

Stanford Economics 266: International Trade
— Lecture 8: Factor Proportions Theory (I) —

Today's Plan

- 1 Factor Proportions Theory
- 2 Ricardo-Viner model
 - 1 Basic environment
 - 2 Comparative statics
- 3 Two-by-Two Heckscher-Ohlin model
 - 1 Basic environment
 - 2 Classical results:
 - 1 Factor Price Equalization Theorem
 - 2 Stolper-Samuelson (1941) Theorem
 - 3 Rybczynski (1965) Theorem

- 1 **Factor Proportions Theory**
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Factor Proportions Theory

- The law of comparative advantage establishes the relationship between relative autarky prices and trade flows
 - But where do relative autarky prices come from?
- Factor proportion theory emphasizes **factor endowment differences**
- **Key elements:**
 - ① Countries differ in terms of factor abundance [i.e. *relative* factor supply]
 - ② Goods differ in terms of factor intensity [i.e. *relative* factor demand]
- Interaction between 1 and 2 will determine differences in relative autarky prices, and in turn, the pattern of trade

Factor Proportions Theory

- In order to shed light on factor endowments as a source of CA, we will assume that:
 - 1 Production functions are identical around the world
 - 2 Households have identical homothetic preferences around the world
- We will first focus on two special models:
 - **Ricardo-Viner** with 2 goods, 1 “mobile” factor (e.g. labor) and 2 “immobile” factors (e.g. sector-specific capital)
 - **Heckscher-Ohlin** with 2 goods and 2 “mobile” factors (e.g. labor and capital)
- The second model is often thought of as a long-run version of the first (Neary 1978)
 - In the case of Heckscher-Ohlin, what is the time horizon such that one can think of total capital as fixed in each country, though freely mobile across sectors?

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Ricardo-Viner Model

Basic environment

- Consider an economy with:
 - Two goods, $g = 1, 2$
 - Three factors with endowments l , k_1 , and k_2
- Output of good g is given by

$$y_g = f^g(l_g, k_g),$$

where:

- l_g is the (endogenous) amount of labor in sector g
- f^g is homogeneous of degree 1 in (l_g, k_g)
- **Comments:**
 - l is a “mobile” factor in the sense that it can be employed in all sectors
 - k_1 and k_2 are “immobile” factors in the sense that they can only be employed in one of them
 - Model is isomorphic to DRS model: $y_g = f^g(l_g)$ with $f_{ll}^g < 0$
 - Payments to specific factors under CRS \equiv profits under DRS

Ricardo-Viner Model

Equilibrium (I): small open economy

- We denote by:
 - p_1 and p_2 the prices of goods 1 and 2
 - w , r_1 , and r_2 the prices of l , k_1 , and k_2
- For now, (p_1, p_2) is exogenously given: **“small open economy”**
 - So no need to look at good market clearing
- **Profit maximization:**

$$p_g f_l^g(l_g, k_g) = w \quad (1)$$

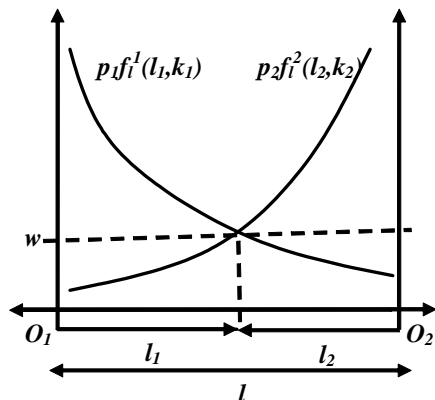
$$p_g f_k^g(l_g, k_g) = r_g \quad (2)$$

- **Labor market clearing:**

$$l = l_1 + l_2 \quad (3)$$

Ricardo-Viner Model

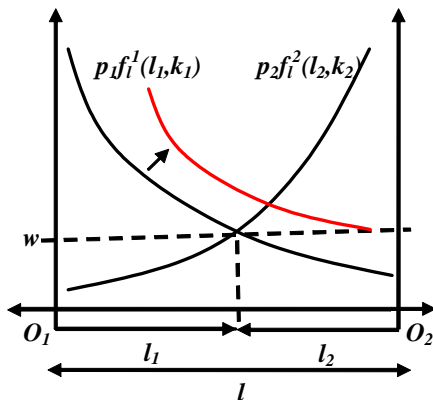
Graphical analysis



- Equations (1) and (3) jointly determine labor allocation and wage
- How do we recover payments to the specific factor from this graph?

Ricardo-Viner Model

Comparative statics



- Consider a TOT shock such that p_1 increases:
 - $w \nearrow$, $l_1 \nearrow$, and $l_2 \searrow$
 - Condition (2) $\Rightarrow r_1/p_1 \nearrow$ whereas r_2 (and a fortiori r_2/p_1) \searrow

Ricardo-Viner Model

Comparative statics

- One can use the same type of arguments to analyze consequences of:
 - Productivity shocks
 - Changes in factor endowments
- In all cases, results are intuitive:
 - “Dutch disease” (Boom in export sectors, Bids up wages, which leads to a contraction in the other sectors)
 - Useful political-economy applications (Grossman and Helpman 1994)
- Easy to extend the analysis to more than 2 sectors:
 - Plot labor demand in one sector vs. rest of the economy

Ricardo-Viner Model

Equilibrium (II): two-country world

- Predictions on the pattern of trade in a two-country world depend on whether differences in factor endowments come from:
 - Differences in the relative supply of specific factors
 - Differences in the relative supply of mobile factors
- Accordingly, any change in factor prices is possible as we move from autarky to free trade (see Feenstra Problem 3.1 p. 98)

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Two-by-Two Heckscher-Ohlin Model

Basic environment

- Consider an economy with:
 - Two goods, $g = 1, 2$,
 - Two factors with endowments l and k
- Output of good g is given by

$$y_g = f^g(l_g, k_g),$$

where:

- l_g, k_g are the (endogenous) amounts of labor and capital in sector g
- f^g is homogeneous of degree 1 in (l_g, k_g)

Two-by-Two Heckscher-Ohlin Model

Back to the dual approach

- $c_g(w, r) \equiv$ unit cost function in sector g

$$c_g(w, r) = \min_{l, k} \{wl + rk \mid f^g(l, k) \geq 1\},$$

where w and r the price of labor and capital

- $a_{fg}(w, r) \equiv$ unit demand for factor f in the production of good g
- Using the Envelope Theorem, it is easy to check that:

$$a_{lg}(w, r) = \frac{dc_g(w, r)}{dw} \quad \text{and} \quad a_{kg}(w, r) = \frac{dc_g(w, r)}{dr}$$

- $A(w, r) \equiv [a_{fg}(w, r)]$ denotes the matrix of total factor requirements

Two-by-Two Heckscher-Ohlin Model

Equilibrium conditions (I): small open economy

- Like in RV model, we first look at the case of a **“small open economy”**

- So no need to look at good market clearing

- **Profit-maximization:**

$$p_g \leq wa_{lg}(w, r) + ra_{kg}(w, r) \text{ for all } g = 1, 2 \quad (4)$$

$$p_g = wa_{lg}(w, r) + ra_{kg}(w, r) \text{ if } g \text{ is produced (in equilib.)} \quad (5)$$

- **Factor market-clearing:**

$$l = y_1 a_{l1}(w, r) + y_2 a_{l2}(w, r) \quad (6)$$

$$k = y_1 a_{k1}(w, r) + y_2 a_{k2}(w, r) \quad (7)$$

Two-by-Two Heckscher-Ohlin Model

Factor Price Equalization

- **Question:**

Can trade in goods be a (perfect) substitute for trade in factors?

- First classical result from the HO literature answers by the affirmative
- To establish this result formally, we'll need the following definition:
- **Definition.** *Factor Intensity Reversal (FIR) does not occur if: (i) $a_{l1}(w, r) / a_{k1}(w, r) > a_{l2}(w, r) / a_{k2}(w, r)$ for all (w, r) ; or (ii) $a_{l1}(w, r) / a_{k1}(w, r) < a_{l2}(w, r) / a_{k2}(w, r)$ for all (w, r) .*

Two-by-Two Heckscher-Ohlin Model

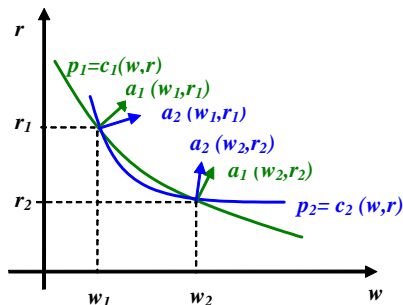
Factor Price Insensitivity (FPI)

- **Lemma** *If both goods are produced in equilibrium and FIR does not occur, then factor prices $\omega \equiv (w, r)$ are uniquely determined by good prices $p \equiv (p_1, p_2)$*
- **Proof:** If both goods are produced in equilibrium, then $p = A'(\omega)\omega$. By Gale and Nikaido (1965), this equation admits a unique solution if $a_{fg}(\omega) > 0$ for all f, g and $\det[A(\omega)] \neq 0$ for all ω , which is guaranteed by no FIR.
- **Comments:**
 - Good prices rather than factor endowments determine factor prices
 - In a closed economy, good prices and factor endowments are, of course, related, but not for a small open economy
 - All economic intuition can be gained by simply looking at Leontieff case
 - Proof already suggests that “dimensionality” will be an issue for FIR

Two-by-Two Heckscher-Ohlin Model

Factor Price Insensitivity (FPI): graphical analysis

- Link between no FIR and FPI can be seen graphically:



- If iso-cost curves cross more than once, then FIR must occur

Heckscher-Ohlin Model

Factor Price Equalization (FPE) Theorem

- The previous lemma directly implies (Samuelson 1949) that:
- **FPE Theorem** *If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices*
- **Comments:**
 - Trade in goods can be a “perfect substitute” for trade in factors
 - Countries with different factor endowments can sustain same factor prices through different allocation of factors across sectors
 - Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries...
 - For next results, we'll maintain assumption that both goods are produced in equilibrium, but won't need free trade and same technology

Heckscher-Ohlin Model

Stolper-Samuelson (1941) Theorem

- **Stolper-Samuelson Theorem** *An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduced the real return to the other factor*
- **Proof:** W.l.o.g. suppose that (i) $a_{l1}(\omega) / a_{k1}(\omega) > a_{l2}(\omega) / a_{k2}(\omega)$ and (ii) $\hat{p}_2 > \hat{p}_1$. Differentiating the zero-profit condition (5), we get

$$\hat{p}_g = \theta_{lg} \hat{w} + (1 - \theta_{lg}) \hat{r}, \quad (8)$$

where $\hat{x} = d \ln x$ and $\theta_{lg} \equiv w a_{lg}(\omega) / c_g(\omega)$. Equation (8) implies

$$\hat{w} \geq \hat{p}_1, \hat{p}_2 \geq \hat{r} \text{ or } \hat{r} \geq \hat{p}_1, \hat{p}_2 \geq \hat{w}$$

By (i), $\theta_{l2} < \theta_{l1}$. So (i) requires $\hat{r} > \hat{w}$. Combining the previous inequalities, we get

$$\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$$

Heckscher-Ohlin Model

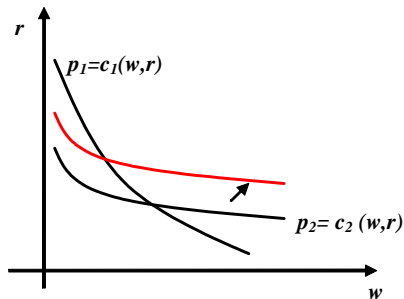
Stolper-Samuelson (1941) Theorem

• Comments:

- Previous “hat” algebra is often referred to “Jones’ (1965) algebra”
- The chain of inequalities $\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$ is referred as a “magnification effect”
- SS predict both winners and losers from change in relative prices
- Like FPI and FPE, SS entirely comes from zero-profit condition (+ no joint production)
- Like FPI and FPE, sharpness of the result hinges on “dimensionality”
- In the empirical literature, people often talk about “Stolper-Samuelson effects” whenever looking at changes in relative factor prices (though changes in relative goods prices are usually hard to observe so it is common to assume constant pass-through of some observable, like tariffs, into goods prices)

Heckscher-Ohlin Model

Stolper-Samuelson (1941) Theorem: graphical analysis



- Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontieff case:
 - In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies

Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem

- Previous results have focused on the implication of *zero profit condition*, Equation (5), for *factor prices*
- We now turn our attention to the implication of *factor market clearing*, Equations (6) and (7), for *factor allocation*
- **Rybczynski Theorem** *An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry*

Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem

- **Proof:** W.l.o.g. suppose that (i)
 $a_{l1}(\omega)/a_{k1}(\omega) > a_{l2}(\omega)/a_{k2}(\omega)$ and (ii) $\hat{k} > \hat{l}$. Differentiating factor market clearing conditions (6) and (7), we get

$$\hat{l} = \lambda_{l1}\hat{y}_1 + (1 - \lambda_{l1})\hat{y}_2 \quad (9)$$

$$\hat{k} = \lambda_{k1}\hat{y}_1 + (1 - \lambda_{k1})\hat{y}_2 \quad (10)$$

where $\lambda_{l1} \equiv a_{l1}(\omega) y_1 / l$ and $\lambda_{k1} \equiv a_{k1}(\omega) y_1 / k$. Equations (8) implies

$$\hat{y}_1 \geq \hat{l}, \hat{k} \geq \hat{y}_2 \text{ or } \hat{y}_2 \geq \hat{l}, \hat{k} \geq \hat{y}_1$$

By (i), $\lambda_{k1} < \lambda_{l1}$. So (ii) requires $\hat{y}_2 > \hat{y}_1$. Combining the previous inequalities, we get

$$\hat{y}_2 > \hat{k} > \hat{l} > \hat{y}_1$$

Two-by-Two Heckscher-Ohlin Model

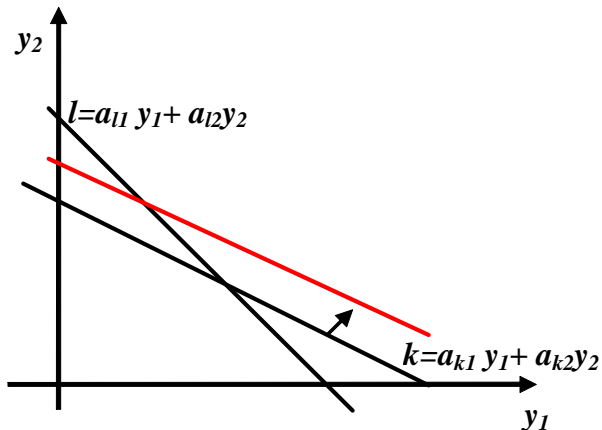
Rybczynski (1965) Theorem

- Like for FPI and FPE Theorems:
 - (p_1, p_2) is exogenously given \Rightarrow factor prices and factor requirements are not affected by changes factor endowments
 - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy
- Like for SS Theorem, we have a “magnification effect”
- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on “dimensionality”

Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem: graphical analysis (I)

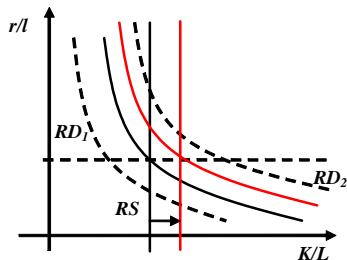
- Since goods prices (and hence factor prices) are fixed, it is as if we were in Leontieff case



Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem: graphical analysis (II)

- Rybczynski effect can also be illustrated using relative factor supply and relative factor demand:



- *Cross-sectoral reallocations* are at the core of HO predictions:
 - For relative factor prices to remain constant, *aggregate* relative demand (weighted sum of RD1 and RD2, with weights depending on output in each sector) must go up. This requires an expansion of the capital intensive sector.