MIT 14.01: Principles of Microeconomics
Sp 2025, Lecture 11: General Equilibrium (part I)

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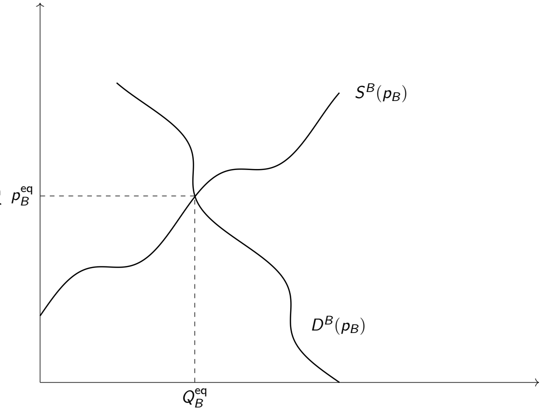
Plan for This Lecture

- Introduction to the study of general equilibrium
- In the context of a pure exchange (no production) economy

General Equilibrium

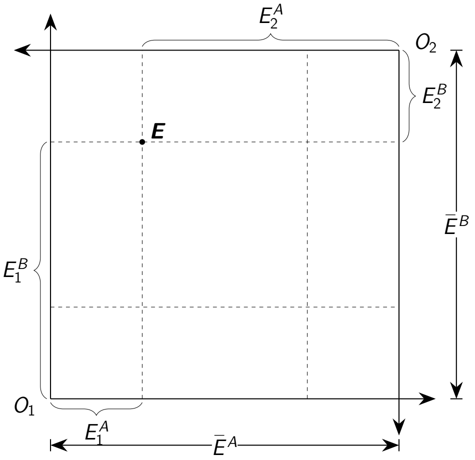
- We studied firms that require inputs ("factors of production", e.g. L and K) to produce and we took the factor prices (e.g. w and r) as exogenous – but what determines factor prices?
- We studied consumers who spend their income y and we took that income as exogenous – but what determines consumer income?
- We studied consumers who demand good B as a function of (p_B, p_A) and good A as a function of (p_B, p_A) – but what jointly determines supply and demand of both products simultaneously?
- Now we begin our study of *general equilibrium*:
  - All goods markets (eg for both A and B) are in equilibrium – rather than one at a time as we did in Lecture #4 – given firm supply (at w, r) and consumer demand (at y)
  - Factor prices (w, r) are determined in *factor market equilibrium* – consumers supply factors and firms demand them
  - Consumers earn their incomes y from supplying factors to firms (i.e. y comes from w and r)

From Partial Equilibrium to General Equilibrium



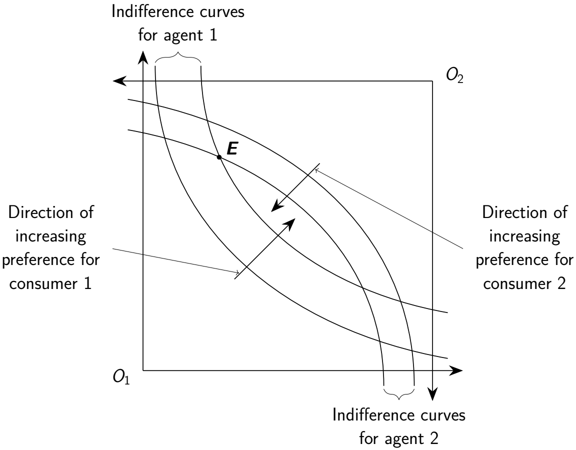
- Remember this figure (from Lecture #4)?
- This described a competitive equilibrium (CE) in one market (for good B) – this was *partial equilibrium*
- And we saw the First Welfare Theorem: conditions under which a CE is Pareto Efficient
- But what happens when there are many markets, all of which depend on each other (e.g $D^B(\cdot)$ depends on p_A, $S^B(\cdot)$ depends on (w, r))?
- We call the supply and demand of many intersecting markets "general equilibrium" (GE)

An Endowment (or Pure Exchange) Economy



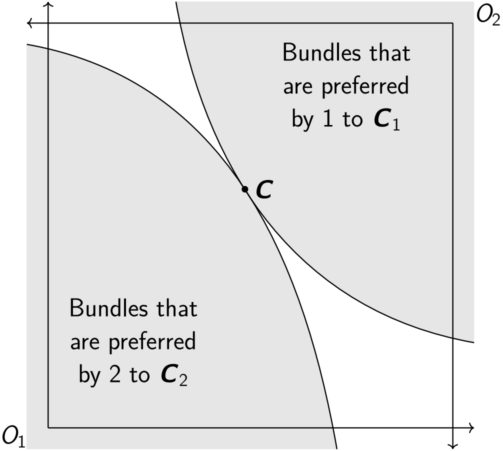
- Let's start with a very simple GE economy in which goods are not created via production
  - Instead, people are just "endowed" with amounts of each good
  - A bit like how we saw a consumer that was endowed with 24 hours of time in Lecture #7
- Two agents ("1" and "2") are each endowed with some amount of each of two goods (A and B):
  - 1: endowed with $E_1^A \geqslant 0$ and $E_1^B \geqslant 0$
  - 2: endowed with $E_1^A \geqslant 0$ and $E_1^B \geqslant 0$
  - Total for economy: endowed with $\bar{E}^A \equiv E_1^A + E_2^A$ and $\bar{E}^B \equiv E_1^B + E_2^B$
- This figure (an "Edgeworth Box") uses a flipped-axis trick to depict the two agents' endowment vectors ($\boldsymbol{E}_1 \equiv (E_1^A, E_1^B)$ and $\boldsymbol{E}_2 \equiv (E_2^A, E_2^B)$) as the point $\boldsymbol{E}$

Preferences Over Allocations Inside the Edgeworth Box



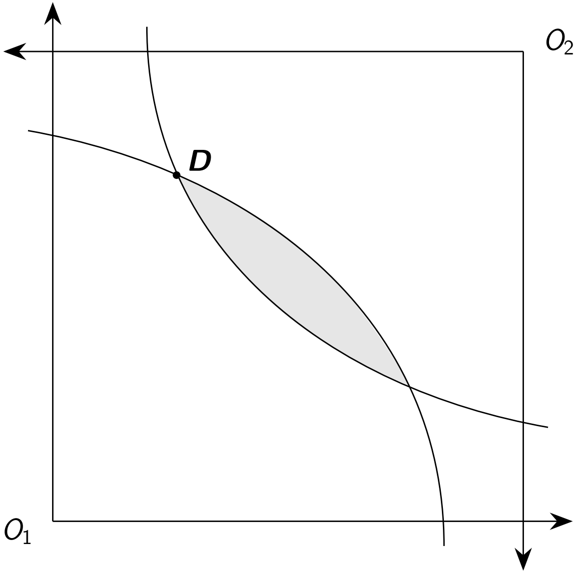
- Suppose that agents 1 and 2 each have some (convex) preferences over each point in the box: the utility they would derive from consuming at that point
- Then 1's indifference curves are convex to O_1 as shown, and 2's indifference curves are convex to (the "flipped axis origin") O_2 as shown

Pareto Efficiency



- Recall the definition of Pareto efficiency (PE) from Lecture #4: an allocation at which it is impossible to make a change that would benefit one agent and do no harm to all other agents
- The point C is an allocation of consumption to each agent that is Pareto efficient
- Note that since the agent's ICs are tangential to each other at C, we also have that $MRS_1 = MRS_2$ at that point (recall $MRS_i \equiv \frac{\partial U_i / \partial C_i^A}{\partial U_i / \partial C_i^B}$ from Lecture #6) – this is necessary (at an interior allocation) for PE

Pareto Inefficiency



- Here, the point ***D*** is not Pareto Efficient
- Two equivalent ways to see this:
  - The two agents' indifference curves are not tangential at ***D*** (i.e $MRS_1 \neq MRS_2$)
  - All of the shaded points would offer Pareto improvements on point ***D***, so ***D*** can't be PE

Pareto (In)efficiency: A Cobb-Douglas Example

- Suppose preferences are: $U_1 = (C_1^A)^\alpha (C_1^B)^{1-\alpha}$ and $U_2 = (C_2^A)^\beta (C_2^B)^{1-\beta}$ and the agents' endowments are
  - $\boldsymbol{E}_1 = (E_1^A, E_1^B) = (4, 2)$
  - $\boldsymbol{E}_2 = (E_2^A, E_2^B) = (2, 4)$

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- Consider the following consumption allocation
  - $\boldsymbol{C}_1 = (C_1^A, C_1^B) = (3, 3)$
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- Is the consumption allocation $\boldsymbol{C} \equiv (\boldsymbol{C}_1, \boldsymbol{C}_2)$ feasible?

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- Is the consumption allocation $\boldsymbol{C} \equiv (\boldsymbol{C}_1, \boldsymbol{C}_2)$ feasible?
  - Yes, since $C_1^A + C_2^A \leqslant E_1^A + E_2^A$ and $C_1^B + C_2^B \leqslant E_1^B + E_2^B$

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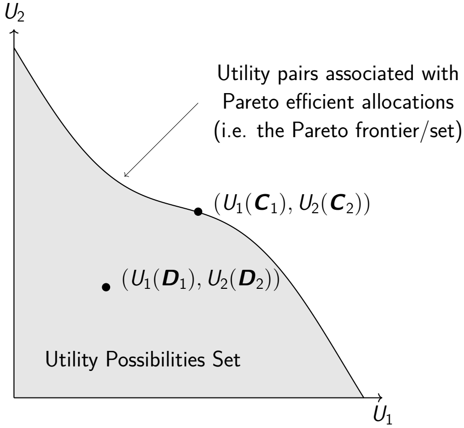
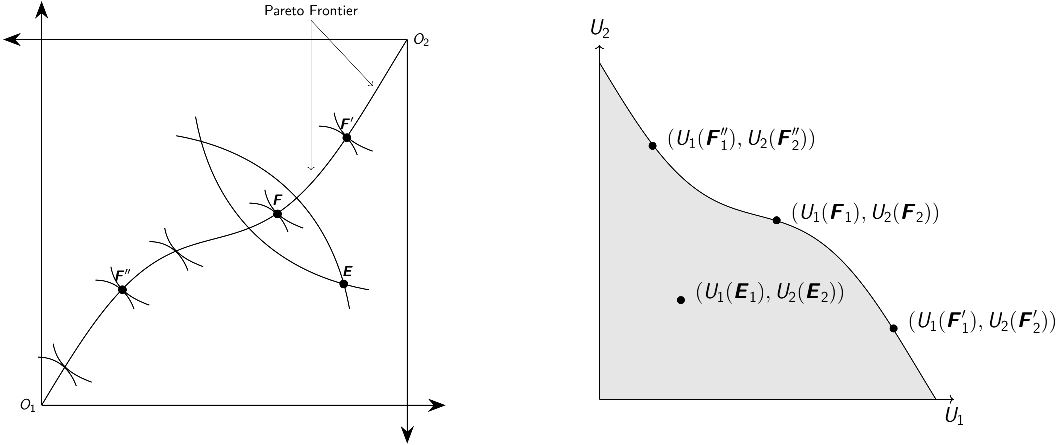
- Is the consumption allocation $\boldsymbol{C}$ Pareto efficient?
  - All (interior) PE allocations have $MRS_1 = MRS_2$
  - Here: $MRS_1 \equiv \frac{\partial U_1/\partial C_1^A}{\partial U_1/\partial C_1^B} = \frac{\alpha}{1-\alpha}\frac{C_1^B}{C_1^A} = \frac{\alpha}{1-\alpha}\frac{3}{3}$ and $MRS_2 \equiv \frac{\partial U_2/\partial C_2^A}{\partial U_2/\partial C_2^B} = \frac{\beta}{1-\beta}\frac{C_2^B}{C_2^A} = \frac{\beta}{1-\beta}\frac{3}{3}$
  - So $\boldsymbol{C}$ is only PE if $\alpha = \beta$

Pareto Frontier



- We can draw the utilities achieved by agents 1 and 2 (U_1 and U_2) if they were to have consumption allocations at every point in the Edgeworth box – this is the shaded Utilities Possibility Set
  - Points outside that set might be fantastic for both 1 and 2, but they are not feasible since they involve consumption allocations that are outside the Edgeworth box
- All the PE allocations (like $\boldsymbol{C}$) are on the upper boundary of that set – they are on the *Pareto frontier/set*, which must slope (strictly) downwards (by def. of PE)
- All other feasible allocations generate utility pairs that are Pareto inefficient (i.e. inside the frontier, like $\boldsymbol{D}$)

Allocations on the Pareto Frontier



Allocations $\boldsymbol{F}$, $\boldsymbol{F}'$ and $\boldsymbol{F}''$ are all on the Pareto frontier (the line at which the indifference curves of agent 1 and 2 are tangential, i.e. where $MRS_1 = MRS_2$). Allocation $\boldsymbol{E}$ is not on this line so it is inside the Pareto frontier. But note that agent 1 nevertheless prefers $\boldsymbol{E}$ to $\boldsymbol{F}''$ (and 2 prefers $\boldsymbol{E}$ to $\boldsymbol{F}'$).

The Pareto Frontier: A Cobb-Douglas Example

- Suppose preferences are: $U_1 = (C_1^A)^\alpha (C_1^B)^{1-\alpha}$ and $U_2 = (C_2^A)^\alpha (C_2^B)^{1-\alpha}$ and the agents' endowments are
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- What does the utility possibilities set look like?

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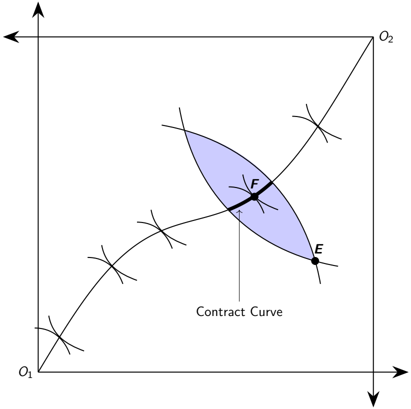
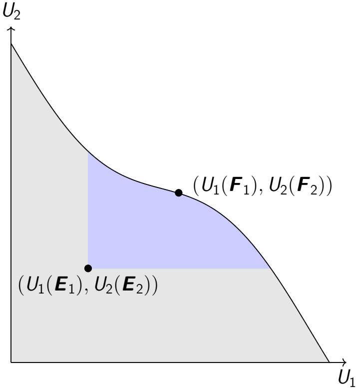
- What does the utility possibilities set look like?
  - The frontier of the set is the set of Pareto efficient points. Above we concluded that (if $\alpha = \beta$, as here) such points satisfy $\frac{C_1^B}{C_1^A} = \frac{C_2^B}{C_2^A}$
  - But the resource constraint means that all allocations must satisfy $C_1^A + C_2^A \leqslant E_1^A + E_2^A \equiv \bar{E}^A$ and $C_1^B + C_2^B \leqslant E_1^B + E_2^B \equiv \bar{E}^B$
  - So frontier points must satisfy $\frac{C_1^B}{C_1^A} = \frac{\bar{E}^B - C_1^B}{\bar{E}^A - C_1^A}$, or $C_1^B = C_1^A \frac{\bar{E}^B}{\bar{E}^A}$
  - So the utilities achieved at these frontier points are:
    - $U_1 = C_1^A \left(\frac{C_1^B}{C_1^A}\right)^{1-\alpha} = C_1^A \left(\frac{\bar{E}^B}{\bar{E}^A}\right)^{1-\alpha}$ and $U_2 = C_2^A \left(\frac{C_2^B}{C_2^A}\right)^{1-\alpha} = (\bar{E}^A - C_1^A) \left(\frac{\bar{E}^B}{\bar{E}^A}\right)^{1-\alpha}$
    - As we vary C_1^A between 0 and $\bar{E}^A$ these two equations describe (U_1, U_2) along frontier
  - Hence the Pareto frontier is the line with slope -1 and intercept (i.e. where $U_2 = 0$) at $U_1 = (\bar{E}^A)^\alpha (\bar{E}^B)^{1-\alpha} = 6^\alpha 6^{1-\alpha}$.
  - The slope of -1, like here, happens whenever agents have identical and homothetic preferences.

Mutually-Beneficial Exchange





Now suppose the agents are endowed with $\boldsymbol{E}$ but they are able to exchange their goods in any way they agree to. The blue shaded "lens" (on the left) is the region of mutually-beneficial post-exchange allocations (the *gains from trade set*). Presumably all exchange that takes place would need to be mutually-beneficial (at least weekly), so it would end up inside (or on) the set. We might even imagine that all opportunities for Pareto improvements would be exploited, so exchange would end up on the Pareto frontier within that set (i.e. on the *contract curve*).

Mutually-Beneficial Exchange: A Cobb-Douglas Example

- Suppose preferences are: $U_1 = (C_1^A)^\alpha (C_1^B)^{1-\alpha}$ and $U_2 = (C_2^A)^\alpha (C_2^B)^{1-\alpha}$, with $\alpha = 1/2$, and the agents' endowments are
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- Consider the following consumption allocation
  - $\boldsymbol{G}_1 = (C_1^A, C_1^B) = (3.5, 2.5)$
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- If $\alpha = \frac{1}{2}$, is the consumption allocation $\boldsymbol{G} \equiv (\boldsymbol{G}_1, \boldsymbol{G}_2)$ one that both agents could arrive at via mutually-beneficial exchange?

Mutually-Beneficial Exchange: A Cobb-Douglas Example

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  - Is $U_1(\boldsymbol{G}_1) > U_1(\boldsymbol{E}_1)$? $U_1(\boldsymbol{G}_1) = \sqrt{\frac{35}{2}} \approx 2.96$, and $U_1(\boldsymbol{E}_1) = 2\sqrt{2} \approx 2.83$. So, yes.
  - Is $U_2(\boldsymbol{G}_2) > U_2(\boldsymbol{E}_2)$? By symmetry, also yes.
  - Then both agents do prefer $\boldsymbol{G}$ to $\boldsymbol{E}$, so it could be arrived at via mutually-beneficial exchange

Mutually-Beneficial Exchange: A Cobb-Douglas Example

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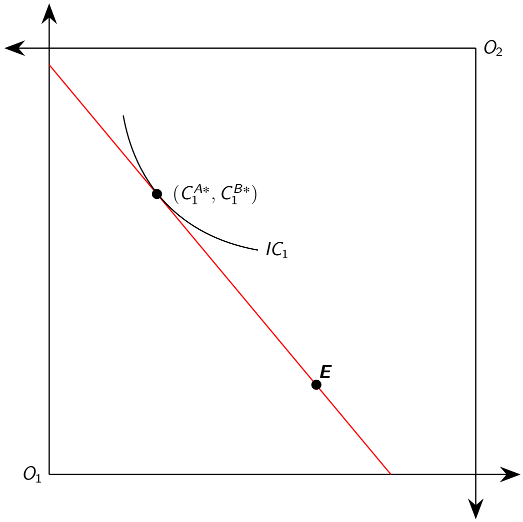
  - Then both agents do prefer $\boldsymbol{G}$ to $\boldsymbol{E}$, so it could be arrived at via mutually-beneficial exchange
- Is $\boldsymbol{G} \equiv (\boldsymbol{G}_1, \boldsymbol{G}_2)$ on the contract curve?
  - No, since we saw earlier that Pareto efficiency requires $\frac{C_1^B}{C_1^A} = \frac{C_2^B}{C_2^A}$

Exchange in a Market



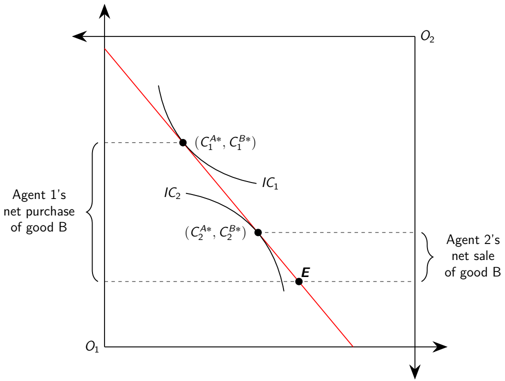
- One form of trading is a *market*: an outsider posts some prices $p = (p_A, p_B)$ and all exchange has to take place at such prices (i.e. all trades must take place "along" the red line, with slope of $-p_B/p_A$)

- What will agent 1 do in such a market? Sell (some of) their endowment bundle to buy best possible consumption bundle:

$$\max_{C_1^A, C_1^B} U(C_1^A, C_1^B)$$
$$\text{s.t.} \quad p_A C_1^A + p_B C_1^B \leqslant p_A E_1^A + p_B E_1^B$$

- Here, optimal choice (C_1^{A*}, C_1^{B*}) involves:
  - Good A: $E_1^A > C_1^{A*}$, so net seller of A
  - Good B: $E_1^B < C_1^{B*}$, so net buyer of B

Exchange in a Market



- Here, both agents optimize facing the same prices. We have:
  - Agent 1 is selling a lot of A and buying a lot of B
  - Agent 2 is selling a little bit of B and buying a little bit of A
- But that can't be an equilibrium! Net supply/demand actions are inconsistent.
- Define *excess demand* for any good at prices $p = (p_A, p_B)$ as:

$$Z_A(p) = (C_1^{A*}(p) - E_1^A) + (C_2^{A*}(p) - E_2^A)$$

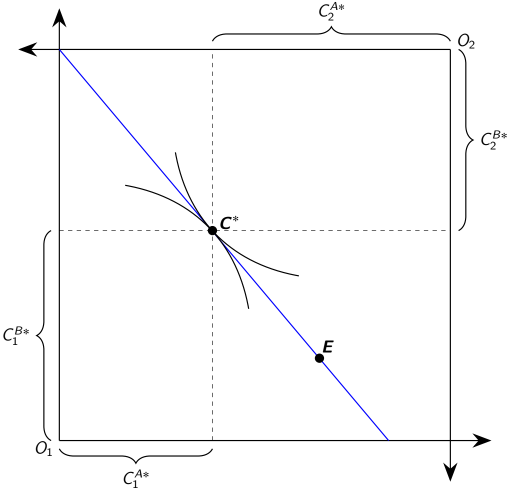
$$Z_B(p) = (C_1^{B*}(p) - E_1^B) + (C_2^{B*}(p) - E_2^B)$$

- This picture has $Z_A(p) < 0$ and $Z_B(p) > 0$

Exchange in a Market: A Cobb-Douglas Example

- Suppose preferences are: $U_1 = (C_1^A)^\alpha (C_1^B)^{1-\alpha}$ and $U_2 = (C_2^A)^\beta (C_2^B)^{1-\beta}$
- If prices are $p = (p_A, p_B)$, what will be the excess demand for each good?
  - Recall that $Z_A(p) = (C_1^{A*}(p) - E_1^A) + (C_2^{A*}(p) - E_2^A) = C_1^{A*}(p) + C_2^{A*}(p) - \bar{E}^A$ etc.
  - And we know from Lecture #6 that (e.g.) agent 1's demand for A will be $C_1^{A*}(p, y_1) = \frac{\alpha y_1}{p_A}$ given income y_1
  - And here income y_1 comes from selling endowments, so $y_1 = p_A E_1^A + p_B E_1^B$
  - So $Z_A(p) = \alpha(E_1^A + \frac{p_B}{p_A} E_1^B) + \beta(E_2^A + \frac{p_B}{p_A} E_2^B) - \bar{E}^A$
  - And $Z_B(p) = (1-\alpha)(\frac{p_A}{p_B} E_1^A + E_1^B) + (1-\beta)(\frac{p_A}{p_B} E_2^A + E_2^B) - \bar{E}^B$

Exchange in a Market



- In this picture, we have found a p^{eq} (slope of blue line) such that "markets clear":
  - $Z_A(p^{\text{eq}}) = 0$ and $Z_B(p^{\text{eq}}) = 0$ (no excess demand or supply for either good)
- This is the definition of a *competitive general equilibrium*:
  - Allocation and price p^{eq} at which all agents optimize while taking prices as given, and all markets clear (i.e. $Z_g(p^{\text{eq}}) = 0$ for all g)
- Two side notes:
  - Only relative prices (the slope, here) are determined, so set $p_g = 1$ for one *numeraire* good
  - *Walras' Law*: if $Z_g(p^{\text{eq}}) = 0$ for all goods g but one then it also $= 0$ for the last good

General Equilibrium: A Cobb-Douglas Example

- Suppose preferences are: $U_1 = (C_1^A)^{\alpha}(C_1^B)^{1-\alpha}$ and $U_2 = (C_2^A)^{\beta}(C_2^B)^{1-\beta}$

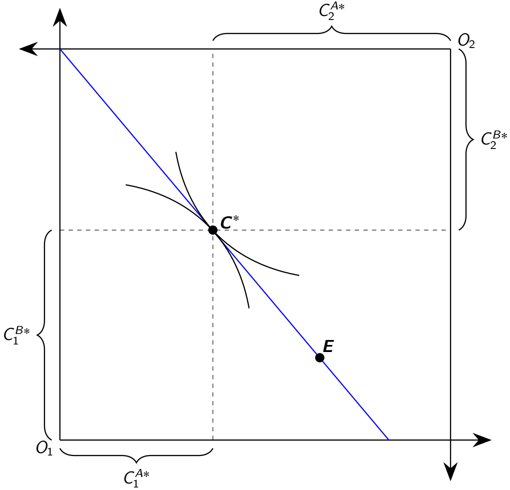
General Equilibrium: A Cobb-Douglas Example

- Suppose preferences are: $U_1 = (C_1^A)^\alpha (C_1^B)^{1-\alpha}$ and $U_2 = (C_2^A)^\beta (C_2^B)^{1-\beta}$

- What is the competitive equilibrium price vector $p^{\text{eq}} = (p_A^{\text{eq}}, p_B^{\text{eq}})$?
  - Recall we can pick one good to be the numeraire, so let's choose $p_B = 1$
  - p^{eq} satisfies $Z_A(p^{\text{eq}}) = 0$ and $Z_B(p^{\text{eq}}) = 0$. But thanks to Walras' Law (i.e. with only 2 goods, if excess demand is zero in one good then it will also be zero in the other good) we only have to solve one of these. Let's do it for good A.
  - We saw above that for any p, $Z_A(p) = \alpha(E_1^A + \frac{p_B}{p_A} E_1^B) + \beta(E_2^A + \frac{p_B}{p_A} E_2^B)$
  - Hence, $p_A^{\text{eq}} = \frac{\alpha E_1^B + \beta E_2^B}{(1-\alpha)E_1^A + (1-\beta)E_2^A}$. This is the unique value of p_A that can achieve $Z_A(p) = 0$ with $p_B = 1$.
  - Bonus: confirm Walras' Law (i.e. check that $Z_B(p_A^{\text{eq}}, 1) = 0$)

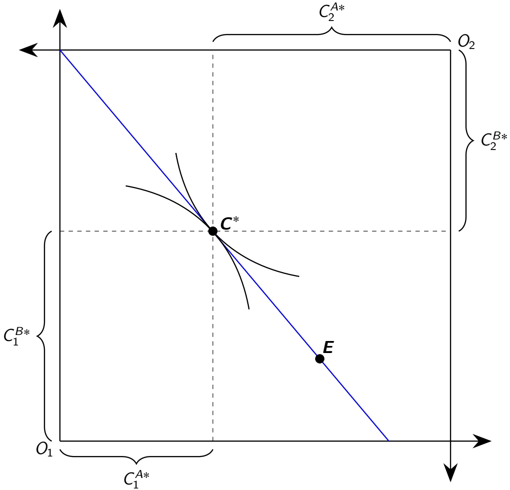
- Intuitively, here, p_A^{eq} is larger when: endowments in good A are low, and/or when good A is highly valued (α or β high); and more so when it is highly valued by those who are rich (e.g. if agent 1's endowments are bigger, then their preferences matter more).

The First Welfare Theorem (Again)



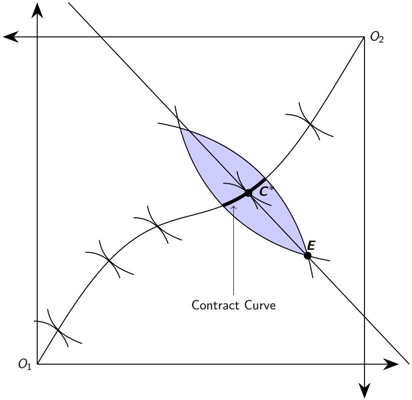
- Recall from Lecture #4 the *First Welfare Theorem*:
  - Under the conditions of no "market failures", a competitive equilibrium (without taxes) is Pareto Efficient
- Now we can see why this is true (even in today's much more general setting):
  - Agent 1 is choosing a consumption allocation $\boldsymbol{C}_1^*$ where $MRS_1 = p_A/p_B$
  - Agent 2 is choosing a consumption allocation $\boldsymbol{C}_2^*$ where $MRS_2 = p_A/p_B$
  - And these choices are mutually consistent (markets clear, $\boldsymbol{C}_1^* + \boldsymbol{C}_2^* = \boldsymbol{E}$), so the two allocations are feasible
  - So feasible allocation has $MRS_1 = MRS_2$, which implies it's a PE allocation

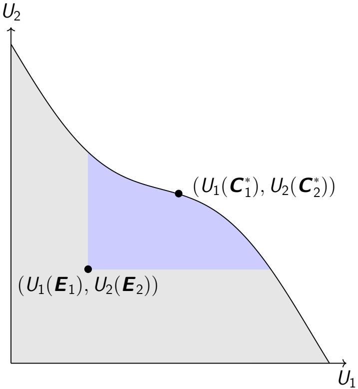
The First Welfare Theorem (and How to Break It)



- The agents (fiercely!) disagree on what makes a good allocation
- However, their disagreements are settled by a rule that says they interact only through the fact that they exchange at prices that they each take as given
- So the "no market failures" condition of the FWT is essentially:
  - Markets: everyone faces the same prices
  - Competitive: agents take prices as given
  - Complete markets: all goods exchanged via such markets, and all interactions work through exchange of goods (i.e. agents affect each other, but only through p)

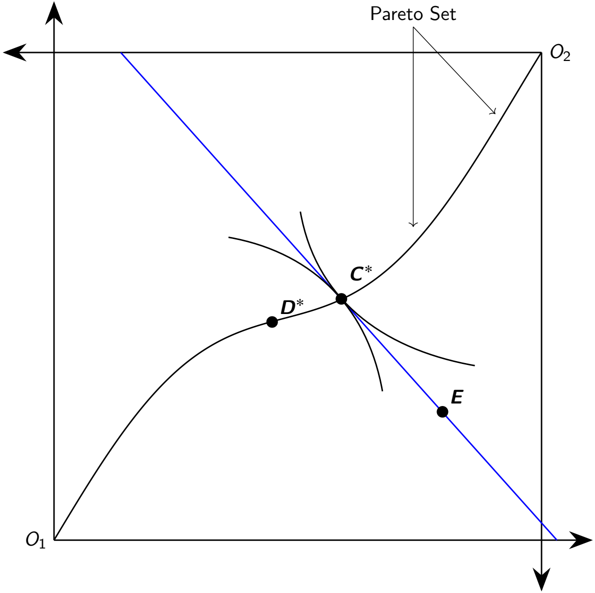
Mutually-Beneficial Exchange and the Competitive Equilibrium





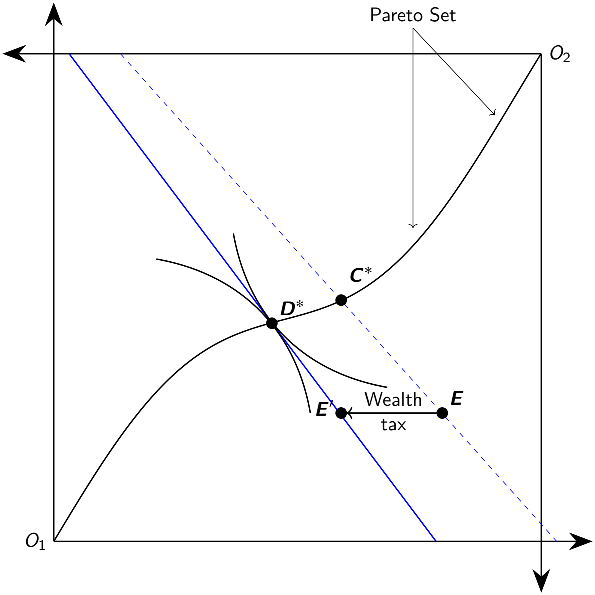
Competitive equilibrium allocation ($\boldsymbol{C}^*$) is Pareto Efficient, so it's on the Pareto frontier. It is the point on the contract curve at which a price line passes through the endowment ($\boldsymbol{E}$) and is tangential to both agents' indifference curves. (*Existence:* Is this guaranteed to be possible? Yes, if preferences are convex. *Uniqueness:* Is there only one place where it can happen? Not necessarily. But typically.)

The Second Welfare Theorem



- Just because the CE allocation ($\boldsymbol{C}^*$) is on the Pareto frontier doesn't mean it's at a "good" point on the frontier
- Suppose we wanted to design a policy that would achieve a different (but still Pareto efficient) allocation like $\boldsymbol{D}^*$ here. What would that policy be?
- *Second welfare theorem*:
  - As long as preferences are convex, any Pareto Efficient allocation can be achieved as a competitive equilibrium with *lump-sum taxes/transfers* (i.e. redistributing the endowment)

The Second Welfare Theorem



- Here we see one example of an endowment tax-and-transfer scheme
- Suppose we wanted to get to the point on the Pareto frontier corresponding to $\boldsymbol{D}^*$, but agents' initial endowments are $\boldsymbol{E}$. What policy could achieve that?
  - Redistribute endowments from $\boldsymbol{E}$ to anywhere on the solid blue line
  - E.g. here: tax agent 1's endowment of good A and transfer it to agent 2 by the amount necessary to move the endowments from $\boldsymbol{E}$ to $\boldsymbol{E}'$
- The CE that obtains with endowments $\boldsymbol{E}'$ would result in a new p^{eq}, and the allocation will be $\boldsymbol{D}^*$ (which is PE)

The Second Welfare Theorem: Comments

- Taxes/transfers like these are a type of "lump-sum" tax/transfer
  - The taxes/subsidies that we saw in Lecture #5 were, by contrast, per-unit or per-value

- The SWT suggests that when lump-sum taxes/transfers are possible, there is no such thing as an *equity-efficiency trade-off* (i.e. a need to prefer some allocation that is Pareto inefficient in order to be more equitable to a particular agent)

- By contrast, as we saw in Lecture #5, non-lump-sum taxes/subsidies will typically lead to Pareto inefficiency. So they do necessitate a trade-off between efficiency and equity goals.

- For this reason, the guidance of the SWT is that redistribution goals should be achieved via policies that are as lump-sum/non-distortionary as possible.
  - E.g. subsidized access to high-quality education rather than income taxation
  - Sometimes goes by moniker "pre-distribution rather than redistribution"

- But of course in practice this can be difficult

Concluding Remarks

- **Key concepts from today's lecture:**
  - Edgeworth box: illustrating endowments, consumption, and exchange
  - Utility possibility set: utilities that are feasible, given total endowments
  - Pareto frontier: the set of Pareto Efficient points in the utility possibility set
  - Mutually-beneficial exchange: "lens" (gains from trade set) of consumption allocations that both agents prefer to the endowment allocation
  - The contract curve: Pareto Efficient points inside the lens
  - Excess demand/ supply for a good: total demand minus total endowments
  - Competitive general equilibrium model for an exchange economy: agents exchange goods taking prices as given, and all markets clear (zero excess demand/supply)
  - Numeraire good: set one good's price to 1 (since only relative prices determined)
  - Walras' Law: in an economy with G goods, if excess demand is zero for $G-1$ goods, then it will also be zero for the last one
  - "No market failures": all interactions between agents occur via competitive markets
  - Second Welfare Theorem: any PE allocation can be achieved as a CE if we have unrestricted lump-sum taxes/transfers (leading to no equity-efficiency trade-off)

- **Next lecture:**
  - GE beyond exchanging endowments: with both production and consumption