Stanford Econ 266: PhD International Trade
— Lecture 14: Gravity Models (Theory) —

Today's Plan

1. The Simplest Gravity Model: Armington
2. Gravity Models and the Gains from Trade: ACR (2012)
3. Beyond ACR's (2012) Equivalence Result: CR (2013)
4. Beyond gravity: Adao, Costinot and Donaldson (2016)

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The Armington Model

The Armington Model: Equilibrium

- Labor endowments

$$L_i \text{ for } i = 1, \ldots n$$

- CES utility $\Rightarrow$ CES price index

$$P_j^{1-\sigma} = \sum_{i=1}^{n} \left(w_i \tau_{ij}\right)^{1-\sigma}$$

- Bilateral trade flows follow **gravity equation**:

$$X_{ij} = \frac{\left(w_i \tau_{ij}\right)^{1-\sigma}}{\sum_{l=1}^{n} \left(w_l \tau_{lj}\right)^{1-\sigma}} w_j L_j$$

- In what follows $\varepsilon \equiv -\frac{d \ln X_{ij}/X_{jj}}{d \ln \tau_{ij}} = \sigma - 1$ denotes the **trade elasticity**
- Trade balance

$$\sum_i X_{ji} = w_j L_j$$

The Armington Model: Welfare Analysis

- **Question:**
  Consider a foreign shock: $L_i \to L_i'$ for $i \neq j$ and $\tau_{ij} \to \tau_{ij}'$ for $i \neq j$. How do foreign shocks affect real consumption, $C_j \equiv w_j / P_j$?

- Shephard's Lemma implies

$$d \ln C_j = d \ln w_j - d \ln P_j = -\sum_{i=1}^{n} \lambda_{ij} \left(d \ln c_{ij} - d \ln c_{jj} \right)$$

  with $c_{ij} \equiv w_i \tau_{ij}$ and $\lambda_{ij} \equiv X_{ij} / w_j L_j$.

- Gravity implies

$$d \ln \lambda_{ij} - d \ln \lambda_{jj} = -\varepsilon \left(d \ln c_{ij} - d \ln c_{jj} \right).$$

The Armington Model: Welfare Analysis

- Combining these two equations yields

$$d \ln C_j = \frac{\sum_{i=1}^{n} \lambda_{ij} \left(d \ln \lambda_{ij} - d \ln \lambda_{jj} \right)}{\varepsilon}.$$

- Noting that $\sum_i \lambda_{ij} = 1 \Longrightarrow \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0$ then

$$d \ln C_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}.$$

- Integrating the previous expression yields (with $\hat{x} \equiv x'/x$)

$$\hat{C}_j = \hat{\lambda}_{jj}^{-1/\varepsilon}.$$

The Armington Model: Welfare Analysis

- In general, predicting $\hat{\lambda}_{jj}$ requires (computer) work
  - We can use "exact hat algebra" as in DEK (Lecture #3)
  - Requires data on initial levels of data $\{\lambda_{ij}, Y_j\}$, and ε
  - Where to get ε? Gravity equation suggests natural (but not the only) way would be to estimate (via OLS):

$$\ln X_{ij} = \alpha_i + \alpha_j - \varepsilon \ln t_{ij} + \nu_{ij}$$

  - With α_i as fixed-effects and t_{ij} some observable and exogenous component of trade costs (that satisfies $\tau_{ij} = t_{ij}\nu_{ij}$).
- But predicting how bad it would be to shut down <u>all</u> trade is easy...
  - In autarky, $\lambda_{jj} = 1$. So

$$C_j^A / C_j = \lambda_{jj}^{1/\varepsilon}$$

  - Thus **gains from trade** can be computed as

$$GT_j \equiv 1 - C_j^A / C_j = 1 - \lambda_{jj}^{1/\varepsilon}$$

The Armington Model: Gains from Trade

- Suppose that we have estimated the trade elasticity using the gravity equation (as described above).
  - Central estimate in the literature is $\varepsilon = 5$; see Head and Mayer (2013) Handbook chapter.
- Using World Input Output Database (2008) to get λ_{jj}, we can then estimate gains from trade:

	λ_{jj}	% GT_j
Canada	0.82	3.8
Denmark	0.74	5.8
France	0.86	3.0
Portugal	0.80	4.4
Slovakia	0.66	7.6
U.S.	0.91	1.8

Cheese, really?

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Motivation

- **New Trade Models**
  - Micro-level data have lead to **new questions** in international trade:
    - How many firms export?
    - How large are exporters?
    - How many products do they export?
  - New models highlight **new margins** of adjustment:
    - From inter-industry to intra-industry to intra-firm reallocations
- **Old question:**
  - How large are the gains from trade (GT)?
- **ACR (AER, 2012) question:**
  - How do new trade models affect the magnitude of GT?

ACR's Main Equivalence Result

- ACR focus on gravity models
  - PC: Armington and Eaton & Kortum '02
  - MC: Krugman '80 and many variations of Melitz '03
- Within that class, welfare changes are ($\hat{x} = x'/x$)

$$\hat{C} = \hat{\lambda}^{1/\varepsilon}$$

- **Two sufficient statistics** for welfare analysis are:
  - Share of domestic expenditure, λ;
  - Trade elasticity, ε
- **Two views** on ACR's result:
  - Optimistic: welfare predictions of Armington model are more robust than you might have thought
  - Pessimistic: within that class of models, micro-level data do not matter

Primitive Assumptions

Preferences and Endowments

- **CES utility**
  - Consumer price index,

$$P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega,$$

- **One factor of production:** labor
  - $L_i \equiv$ labor endowment in country i
  - $w_i \equiv$ wage in country i

Primitive Assumptions

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = \underbrace{q w_i \tau_{ij} \alpha_{ij}(\omega)\, t^{\frac{1}{1-\sigma}}}_{\text{variable cost}} + \underbrace{w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega)\, m_{ij}(t)}_{\text{fixed cost}},$$

q : quantity,
τ_{ij} : iceberg transportation cost,
$\alpha_{ij}(\omega)$: good-specific heterogeneity in variable costs,
ξ_{ij} : fixed cost parameter,
$\phi_{ij}(\omega)$: good-specific heterogeneity in fixed costs.

Primitive Assumptions

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = q w_i \tau_{ij} \alpha_{ij}(\omega) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega) m_{ij}(t)$$

$m_{ij}(t)$: cost for endogenous destination specific technology choice, t,

$$t \in [\underline{t}, \overline{t}] \ , \ m'_{ij} > 0, \ m''_{ij} \geq 0$$

- Heterogeneity across goods

$$G_j(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n) \equiv \{\omega \in \Omega \mid \alpha_{ij}(\omega) \leq \alpha_i, \ \phi_{ij}(\omega) \leq \phi_i, \ \forall i\}$$

Primitive Assumptions

Market Structure

- **Perfect competition**
  - Firms can produce any good.
  - No fixed exporting costs.
- **Monopolistic competition**
  - Either firms in i can pay $w_i F_i$ for monopoly power over a random good.
  - Or exogenous measure of firms, $\overline{N}_i < \overline{N}$, receive monopoly power.
- Let N_i be the measure of goods that can be produced in i
  - Perfect competition: $N_i = \overline{N}$
  - Monopolistic competition: $N_i < \overline{N}$

Macro-Level Restrictions

Trade is Balanced

- Bilateral trade flows are

$$X_{ij} = \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij}(\omega)\, d\omega$$

- **R1:** *For any country j,*

$$\sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji}$$

  - Trivial if perfect competition or $\beta = 0$.
  - Non trivial if $\beta > 0$.

Macro-Level Restrictions

Profit Share is Constant

- **R2:** *For any country j,*

$$\Pi_j / \left(\sum_{i=1}^{n} X_{ji} \right) \text{ is constant}$$

where Π_j : aggregate profits gross of entry costs, $w_j F_j$, (if any)

  - Trivial under perfect competition.
  - Direct from CES preferences in Krugman (1980).
  - Non-trivial in more general environments.

Macro-Level Restrictions

CES Import Demand System

- *Import demand system* defined as:

$$(\mathbf{w}, \mathbf{N}, \boldsymbol{\tau}) \rightarrow \mathbf{X}$$

- **R3:**

$$\varepsilon_j^{ii'} \equiv \partial \ln \left(X_{ij} / X_{jj} \right) / \partial \ln \tau_{i'j} = \begin{cases} \varepsilon < 0 & i = i' \neq j \\ 0 & \textit{otherwise} \end{cases}$$

- Note: symmetry and separability.

Macro-Level Restrictions

Comments on CES Import Demand System

- The *trade elasticity* ε is an *upper-level* elasticity: it combines
  - $x_{ij}(\omega)$ (*intensive margin*)
  - Ω_{ij} (*extensive margin*).
- R3 $\implies$ complete specialization.
- R1-R3 are not necessarily independent
  - E.g., if $\beta = 0$ then R3 $\implies$ R2.

Macro-Level Restrictions

Strong CES Import Demand System (AKA Gravity)

- **R3':** The IDS satisfies

$$X_{ij} = \frac{\chi_{ij} \cdot M_i \cdot (w_i \tau_{ij})^{\varepsilon} \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot M_{i'} \cdot (w_{i'} \tau_{i'j})^{\varepsilon}}$$

  where χ_{ij} is independent of $(\mathbf{w}, \mathbf{M}, \boldsymbol{\tau})$.

- Same restriction on $\varepsilon_j^{ii'}$ as R3 but, but additional structural relationships (R3' $\Rightarrow$ R3 but converse not true)

Welfare results

- State of the world economy:

$$\mathbf{Z} \equiv (\mathbf{L}, \boldsymbol{\tau}, \boldsymbol{\xi})$$

- *Foreign shocks*: a change from $\mathbf{Z}$ to $\mathbf{Z}'$ with no domestic change.
  - Effects of a domestic change in L_i would be different between PC and MC models (due to the home-market effect which is at work in the MC models but not in the PC models).

Equivalence (I)

- **Proposition 1:** *Suppose that R1-R3 hold. Then for any foreign shock it must be true that*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}.$$

- Implication: $\widehat{\lambda}_{jj}$ acts as (along with elasticity ε) sufficient statistics for fully global, GE welfare analysis
- Note that it is still true that for any of these models we could estimate ε from OLS gravity regression:

$$\ln X_{ij} = \alpha_i + \alpha_j - \varepsilon \ln t_{ij} + \nu_{ij}$$

- New margins affect structural interpretation of ε
  - ...and composition of gains from trade (GT)...
  - ... but size of GT is the same.

Gains from Trade Revisited

- Proposition 1 is an *ex-post* result (based on seeing $\widehat{\lambda}_{jj}$).
- A simple *ex-ante* result (for the case of going to autarky):
- **Corollary 1:** *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}.$$

Equivalence (II)

- A stronger ex-ante result for **variable trade costs** (and things that act like variable trade costs, like productivity shocks; but not for other types of foreign shocks) under R1-**R3'**:
- **Proposition 2:** *Suppose that R1-R3' hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$$

where

$$\widehat{\lambda}_{jj} = \left[\sum_{i=1}^{n} \lambda_{ij} \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}\right]^{-1},$$

and

$$\widehat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij} \hat{w}_j Y_j \left(\hat{w}_i \hat{\tau}_{ij}\right)^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j}\right)^{\varepsilon}}.$$

- So: ε and $\{\lambda_{ij}\}$ are sufficient to predict $\widehat{W}_j$ (ex-ante) from $\hat{\tau}_{ij}$, $i \neq j$.

Taking Stock

- ACR consider models featuring:
  - (i) CES preferences;
  - (ii) one factor of production;
  - (iii) linear cost functions; and
  - (iv) perfect or monopolistic competition;

  with three macro-level restrictions:
  - (i) trade is balanced;
  - (ii) aggregate profits are a constant share of aggregate revenues; and
  - (iii) a CES import demand system.
- Equivalence for ex-post welfare changes and GT
  - under R3' equivalence carries to ex-ante welfare changes
  - So if R3' is satisfied, then models in this ACR class agree on all implications of changes in variable trade costs

A note on methodology

- ACR set out to answer the question of whether different trade models predict different GT
- In some of these cases, tempting to think that it is easy to rank GT across these models.
  - E.g., following Melitz and Redding (AER, 2015), note that since Krugman is a strictly nested special case of Melitz for which the firm productivity distribution is degenerate, the Melitz model is Krugman plus an additional margin of adjustment.
  - And since both models are efficient (i.e. correspond to planner's problem, so equilibrium is maximizing something), an additional margin of adjustment guarantees that the damage done by a negative shock (e.g. move to autarky) is less bad in Melitz than in Krugman.
- But note that this is not the ACR thought experiment.
  - ACR's thought experiment is to ask about GT (or any other counterfactual), conditional on the data $\{\lambda_{ij}, Y_j\}$ and elasticity ε we have.

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Departing from ACR's (2012) Equivalence Result

- Costinot and Rodriguez-Clare (Handbook of International Econ, 2013) has nice discussion of general cases.
- **Other Gravity Models:**
  - Multiple Sectors
  - Tradable Intermediate Goods
  - Multiple Factors
  - Variable Markups (ACDR 2012)

Multiple sectors, GT

- Nested CES: Upper level EoS ρ and lower level EoS ε_s
- Recall gains for Canada of 3.8%. Now gains can be much higher: $\rho = 1$ implies $GT = 17.4\%$

Tradable intermediates, GT

- Set $\rho = 1$, add tradable intermediates with Input-Output structure
- Labor shares are $1 - \alpha_{j,s}$ and input shares are $\alpha_{j,ks}$ $(\sum_k \alpha_{j,ks} = \alpha_{j,s})$

Tradable intermediates, GT

	$\%\ GT_j$	$\%\ GT_j^{MS}$	$\%\ GT_j^{IO}$
Canada	3.8	17.4	30.2
Denmark	5.8	30.2	41.4
France	3.0	9.4	17.2
Portugal	4.4	23.8	35.9
U.S.	1.8	4.4	8.3

Multiple sectors: a combination of micro and macro features

- In Krugman, free entry $\Rightarrow$ scale effects associated with total employment
- In Melitz, additional scale effects associated with sales in each market
- In both models, trade may affect entry and fixed costs
- All these effects do not play a role in the one sector model
- With multiple sectors and traded intermediates, these effects come back

Gains from Trade

"MS" = multiple sectors; "IO" = tradable intermediates

.....	Canada	China	Germany	Romania	US
Aggregate	**3.8**	**0.8**	**4.5**	**4.5**	**1.8**

Gains from Trade

"MS" = multiple sectors; "IO" = tradable intermediates

.....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	**17.4**	**4.0**	**12.7**	**17.7**	**4.4**

Gains from Trade

"MS" = multiple sectors; "IO" = tradable intermediates

.....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	**15.3**	**4.0**	**17.6**	**12.7**	**3.8**

Gains from Trade

"MS" = multiple sectors; "IO" = tradable intermediates

.....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	15.3	4.0	17.6	12.7	3.8
MS, IO, PC	**29.5**	**11.2**	**22.5**	**29.2**	**8.0**

Gains from Trade

"MS" = multiple sectors; "IO" = tradable intermediates

.....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	15.3	4.0	17.6	12.7	3.8
MS, IO, PC	29.5	11.2	22.5	29.2	8.0
MS, IO, MC (Krugman)	**33.0**	**28.0**	**41.4**	**20.8**	**8.6**

Gains from Trade

"MS" = multiple sectors; "IO" = tradable intermediates

.....	Canada	China	Germany	Romania	US
Aggregate	3.8	0.8	4.5	4.5	1.8
MS, PC	17.4	4.0	12.7	17.7	4.4
MS, MC	15.3	4.0	17.6	12.7	3.8
MS, IO, PC	29.5	11.2	22.5	29.2	8.0
MS, IO, MC (Krugman)	33.0	28.0	41.4	20.8	8.6
MS, IO, MC (Melitz)	**39.8**	**77.9**	**52.9**	**20.7**	**10.3**

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Adao, Costinot and Donaldson (AER, 2016)

- Without access to (much) quasi-experimental variation, traditional approach in the field has been to model everything: demand-side, supply-side, market structure, trade costs
  - E.g. #1: "Old CGE": GTAP model [13,000 structural parameters]
  - E.g. #2: "New CGE": EK model [1 key parameter]

- **Question:** Can we relax EK's strong functional form assumptions without circling back to GTAP's 13,000 parameters?

ACD (2016): 4 Steps

1. For many counterfactual questions, neoclassical models are exactly equivalent to a *reduced factor exchange economy*
   - *Reduced factor demand system* sufficient for counterfactual analysis
2. Nonparametric generalization of standard gravity tools:
   - Dekle, Eaton and Kortum (2008): exact hat algebra
   - Arkolakis, Costinot, and Rodriguez-Clare (2012): welfare gains
   - Head and Ries (2001): trade costs
3. Reduced factor demand system is nonparametrically identified under standard assumptions about data/orthogonality
4. (Not today) Empirical application: What was the impact of China's integration into the world economy in the past two decades?
   - Departures from CES modeled in the spirit of BLP (1995)

Neoclassical Trade Model

- $i = 1, ..., I$ countries
- $k = 1, ..., K$ goods
- $n = 1, ..., N$ factors
- Goods consumed in country i:

$$\boldsymbol{q_i} \equiv \{q_{ji}^k\}$$

- Factors used in country i to produce good k for country j:

$$\boldsymbol{l_{ij}^k} \equiv \{l_{ij}^{nk}\}$$

Neoclassical Trade Model

- Preferences: $u_i = u_i(\boldsymbol{q_i})$
  - Representative consumer (driven by data from "country" i)
- Technology: $q_{ij}^k = f_{ij}^k(\boldsymbol{l_{ij}^k})$
  - Non-increasing returns to scale. No joint production.
  - Extensions in paper to include (global/domestic) input-output linkages and tariffs/taxes/subsidies.
- Factor endowments: $\nu_i^n > 0$
  - Defined as the (set of imperfectly substitutable) inputs to production that are in fixed supply.

Competitive Equilibrium

A $\boldsymbol{q} \equiv \{\boldsymbol{q_i}\}$, $\boldsymbol{l} \equiv \{\boldsymbol{l_i}\}$, $\boldsymbol{p} \equiv \{\boldsymbol{p_i}\}$, and $\boldsymbol{w} \equiv \{\boldsymbol{w_i}\}$ such that:

1. Consumers maximize their utility:

$$\boldsymbol{q_i} \in \text{argmax}_{\tilde{\boldsymbol{q}}_i} u_i(\tilde{\boldsymbol{q}}_i)$$
$$\sum_{j,k} p_{ji}^k \tilde{q}_{ji}^k \leq \sum_n w_i^n \nu_i^n \text{ for all } i;$$

2. Firms maximize their profits:

$$\boldsymbol{l_{ij}^k} \in \text{argmax}_{\tilde{\boldsymbol{l}}_{ij}^k} \{p_{ij}^k f_{ij}^k(\tilde{\boldsymbol{l}}_{ij}^k) - \sum_n w_i^n \tilde{l}_{ij}^{nk}\} \text{ for all } i, j, \text{ and } k;$$

3. Goods markets clear:

$$q_{ij}^k = f_{ij}^k(\boldsymbol{l_{ij}^k}) \text{ for all } i, j, \text{ and } k;$$

4. Factors markets clear:

$$\sum_{j,k} l_{ij}^{nk} = \nu_i^n \text{ for all } i \text{ and } n.$$

Reduced Exchange Model

- Fictitious endowment economy in which consumers directly exchange factor services
  - Taylor (1938), Rader (1972), Wilson (1980), Mas-Colell (1991)

- *Reduced preferences* over primary factors of production:

$$U_i(\boldsymbol{L_i}) \equiv \max_{\tilde{\boldsymbol{q}}_i, \tilde{\boldsymbol{l}}_i} u_i(\tilde{\boldsymbol{q}}_i)$$
$$\tilde{q}_{ji}^k \leq f_{ji}^k(\tilde{\boldsymbol{l}}_{ji}^k) \text{ for all } j \text{ and } k,$$
$$\sum_k \tilde{l}_{ji}^{nk} \leq L_{ji}^n \text{ for all } j \text{ and } n,$$

- Easy to check that $U_i(\cdot)$ is strictly increasing and quasiconcave.
  - Not necessarily strictly quasiconcave, even if $u_i(\cdot)$ is.
  - Example: H-O model inside FPE set.

Reduced Equilibrium

Corresponds to $\boldsymbol{L} \equiv \{\boldsymbol{L_i}\}$ and $\boldsymbol{w} \equiv \{\boldsymbol{w_i}\}$ such that:

1. Consumers maximize their reduced utility:

$$\boldsymbol{L_i} \in \text{argmax}_{\tilde{\boldsymbol{L}}_i} U_i(\tilde{\boldsymbol{L}}_i)$$
$$\sum_{j,n} w_j^n \tilde{L}_{ji}^n \leq \sum_n w_i^n \nu_i^n \text{ for all } i;$$

2. Factor markets clear:

$$\sum_j L_{ij}^n = \nu_i^n \text{ for all } i \text{ and } n.$$

Equivalence

- **Proposition 1**: *For any competitive equilibrium,* $(\boldsymbol{q}, \boldsymbol{l}, \boldsymbol{p}, \boldsymbol{w})$*, there exists a reduced equilibrium,* $(\boldsymbol{L}, \boldsymbol{w})$*, with:*
  1. *the same factor prices,* $\boldsymbol{w}$*;*
  2. *the same factor content of trade,* $L_{ji}^{n} = \sum_k l_{ji}^{nk}$ *for all* i*,* j*, and* n*;*
  3. *the same welfare levels,* $U_i(\boldsymbol{L_i}) = u_i(\boldsymbol{q_i})$ *for all* i*.*

  Conversely, for any reduced equilibrium, $(\boldsymbol{L}, \boldsymbol{w})$*, there exists a competitive equilibrium,* $(\boldsymbol{q}, \boldsymbol{l}, \boldsymbol{p}, \boldsymbol{w})$*, such that 1-3 hold.*

Equivalence

- **Comments:**
  - Proof is similar to First and Second Welfare Theorems. Key distinction is that standard Welfare Theorems go from CE to *global* planner's problem, whereas RE remains a decentralized equilibrium (but one in which countries fictitiously trade factor services and budget is balanced country by country).
  - Key implication of Prop. 1: If one is interested in the factor content of trade, factor prices and/or welfare, then one can always study a RE instead of a CE. One doesn't need *direct* knowledge of primitives u and f but only of how these *indirectly* shape U.

Reduced Counterfactuals

- Suppose that the reduced utility function over primary factors in this economy can be parametrized as

$$U_i(\boldsymbol{L_i}) \equiv \bar{U}_i(\{L_{ji}^n / \tau_{ji}^n\}),$$

where $\tau_{ji}^n > 0$ are exogenous preference shocks

- **Counterfactual question:** *What are the effects of a change from* $(\boldsymbol{\tau}, \boldsymbol{\nu})$ *to* $(\boldsymbol{\tau'}, \boldsymbol{\nu'})$ *on trade flows, factor prices, and welfare?*

Reduced Factor Demand System

- Start from factor demand = solution of reduced UMP:

$$\boldsymbol{L_i}(\boldsymbol{w}, y_i|\boldsymbol{\tau_i})$$

- Compute associated expenditure shares:

$$\chi_{\boldsymbol{i}}(\boldsymbol{w}, y_i|\boldsymbol{\tau_i}) \equiv \{\{x_{ji}^n\}|x_{ji}^n = w_j^n L_{ji}^n / y_i \text{ for some } \boldsymbol{L_i} \in \boldsymbol{L_i}(\boldsymbol{w}, y_i|\boldsymbol{\tau_i})\}$$

- Rearrange in terms of *effective factor prices*, $\boldsymbol{\omega_i} \equiv \{w_j^n \tau_{ji}^n\}$:

$$\chi_{\boldsymbol{i}}(\boldsymbol{w}, y_i|\boldsymbol{\tau_i}) \equiv \chi_{\boldsymbol{i}}(\boldsymbol{\omega_i}, y_i)$$

Reduced Equilibrium

- In this notation, RE is:

$$\boldsymbol{x_i} \in \boldsymbol{\chi_i}(\boldsymbol{\omega_i}, y_i), \text{ for all } i,$$
$$\sum_j x_{ij}^n y_j = y_i^n, \text{ for all } i \text{ and } n$$

- **Gravity model (i.e. ACR):** Reduced factor demand system is CES

$$\chi_{ji}(\boldsymbol{\omega_i}, y_i) = \frac{(\omega_{ji})^\epsilon}{\sum_l (\omega_{li})^\epsilon}, \text{ for all } j \text{ and } i$$

Exact Hat Algebra

- Start from the counterfactual equilibrium:

$$\boldsymbol{x_i'} \in \boldsymbol{\chi_i}(\boldsymbol{\omega_i'}, y_i') \text{ for all } i,$$
$$\sum_j (x_{ij}^n)' y_j' = (y_i^n)', \text{ for all } i \text{ and } n.$$

- Rearrange in terms of proportional changes:

$$\{\hat{x}_{ji}^n x_{ji}^n\} \in \boldsymbol{\chi_i}(\{\hat{w}_j^n \hat{\tau}_{ji}^n \omega_{ji}^n\}, \sum_n \hat{w}_i^n \hat{v}_i^n y_i^n) \text{ for all } i,$$
$$\sum_j \hat{x}_{ij}^n x_{ij}^n (\sum_n \hat{w}_j^n \hat{v}_j^n y_j^n) = \hat{w}_i^n \hat{v}_i^n y_i^n, \text{ for all } i \text{ and } n.$$

Counterfactual Trade Flows and Factor Prices

- **A1 [Invertibility].** *In any country i, for any $\boldsymbol{x} > 0$ and $y > 0$, there exists a unique vector of factor prices (up to normalization) such that $\boldsymbol{x} \in \chi_{\boldsymbol{i}}(\boldsymbol{\omega}, y_i)$, which we denote $\chi_{\boldsymbol{i}}{}^{-1}(\boldsymbol{x}, y)$.*

- Sufficient conditions:
  - A1 holds if $\chi_{\boldsymbol{i}}$ satisfies connected substitutes property (Arrow and Hahn 1971, Howitt 1980, and Berry, Gandhi and Haile 2013)
  - $\chi_{\boldsymbol{i}}$ satisfies connected substitutes property in a Ricardian economy if preferences satisfy connected substitutes property

Counterfactual Trade Flows and Factor Prices

- **Proposition 2** *Under A1, proportional changes in expenditure shares and factor prices,* $\hat{\boldsymbol{x}}$ *and* $\hat{\boldsymbol{w}}$*, caused by proportional changes in preferences and endowments,* $\hat{\boldsymbol{\tau}}$ *and* $\hat{\boldsymbol{v}}$*, solve*

$$\{\hat{x}_{ji}^n x_{ji}^n\} \in \boldsymbol{\chi_i}(\{\hat{w}_j^n \hat{\tau}_{ji}^n \omega_{ji}^n\}, \sum_n \hat{w}_i^n \hat{v}_i^n y_i^n) \ \forall \ i,$$

$$\sum_j \hat{x}_{ij}^n x_{ij}^n (\sum_n \hat{w}_j^n \hat{v}_j^n y_j^n) = \hat{w}_i^n \hat{v}_i^n y_i^n \ \forall \ i \text{ and } n.$$

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Welfare

- Equivalent variation for country i associated with change from $(\boldsymbol{\tau}, \boldsymbol{\nu})$ to $(\boldsymbol{\tau'}, \boldsymbol{\nu'})$, expressed as fraction of initial income:

$$\Delta W_i = (e_i(\boldsymbol{\omega_i}, U_i')) - y_i)/y_i,$$

with U_i' = counterfactual utility and e_i = expenditure function,

$$e_i(\boldsymbol{\omega_i}, U_i') \equiv \min_{\tilde{\boldsymbol{L}}_i} \sum \omega_{ji}^n L_{ji}^n$$
$$\bar{U}_i(\tilde{\boldsymbol{L}}_i) \geq U_i'.$$

Integrating Below Factor Demand Curves

- To go from $\boldsymbol{\chi_i}$ to ΔW_i, solve system of ODEs
- For any selection $\{x_{ji}^n(\boldsymbol{\omega}, y)\} \in \boldsymbol{\chi_i}(\boldsymbol{\omega}, y)$, Envelope Theorem:

$$\frac{d \ln e_i(\boldsymbol{\omega}, U_i')}{d \ln \omega_j^n} = x_{ji}^n(\boldsymbol{\omega}, e_i(\boldsymbol{\omega}, U_i')) \text{ for all } j \text{ and } n.$$

- Budget balance in the counterfactual equilibrium

$$e_i(\boldsymbol{\omega_i'}, U_i') = y_i'.$$

Counterfactual Welfare Changes

- **Proposition 3** *Under A1, equivalent variation associated with change from* $(\boldsymbol{\tau}, \boldsymbol{\nu})$ *to* $(\boldsymbol{\tau'}, \boldsymbol{\nu'})$ *is*

$$\Delta W_i = (e(\{(\chi_{ji}^n)^{-1}(\boldsymbol{x_i}, y_i)\}, U_i') - y_i)/y_i,$$

where $e(\cdot, U_i')$ *is the unique solution of (1) and (2).*

Application to Neoclassical Trade Models

- Suppose that technology in neoclassical trade model satisfies:

$$f_{ij}^k(\boldsymbol{l_{ij}^k}) \equiv \bar{f}_{ij}^k(\{l_{ij}^{nk}/\tau_{ij}^n\}), \text{ for all } i, j, \text{ and } k,$$

- Reduced utility function over primary factors of production:

$$\begin{aligned} U_i(\boldsymbol{L_i}) \equiv \max_{\boldsymbol{\tilde{q}_i}, \boldsymbol{\tilde{l}_i}} & u_i(\boldsymbol{\tilde{q}_i}) \\ & \tilde{q}_{ji}^k \leq \bar{f}_{ji}^k(\{\tilde{l}_{ji}^{nk}/\tau_{ji}^n\}) \text{ for all } j \text{ and } k, \\ & \sum_k \tilde{l}_{ji}^{nk} \leq L_{ji}^n \text{ for all } j \text{ and } n. \end{aligned}$$

- Change of variable: $U_i(L_i) \equiv \bar{U}_i(\{L_{ji}^n/\tau_{ji}^n\}) \Rightarrow$ factor-augmenting productivity shocks in CE = preference shocks in RE
  - NB: $\hat{\boldsymbol{\tau}}$ cannot depend on k.
  - However, $\boldsymbol{\tau}$ can do so freely.
  - And can always allow for $\hat{\tau}_{ji}^{nk}$ by defining a new factor that is specific to sector k (with arbitrage condition across sectors).

Taking Stock

Given data on expenditure shares and factor payments, $\{x_{ji}^n, y_i^n\}$, if one knows factor demand system, χ_i, then:

1. Can compute exact, fully GE change due to counterfactual move from (τ, ν) to $(\tau', \nu'))$ in:
   - Factor prices
   - Factor content of trade (and hence total volume of trade)
   - Welfare
2. Proposition 1 $\Rightarrow$ will get same answer as any neoclassical economy with the same χ_i.
3. Standard gravity "toolkit"—developed for CES factor demands—extends nonparametrically to any invertible factor demand system

Identification Assumptions: Shocks

- Data generated by neoclassical trade model at different dates t
- At each date, preferences and technology such that:

$$u_i(\boldsymbol{q_{i,t}}) = \bar{u}(\{q^k_{ji,t}/\theta_{ji}\}), \text{ for all } i,$$
$$f^k_{ij,t}(\boldsymbol{l^k_{ij,t}}) = \bar{f}^k_i(\{l^{nk}_{ij,t}/\tau^n_{ij,t}\}), \text{ for all } i, j, \text{ and } k.$$

- **A2. [Price heterogeneity]** *In any country i and at any date t, there exists $\boldsymbol{\omega_{i,t}} \equiv \{w^n_{j,t}\theta_{ji}\tau^n_{ji,t}\}$, such that factor demand can be expressed as $\bar{\chi}(\boldsymbol{\omega_{i,t}}, y_{i,t})$.*

Identification Assumptions: Exogeneity

- Observables:
  1. $x^n_{ji,t}$: factor expenditure shares (normal FCT data in principle; but non-trivial aggregation bias issues in practice)
  2. $y^n_{i,t}$: factor payments
  3. $z^n_{ji,t}$: factor price shifters (e.g. observable shifter of trade costs)

- Effective factor prices, $\omega_{ji,t}$, unobservable, but related to $z^n_{ji,t}$:

$$\ln\omega^n_{ji,t} = \ln z^n_{ji,t} + \varphi^n_{ji} + \xi^n_{j,t} + \epsilon^n_{ji,t}, \text{ for all } i,\ j,\ n, \text{ and } t$$

- **A3. [Exogeneity]** $E[\epsilon_{\boldsymbol{i,t}} | \ln \boldsymbol{z_{i,t}}, \boldsymbol{d_{i,t}}] = 0$, *where* $\boldsymbol{d_{i,t}}$ *is a vector of importer-exporter-factor dummies and exporter-factor-year dummies with* i *= importer and* t *= year for all dummies.*

Identification Assumptions: Completeness

- Following Newey and Powell (2003), we conclude by imposing the following completeness condition.

- **A4. [Completeness]** *For any* $g(\boldsymbol{x_{i,t}}, y_{i,t})$, $E[g(\boldsymbol{x_{i,t}}, y_{i,t}) | \ln \boldsymbol{z_{i,t}}, \boldsymbol{d_{i,t}}] = 0$ *implies* $g(\boldsymbol{x_{i,t}}, y_{i,t}) = 0$.

- A4 = rank condition in estimation of parametric models.

Identifying Factor Demand

- A1 $\Rightarrow$

$$\omega^n_{ji,t} = (\chi^n_{ji})^{-1}(\boldsymbol{x_{i,t}}, y_{i,t}).$$

- Taking logs and using definition of $\epsilon^n_{ji,t}$:

$$\epsilon^n_{ji,t} = \ln(\chi^n_{ji})^{-1}(\boldsymbol{x_{i,t}}, y_{i,t}) - \ln z^n_{ji,t} - \varphi^n_{ji} - \xi^n_{j,t}.$$

- A2 $\Rightarrow$

$$\epsilon^n_{ji,t} = \ln(\bar{\chi}^n_{j})^{-1}(\boldsymbol{x_{i,t}}, y_{i,t}) - \ln z^n_{ji,t} - \varphi^n_{ji} - \xi^n_{j,t}.$$

- Using A3, we obtain the following moment condition

$$E[\ln(\bar{\chi}^n_{j})^{-1}(\boldsymbol{x_{i,t}}, y_{i,t}) - \ln z^n_{ji,t} - \varphi^n_{ji} - \xi^n_{j,t} | \ln \boldsymbol{z_{i,t}}, \boldsymbol{d_{i,t}}] = 0.$$

- A4 $\Rightarrow$ unique solution $(\bar{\chi}^n_{j})^{-1}$ to (3) (up to a normalization)

Identifying Factor Demand

- Once the inverse factor demand is known, both factor demand and effective factor prices are known as well, with prices being uniquely pinned down (up to a normalization).

- **Proposition 4** *Suppose that A1-A4 hold. Then effective factor prices* $\{\omega_{i,t}\}$ *and the factor demand system* $\bar{\chi}$ *are identified (up to a normalization).*