Economics 266: International Trade
— Lecture 4: Ricardian Theory (II)—

"Putting Ricardo to Work"

(Title of nice survey by Eaton and Kortum (JEP, 2012))

- Ricardian model has long been perceived as a useful pedagogic tool, with little empirical content...
  - Great to explain undergrads why there are gains from trade
  - But grad students should study richer models (e.g. Feenstra's graduate textbook—edition 1, from 2003—had a total of 3 pages on the Ricardian model!)
- Eaton and Kortum (Ecta, 2002) has lead to "Ricardian revival"
  - Same basic idea as in Wilson (1980): Is it necessary to know about the pattern of trade to do certain types of counterfactual analysis?
  - But more structure: Small number of parameters, so well-suited for quantitative work.
- **Goals of this lecture:**
  1. Present EK model
  2. Discuss estimation of its key parameters
  3. Introduce tools for welfare and counterfactual analysis

Basic Assumptions

- N countries, $i = 1, ..., N$
- Continuum of goods $u \in [0, 1]$
- Preferences are CES with elasticity of substitution σ:

$$U_i = \left(\int_0^1 q_i(u)^{(\sigma-1)/\sigma} du \right)^{\sigma/(\sigma-1)},$$

- One factor of production (labor)
- There may also be intermediate goods (more on that later)
- $c_i \equiv$ unit cost of the "common input" used in production of all goods
    - Without intermediate goods, c_i is equal to "wage" w_i in country i

Basic Assumptions (Cont.)

- Constant returns to scale:
  - $Z_i(u)$ denotes productivity of (any) firm producing u in country i
  - $Z_i(u)$ is drawn independently (across goods and countries) from a **Fréchet distribution**:

$$\Pr(Z_i \leq z) = F_i(z) = e^{-T_i z^{-\theta}},$$

    with $\theta > \sigma - 1$ (important restriction, see below)
  - Since goods are symmetric except for productivity, we can forget about index u and keep track of goods through $\boldsymbol{Z} \equiv (Z_1, ..., Z_N)$.

- Trade is subject to "iceberg" (Samuelson, 1954) costs $d_{ni} \geq 1$
  - d_{ni} units need to be shipped from i so that 1 unit makes it to n; transport costs, but no real resources used in transport.

- All markets are perfectly competitive

Four Key Results

A - The Price Distribution

- Let $P_{ni}(Z_i) \equiv c_i d_{ni} / Z_i$ be the unit cost at which country i can serve a good $\boldsymbol{Z}$ to country n and let $G_{ni}(p) \equiv \Pr(P_{ni}(Z_i) \leq p)$. Then:

$$G_{ni}(p) = \Pr\left(Z_i \geq c_i d_{ni} / p\right) = 1 - F_i(c_i d_{ni} / p)$$

- Let $P_n(\boldsymbol{Z}) \equiv \min\{P_{n1}(\boldsymbol{Z}), ..., P_{nN}(\boldsymbol{Z})\}$ and let $G_n(p) \equiv \Pr(P_n(\boldsymbol{Z}) \leq p)$ be the price distribution in country n. Then:

$$G_n(p) = 1 - \exp[-\Phi_n p^{\theta}]$$

  where

$$\Phi_n \equiv \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta}$$

- To show this, note that (suppressing notation $\boldsymbol{Z}$ from here onwards)

$$\begin{aligned}\Pr(P_n \leq p) &= 1 - \Pi_i \Pr(P_{ni} \geq p) \\ &= 1 - \Pi_i \left[1 - G_{ni}(p)\right]\end{aligned}$$

- But using

$$G_{ni}(p) = 1 - F_i(c_i d_{ni}/p)$$

then

$$\begin{aligned}1 - \Pi_i \left[1 - G_{ni}(p)\right] &= 1 - \Pi_i F_i(c_i d_{ni}/p) \\ &= 1 - \Pi_i e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}} \\ &= 1 - e^{-\Phi_n p^{\theta}}\end{aligned}$$

- Consider a particular good. Country n buys the good from country i if $i = \arg\min\{p_{n1}, ..., p_{nN}\}$. The probability of this event is simply country i's contribution to country n's price parameter Φ_n,

$$\pi_{ni} = \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}$$

- To show this, note that

$$\pi_{ni} = \Pr\left(P_{ni} \leq \min_{s \neq i} P_{ns}\right)$$

- If $P_{ni} = p$, then the probability that country i is the least cost supplier to country n is equal to the probability that $P_{ns} \geq p$ for all $s \neq i$

Four Key Results

B - The Allocation of Purchases (Cont.)

- The previous probability is equal to

$$\Pi_{s\neq i}\Pr(P_{ns}\geq p)=\Pi_{s\neq i}\left[1-G_{ns}(p)\right]=e^{-\Phi_n^{-i}p^{\theta}}$$

where

$$\Phi_n^{-i}=\sum_{s\neq i}T_i\left(c_i d_{ni}\right)^{-\theta}$$

- Now we integrate over this for all possible p's times the density $dG_{ni}(p)$ to obtain

$$\begin{aligned}
&\int_0^{\infty}e^{-\Phi_n^{-i}p^{\theta}}T_i\left(c_i d_{ni}\right)^{-\theta}\theta p^{\theta-1}e^{-T_i\left(c_i d_{ni}\right)^{-\theta}p^{\theta}}dp\\
=&\left(\frac{T_i\left(c_i d_{ni}\right)^{-\theta}}{\Phi_n}\right)\int_0^{\infty}\theta\Phi_n e^{-\Phi_n p^{\theta}}p^{\theta-1}dp\\
=&\ \pi_{ni}\int_0^{\infty}dG_n(p)dp=\pi_{ni}
\end{aligned}$$

- The price of a good that country n actually buys from any country i also has the distribution $G_n(p)$.
- To show this, note that if country n buys a good from country i it means that i is the least cost supplier. If the price at which country i sells this good in country n is q, then the probability that i is the least cost supplier is

$$\Pi_{s \neq i} \Pr(P_{ni} \geq q) = \Pi_{s \neq i} \left[1 - G_{ns}(q)\right] = e^{-\Phi_n^{-i} q^{\theta}}$$

- Then the joint probability that country i has a unit cost q of delivering the good to country n **and** is the the least cost supplier of that good in country n is then

$$e^{-\Phi_n^{-i} q^{\theta}} dG_{ni}(q)$$

- Integrating this probability $e^{-\Phi_n^{-i}q^\theta}dG_{ni}(q)$ over all prices $q \leq p$ and using $G_{ni}(q) = 1 - e^{-T_i(c_i d_{ni})^{-\theta}p^\theta}$ then

$$
\begin{aligned}
& \int_0^p e^{-\Phi_n^{-i}q^\theta}dG_{ni}(q) \\
= & \int_0^p e^{-\Phi_n^{-i}q^\theta}\theta T_i(c_i d_{ni})^{-\theta}q^{\theta-1}e^{-T_i(c_i d_{ni})^{-\theta}p^\theta}dq \\
= & \left(\frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}\right)\int_0^p e^{-\Phi_n q^\theta}\theta\Phi_n q^{\theta-1}dq \\
= & \pi_{ni}G_n(p)
\end{aligned}
$$

- Given that $\pi_{ni} \equiv$ probability that for any particular good country i is the least cost supplier in n, the conditional distribution of the price charged by i in n for the goods that i actually sells in n is

$$
\frac{1}{\pi_{ni}}\int_0^p e^{-\Phi_n^{-i}q^\theta}dG_{ni}(q) = G_n(p)
$$

- Hence, we see that in Eaton and Kortum (2002):
  1. All the adjustment is at the extensive margin: countries that are more distant, have higher costs, or lower $T's$, simply sell a smaller range of goods, but the average price charged is the same.
  2. The share of spending by country n on goods from country i is the same as the probability π_{ni} calculated above.

- We will establish (in Lectures 10-11) a similar property in models of monopolistic competition with Pareto distributions of firm-level productivity

Four Key Results

D - The Price Index

- The exact price index for a CES utility with elasticity of substitution $\sigma < 1 + \theta$, defined as

$$p_n \equiv \left(\int_0^1 p_n(u)^{1-\sigma} du \right)^{1/(1-\sigma)},$$

is given by

$$p_n = \gamma \Phi_n^{-1/\theta}$$

where

$$\gamma = \left[\Gamma \left(\frac{1-\sigma}{\theta} + 1 \right) \right]^{1/(1-\sigma)},$$

where Γ is the Gamma function, *i.e.* $\Gamma(a) \equiv \int_0^\infty x^{a-1} e^{-x} dx$.

- To show this, note that

$$\begin{aligned} p_n^{1-\sigma} &= \int_0^1 p_n(u)^{1-\sigma} du \\ = \int_0^\infty p^{1-\sigma} dG_n(p) &= \int_0^\infty p^{1-\sigma} \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta} dp. \end{aligned}$$

- Defining $x = \Phi_n p^\theta$, then $dx = \Phi_n \theta p^{\theta-1}$, $p^{1-\sigma} = (x/\Phi_n)^{(1-\sigma)/\theta}$, and

$$\begin{aligned} p_n^{1-\sigma} &= \int_0^\infty (x/\Phi_n)^{(1-\sigma)/\theta} e^{-x} dx \\ &= \Phi_n^{-(1-\sigma)/\theta} \int_0^\infty x^{(1-\sigma)/\theta} e^{-x} dx \\ &= \Phi_n^{-(1-\sigma)/\theta} \Gamma\left(\frac{1-\sigma}{\theta} + 1\right) \end{aligned}$$

- This implies $p_n = \gamma \Phi_n^{-1/\theta}$ with $\frac{1-\sigma}{\theta} + 1 > 0$ or $\sigma - 1 < \theta$ for gamma function to be well defined

Equilibrium

- Let X_{ni} be total spending in country n on goods from country i
- Let $X_n \equiv \sum_i X_{ni}$ be country n's total spending
- We know that $X_{ni}/X_n = \pi_{ni}$, so

$$X_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n} X_n \qquad (*)$$

- Suppose that there are no intermediate goods so that $c_i = w_i$.
- In equilibrium, total income in country i must be equal to total spending on goods from country i so

$$w_i L_i = \sum_n X_{ni}$$

- Trade balance further requires $X_n = w_n L_n$ so that

$$w_i L_i = \sum_n \frac{T_i(w_i d_{ni})^{-\theta}}{\sum_j T_j(w_j d_{nj})^{-\theta}} w_n L_n$$

Equilibrium (Cont.)

- This provides system of $N-1$ independent equations (Walras' Law) that can be solved for wages $(w_1, ..., w_N)$ up to a choice of numeraire
  - We now know that this is unique: Scarf and Wilson (2003), Alvarez and Lucas (2007), Allen and Arkolakis (2015)
- Everything is as if countries were exchanging labor (as we expected from Wilson, 1980).
  - Fréchet distributions just imply that labor demands are iso-elastic
  - Begs the question: might there be other Ricardian microfoundations under which the global labor demand system is isoelastic? And what happens if it's not isoelastic? See Lectures 14-15.

Equilibrium (Cont.)

- Under frictionless trade ($d_{ni} = 1$ for all n, i) previous system implies

$$w_i^{1+\theta} = \frac{T_i}{L_i} \frac{\sum_n w_n L_n}{\sum_j T_j w_j^{-\theta}}$$

and hence (for any 2 countries, i and j), writing this as "relative labor demand = relative labor supply"

$$\left(\frac{w_i}{w_j}\right)^{1+\theta} \frac{T_j}{T_i} = \frac{L_i}{L_j}$$

- Compare with similar equation in DFS (1977), in Lecture 3, but with symmetric Cobb-Douglas prefs (i.e. $\theta(\tilde{z}) = \tilde{z}$):

$$\begin{aligned} \frac{w}{w^*} \frac{1-\tilde{z}}{\tilde{z}} &= \frac{L^*}{L} \\ \Rightarrow \left(\frac{w}{w^*} \frac{1 - A^{-1}\left(\frac{w}{w^*}\right)}{A^{-1}\left(\frac{w}{w^*}\right)} \right)^{-1} &= \frac{L}{L^*} \end{aligned}$$

The Gravity Equation

- Letting $Y_i = \sum_n X_{ni}$ be country $i's$ total sales, then

$$Y_i = \sum_n \frac{T_i \left(c_i d_{ni}\right)^{-\theta} X_n}{\Phi_n} = T_i c_i^{-\theta} \Omega_i^{-\theta}$$

  where

$$\Omega_i^{-\theta} \equiv \sum_n \frac{d_{ni}^{-\theta} X_n}{\Phi_n}$$

- Solving $T_i c_i^{-\theta}$ from $Y_i = T_i c_i^{-\theta} \Omega_i^{-\theta}$ and plugging into (*) we get

$$X_{ni} = \frac{X_n Y_i {d_{ni}}^{-\theta} \Omega_i^{\theta}}{\Phi_n}$$

- Using $p_n = \gamma \Phi_n^{-1/\theta}$ we can then get

$$X_{ni} = \gamma^{-\theta} X_n Y_i {d_{ni}}^{-\theta} \left(p_n \Omega_i\right)^{\theta}$$

- This is the **Gravity Equation**, with bilateral resistance d_{ni} and multilateral resistance terms p_n (inward) and Ω_i (outward).

- From (*) we also get that country i's share in country n's expenditures normalized by its own share is

$$S_{ni} \equiv \frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\Phi_i}{\Phi_n} d_{ni}^{-\theta} = \left(\frac{p_i d_{ni}}{p_n} \right)^{-\theta}$$

- This shows the importance of trade costs and comparative advantage in determining trade volumes. Note that if there are no trade barriers (i.e, frictionless trade), then $S_{ni} = 1$.
- Letting $B_{ni} \equiv \left(\frac{X_{ni}}{X_{ii}} \cdot \frac{X_{in}}{X_{nn}} \right)^{1/2}$ then

$$B_{ni} = (S_{ni} S_{in})^{1/2} = \left(d_{ni}^{-\theta} d_{in}^{-\theta} \right)^{1/2}$$

- Under symmetric trade costs (i.e., $d_{ni} = d_{in}$) then $B_{ni}^{-1/\theta} = d_{ni}$ can be used as a measure of trade costs. (Later we will call this the Head and Ries (AER, 2001) index of trade costs.)

We can also see how B_{ni} varies with physical distance—perhaps a plausible proxy for d_{ni}—between n and i:

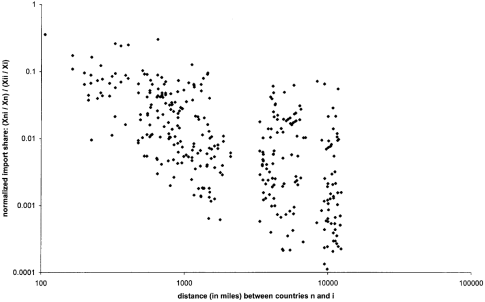

FIGURE 1.—Trade and geography.

How to Estimate θ, "The Trade Elasticity"?

- As we will see, θ, often referred to as "the trade elasticity," is the key structural parameter for welfare and counterfactual analysis in EK model

- In order to estimate θ directly from $B_{ni} = d_{ni}^{-\theta}$ we need a measure of d_{ni}, not just a proxy (like distance).
  - Negative relationship in Figure 1 could come from strong effect of proxy variable (distance) on d_{ni} or from mild CA (high θ), so θ not identified.

How to Estimate the Trade Elasticity?

- EK use price data to measure $p_i d_{ni} / p_n$, and then use fact that $S_{ni} = \left(\frac{p_i d_{ni}}{p_n} \right)^{-\theta}$.
- They use retail prices in 19 OECD countries for 50 manufactured products from the UNICP 1990 benchmark study.
- They interpret these data as a sample of the prices $p_i(j)$ of individual goods j in the model.
- They note that for goods that n imports from i we should have $p_n(j) / p_i(j) = d_{ni}$, whereas goods that n doesn't import from i can have $p_n(j) / p_i(j) \leq d_{ni}$.
- Since every country in the sample does import manufactured goods from every other, then $\max_j \{ p_n(j) / p_i(j) \}$ should be equal to d_{ni}.
- To deal with measurement error, they actually use the second highest $p_n(j) / p_i(j)$ as a measure of d_{ni}.

How to Estimate the Trade Elasticity?

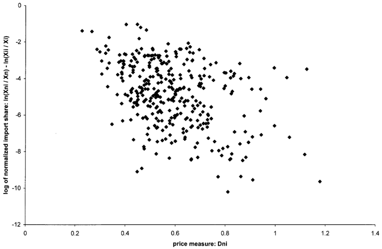

FIGURE 2.—Trade and prices.

- Let $r_{ni}(j) \equiv \ln p_n(j) - \ln p_i(j)$. EK calculate $\ln(p_n/p_i)$ as the mean across j of $r_{ni}(j)$. Then they measure $\ln(p_i d_{ni}/p_n)$ by

$$D_{ni} = \frac{\max 2_j\{r_{ni}(j)\}}{\sum_j r_{ni}(j)/50}$$

- Given $S_{ni} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$ they estimate θ from $\ln(S_{ni}) = -\theta D_{ni}$. Method of moments: $\theta = 8.28$. OLS with zero intercept: $\theta = 8.03$.

Alternative Strategies

- Simonovska and Waugh (2011) argue that EK's procedure suffers from upward bias:
  - Since EK are only considering 50 goods (real world has more), maximum price gap may still be strictly lower than trade cost.
  - If we underestimate trade costs, we overestimate trade elasticity
  - Simulation based method of moments leads to a θ closer to 4.
- An alternative approach is to use tariffs (Caliendo and Parro, 2011). If $d_{ni} = t_{ni}\tau_{ni}$ where t_{ni} is one plus the ad-valorem tariff (they actually do this for each 2 digit industry) and τ_{ni} is assumed to be symmetric, then:

$$\frac{X_{ni}X_{ij}X_{jn}}{X_{nj}X_{ji}X_{in}} = \left(\frac{d_{ni}d_{ij}d_{jn}}{d_{nj}d_{ji}d_{in}}\right)^{-\theta} = \left(\frac{t_{ni}t_{ij}t_{jn}}{t_{nj}t_{ji}t_{in}}\right)^{-\theta}.$$

- They can then run an OLS regression and recover θ. (In practice, this is done over time and Their preferred specification leads to an estimate of 8.22.

Gains from Trade

- Consider again the case where $c_i = w_i$
- From (*), we know that

$$\pi_{nn} = \frac{X_{nn}}{X_n} = \frac{T_n w_n^{-\theta}}{\Phi_n}$$

- We also know that $p_n = \gamma \Phi_n^{-1/\theta}$, so

$$\omega_n \equiv w_n / p_n = \gamma^{-1} T_n^{1/\theta} \pi_{nn}^{-1/\theta}.$$

- Under autarky we have $\omega_n^A = \gamma^{-1} T_n^{1/\theta}$, hence the **gains from trade** are given by

$$GT_n \equiv \omega_n / \omega_n^A = \pi_{nn}^{-1/\theta}$$

- Trade elasticity θ and share of expenditure on domestic goods π_{nn} are sufficient statistics to compute GT

Gains from Trade (Cont.)

- A typical value for π_{nn} (manufacturing) is 0.7. With $\theta = 5$ this implies $GT_n = 0.7^{-1/5} = 1.074$ or 7.4% gains. Belgium has $\pi_{nn} = 0.2$, so its gains are $GT_n = 0.2^{-1/5} = 1.38$ or 38%.
- One can also use the previous approach to measure the welfare gains associated with any foreign shock, not just moving to autarky:

$$\omega_n' / \omega_n = \left(\pi_{nn}' / \pi_{nn} \right)^{-1/\theta}$$

- For more general counterfactual scenarios, however, one needs to know both π_{nn}' and π_{nn}. (In autarky we knew that $\pi_{nn} = 1$.)

Adding an Input-Output Loop

- Now imagine that intermediate goods are used to produce a composite good with a CES production function with elasticity $\sigma > 1$.
    - This composite good can be either consumed or used to produce intermediate goods (input-output loop).
- Each intermediate good is produced from labor and the composite good with a Cobb-Douglas technology with labor share β. We can then write $c_i = w_i^{\beta} p_i^{1-\beta}$.

Adding an Input-Output Loop (Cont.)

- The analysis above implies

$$\pi_{nn} = \gamma^{-\theta} T_n \left(\frac{c_n}{p_n} \right)^{-\theta}$$

  and hence

$$c_n = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n$$

- Using $c_n = w_n^{\beta} p_n^{1-\beta}$ this implies

$$w_n^{\beta} p_n^{1-\beta} = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n$$

  so

$$w_n / p_n = \gamma^{-1/\beta} T_n^{-1/\theta\beta} \pi_{nn}^{-1/\theta\beta}$$

- The gains from trade are now

$$\omega_n / \omega_n^A = \pi_{nn}^{-1/\theta\beta}$$

- Standard value for β is $1/2$ (Alvarez and Lucas, 2007). For $\pi_{nn} = 0.7$ and $\theta = 5$ this implies $GT_n = 0.7^{-2/5} = 1.15$ or 15% gains.

Adding Non-Tradables

- Assume now that the composite good cannot be consumed directly.
- Instead, it can either be used to produce intermediates (as above) or to produce a consumption good (together with labor).
- The production function for the consumption good is Cobb-Douglas with labor share α.
- This consumption good is assumed to be non-tradable.

Adding Non-Tradables (Cont.)

- The price index computed above is now p_{gn}, but we care about $\omega_n \equiv w_n / p_{fn}$, where

$$p_{fn} = w_n^{\alpha} p_{gn}^{1-\alpha}$$

- This implies that

$$\omega_n = \frac{w_n}{w_n^{\alpha} p_{gn}^{1-\alpha}} = (w_n / p_{gn})^{1-\alpha}$$

- Thus, the gains from trade are now

$$\omega_n / \omega_n^A = \pi_{nn}^{-\eta/\theta}$$

  where

$$\eta \equiv \frac{1-\alpha}{\beta}$$

- Alvarez and Lucas argue that $\alpha = 0.75$ (share of labor in services). Thus, for $\pi_{nn} = 0.7$, $\theta = 5$ and $\beta = 0.5$, this implies $GT_n = 0.7^{-1/10} = 1.036$ or 3.6% gains

Comparative statics (Dekle, Eaton and Kortum, 2008)

See also Rutherford (1994), "Lecture Notes on Constant Elasticity Functions"

- Go back to the simple EK model above ($\alpha = 0$, $\beta = 1$). We have

$$\begin{aligned} X_{ni} &= \frac{T_i (w_i d_{ni})^{-\theta} X_n}{\sum_{i=1}^{N} T_i (w_i d_{ni})^{-\theta}} \\ \sum_n X_{ni} &= w_i L_i \end{aligned}$$

- As we have already established, this leads to a system of non-linear equations to solve for wages,

$$w_i L_i = \sum_n \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_k T_k \left(w_k d_{nk}\right)^{-\theta}} w_n L_n.$$

Comparative statics (Dekle, Eaton and Kortum, 2008)

- Consider any shock to labor endowments ($L_i \to L_i'$), trade costs ($d_{ni} \to d_{ni}'$), or productivity ($T_i \to T_i'$).
- Two methods for solving for the change in endogenous variables:
  1. As in EK (2002): estimate or calibrate θ and L_i, L_i', d_{ni}, d_{ni}', T_i and T_i'. Then solve for the endogenous variables at old and new equilibrium values.
  2. As in DEK (2008): If the initial equilibrium corresponds to a setting where all endogenous variables (matrix of X_{ni} values) are observed (and no measurement error), and have estimate of θ, then can instead solve for changes in endogenous variables. Advantages:
     - Often simpler to solve for one set of changes than two sets of levels.
     - Often simpler to explain to audience.
     - Model fits data in observed pre-shock year exactly.
- NB: DEK procedure creates impression that hard and often controversial task of estimating many (L, d, T) parameters has been avoided, but that's not true.

Comparative statics (Dekle, Eaton and Kortum, 2008)

- To see how it works, note that trade shares are (from *)

$$\pi_{ni} = \frac{T_i \left(w_i d_{ni}\right)^{-\theta}}{\sum_k T_k \left(w_k d_{nk}\right)^{-\theta}} \text{ and } \pi'_{ni} = \frac{T'_i \left(w'_i d'_{ni}\right)^{-\theta}}{\sum_k T'_k \left(w'_k d'_{nk}\right)^{-\theta}}.$$

- Letting $\hat{x} \equiv x'/x$, then we have

$$\begin{aligned}
\hat{\pi}_{ni} &= \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k T'_k \left(w'_k d'_{nk}\right)^{-\theta} / \sum_j T_j \left(w_j d_{nj}\right)^{-\theta}} \\
&= \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta} T_k \left(w_k d_{nk}\right)^{-\theta} / \sum_j T_j \left(w_j d_{nj}\right)^{-\theta}} \\
&= \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta}}.
\end{aligned}$$

Comparative statics (Dekle, Eaton and Kortum, 2008)

- On the other hand, for equilibrium we have

$$w_i' L_i' = \sum_n \pi_{ni}' w_n' L_n' = \sum_n \hat{\pi}_{ni} \pi_{ni} w_n' L_n'$$

- Letting $Y_n \equiv w_n L_n$ and using the result above for $\hat{\pi}_{ni}$ we get

$$\hat{w}_i \hat{L}_i Y_i = \sum_n \frac{\pi_{ni} \hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta}} \hat{w}_n \hat{L}_n Y_n$$

- This forms a system of N equations in N unknowns, $\hat{w}_i$, from which we can get $\hat{w}_i$ as a function of shocks and initial observables (establishing some numeraire). Here π_{ni} and Y_i are data (obtainable from X_{ni} matrix) and we know $\hat{d}_{ni}$, $\hat{T}_i$, $\hat{L}_i$, as well as θ.

Comparative statics (Dekle, Eaton and Kortum, 2008)

- To compute the implications for welfare of a foreign shock, simply impose that $\hat{L}_n = \hat{T}_n = 1$, solve the system above to get $\hat{w}_i$ and get the implied $\hat{\pi}_{nn}$ through

$$\hat{\pi}_{ni} = \frac{\hat{T}_i \left(\hat{w}_i \hat{d}_{ni}\right)^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k \left(\hat{w}_k \hat{d}_{nk}\right)^{-\theta}}.$$

  and use the formula to get

$$\hat{\omega}_n = \hat{\pi}_{nn}^{-1/\theta}$$

- Of course, if it is not the case that $\hat{L}_n = \hat{T}_n = 1$ (i.e. there is some "domestic" component to the shock too), then one can still use this approach, since in general we have:

$$\hat{\omega}_n = \left(\hat{T}_n\right)^{1/\theta} \hat{\pi}_{nn}^{-1/\theta}$$

Some Examples of Extensions of EK (2002)

But there are many others...

- **Bertrand Competition**: Bernard, Eaton, Jensen, and Kortum (2003)
  - The same (Extreme Value Theory) tricks that EK (2002) show work for characterizing the lowest price work for finding the second-lowest, etc.
  - Bertrand competition $\Rightarrow$ variable markups at the firm-level
  - Measured productivity varies across firms $\Rightarrow$ one can use firm-level data to calibrate model

- **Multiple Sectors**: Costinot, Donaldson, and Komunjer (2012)
  - $T_i^k \equiv$ fundamental productivity in country i and sector k
  - One can use EK's machinery to study pattern of trade, not just volumes
  - More in next lecture (on empirics of Ricardian models)

- **Non-homothetic preferences:** Fieler (2011)
  - Rich and poor countries have different expenditure shares
  - Combined with differences in θ^k across sectors k, one can explain pattern of North-North, North-South, and South-South trade