^*) and consumers (e.g. C_A^*), and where those quantities are mutually consistent (e.g. $Q_A^* = C_A^*$)

Is This a "Good" Way to Organize Production and Consumption?



- *First Welfare Theorem for a Production Economy*:
  - Absent factor or goods market failures (taxes, missing markets, etc.) any competitive general equilibrium with production will be Pareto Efficient
- We've seen above how competitive factor markets part leads to production efficiency. New feature here is that since consumers and firms face same goods prices, we have (if 2 consumers, as shown here):

$$MRS_1 = MRS_2 = -\frac{p_A}{p_B} = MRT$$

- Each utility trade-off is equal to the technological trade-off (from the PPF)

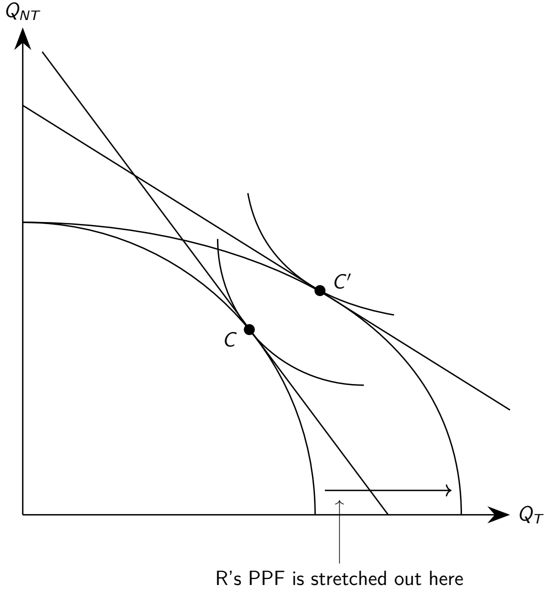
First Welfare Theorem: Comments

- As usual, just because an allocation is Pareto Efficient doesn't mean it's "good"
  - Some social objective function might want to sacrifice PE in favor of equality/redistribution.

- But there is something different about production efficiency
  - This is all about arranging production in a way that it is impossible to produce "more with less"
  - It would be weird for a society to deliberately limit its productivity (akin to producing some good and then destroying it!) in the name of redistribution
  - For this reason, economic policy usually favors taxes on final products over taxes on input use. The logic: at least make sure we're on the PPF; then, if needed, consider departing from PE (e.g. $MRS_1 \neq MRS_2$, or $MRS \neq MRT$) for the sake of equality/redistribution.

- As in Lecture #11, there is a *Second Welfare Theorem for a Production Economy*
  - If technologies and preferences are convex, any PE allocation can be achieved as a competitive general equilibrium with lump-sum taxes/transfers of factor endowments

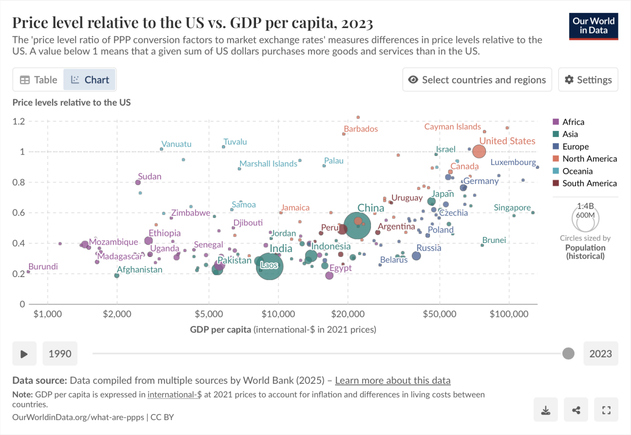
Why Are Haircuts So Cheap in Low-Income Countries?



- Suppose country R is more productive than country P, but more so in good T (than in good NT)
  - So R's PPF is stretched out (relative to P's PPF) more in the Q_T direction than the Q_{NT} direction (extreme version shown here)
- But the two countries' preferences are similar (and homothetic)
- Then (as you can see here) equilibrium prices will feature:

$$\left(\frac{p_{NT}}{p_T}\right)^R > \left(\frac{p_{NT}}{p_T}\right)^P$$

Why Are Haircuts So Cheap in Low-Income Countries?

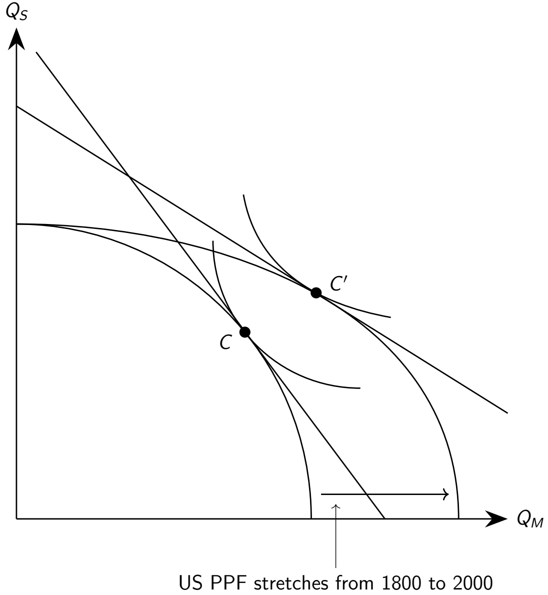


- If the T good is tradable and the NT good is not (e.g. haircuts are not very tradable) then international arbitrage will imply that $(p_T)^R \approx (p_T)^P$
- And hence we expect to see

$$(p_{NT})^R > (p_{NT})^P$$

- This is known as the "Balassa- Samuelson effect".
- This fig. shows data on prices of "everything" (some avg. of p_T and p_{NT}).

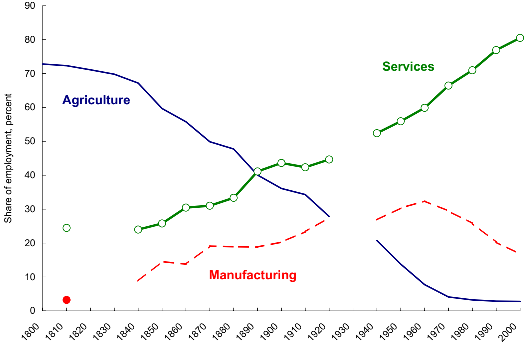
Where Did All the Manufacturing (and Agriculture) Jobs Go?



- Same idea but now one economy at two points in time (e.g. US at 1800 and 2025)
- Suppose that productivity growth in the services sector (finance, health, education, hospitality, ...) has been slower than that in the merchandise sector (agriculture, manufacturing, mining, ...)
  - So PPF stretches out more in the Q_M direction than the Q_S direction (extreme version shown here)
- Then (with US prefs. that are homothetic and not changing over time) equilibrium prices will feature:

$$\left(\frac{p_S}{p_M}\right)^{2025} > \left(\frac{p_S}{p_M}\right)^{1900}$$

Where Did All the Manufacturing (and Agriculture) Jobs Go?



Iscan (*B.E.J. Macro*, 2010)

- If demand elasticities for these broad categories are inelastic (which they seem to be) then this implies:

$$\left(\frac{Q_S p_S}{Q_M p_M}\right)^{2025} > \left(\frac{Q_S p_S}{Q_M p_M}\right)^{1900}$$

- And so if most costs are labor and profits are low (i.e $Q_f p_f \approx wL_f$, for $f = M$ or S):

$$\left(\frac{L_S}{L_M}\right)^{2025} > \left(\frac{L_S}{L_M}\right)^{1900}$$

- This relates to "Baumol's cost disease":
    - Sector with slower productivity growth ends up taking over the economy (and hence aggregate growth has to slow)

Concluding Remarks

- **Key concepts from today's lecture:**
  - Factor allocation to firms in a production Edgeworth box
  - Production efficiency: cannot produce more of a good without making less of others
  - Production possibility frontier (PPF): set of productively efficient output bundles
  - Marginal rate of transformation (MRT): slope of PPF
  - Factor market equilibrium (given goods prices): firms' unconditional demands for factors equal factor supply
  - FWT for production: (with no market failures or taxes) competitive factor markets leads to production efficiency
  - Each point on PPF has a different relative factor price (i.e. w/r)
  - Full GE with production: goods+factor prices at which goods+factor markets clear
  - FWT for production economy: (with no market failures or taxes) competitive goods and factor market equilibrium is Pareto Efficient (i.e. $MRT = MRS_i$ for all i)
  - SWT with production: lump-sum transfers of factors can achieve and PE outcome
  - Balassa-Samuelson effect: relatively low prices of non-tradables in poor countries
  - Baumol's cost disease: slower productivity growth in services means services becomes larger share of economy
- **Next lecture:** International/inter-regional trade: two interacting GE economies